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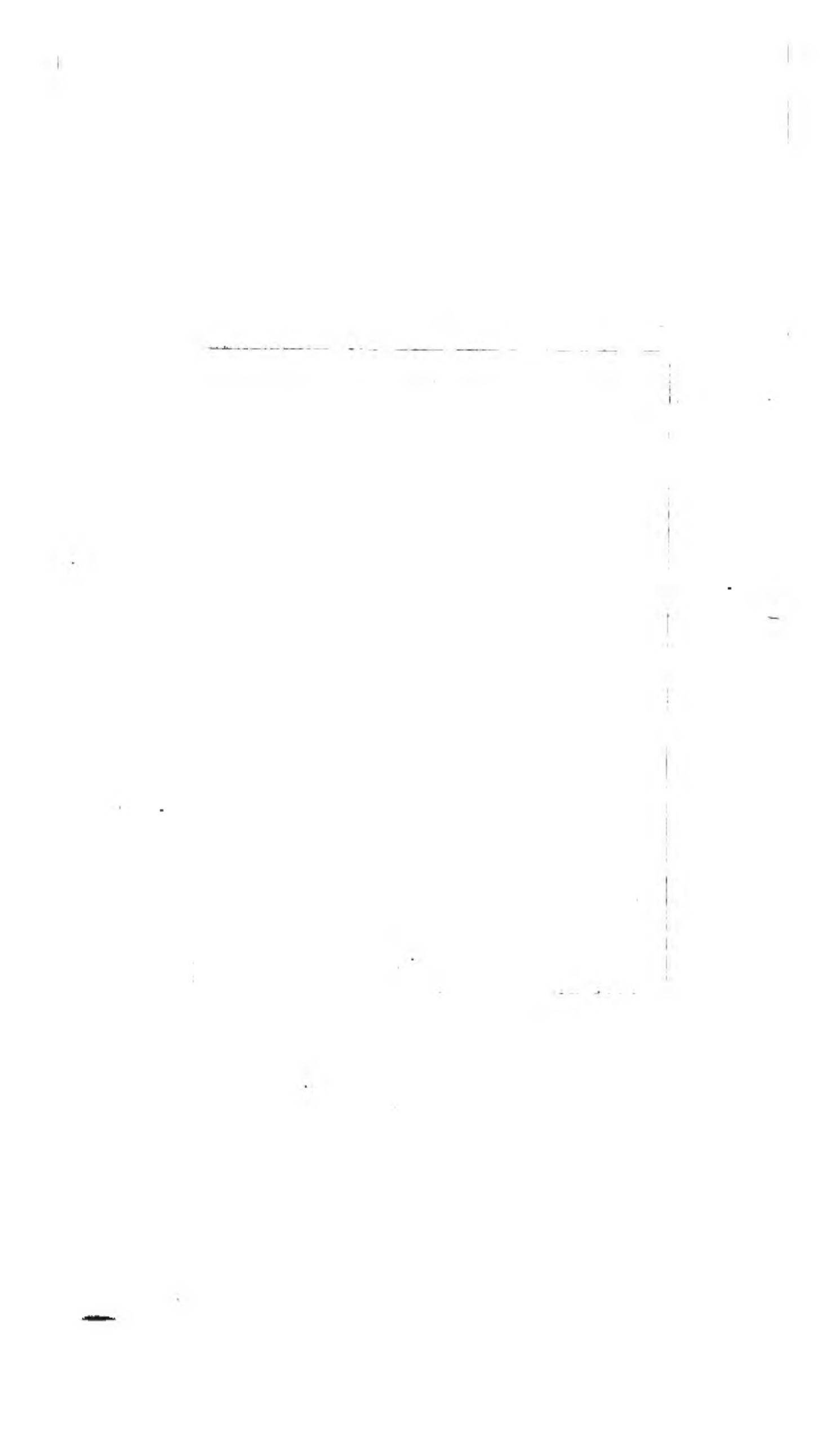
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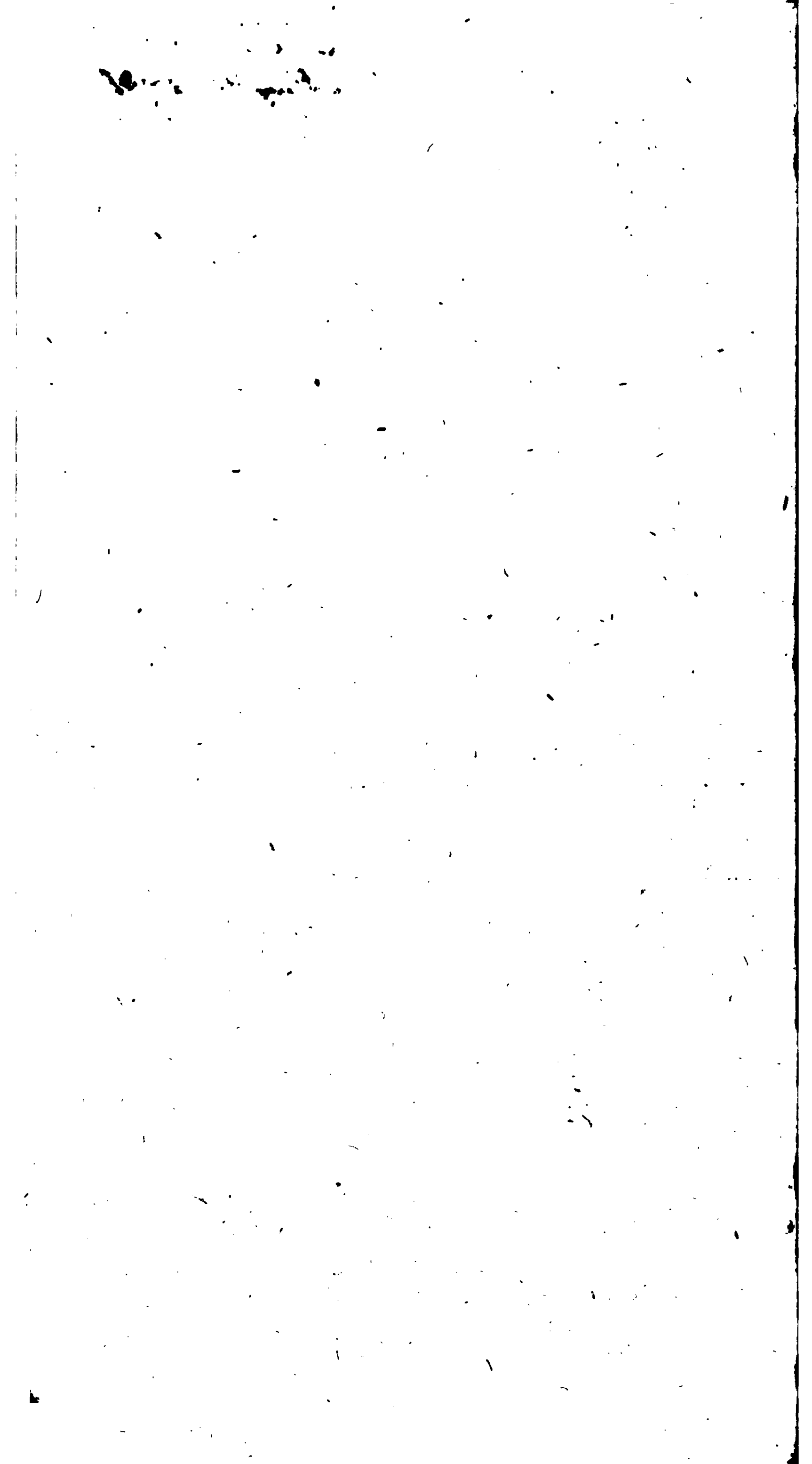








*S. J. Orizans*  
*May 31. 1891*



A  
SYNOPSIS  
OF  
PRACTICAL MATHEMATICS.

CONTAINING

PLAIN TRIGONOMETRY, MENSURATION OF HEIGHTS,  
DISTANCES, SURFACES, AND SOLIDS; SURVEYING  
OF LAND, GAUGING, NAVIGATION, AND GUNNERY.

WITH

TABLES OF THE LOGARITHMS OF NUMBERS,  
AND OF SINES AND TANGENTS.

FOR THE USE OF SCHOOLS AND MEN OF BUSINESS.

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BY ALEXANDER EWING,  
TEACHER OF MATHEMATICS, EDINBURGH.

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FOURTH EDITION.

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Gift

Professor William H. Butler

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## P R E F A C E.

**M**ATHEMATICS are studied either by gentlemen of birth and fortune, as a necessary part of genteel education, or by those in the middle rank of life, in order to qualify them for the employments or professions which they intend to follow.

The views of these classes are as different as their stations. The gentleman, by studying a system of theory, and such of its applications as are necessary in his other studies, may have all his ends answered. But the soldier, sailor, engineer, surveyor, and man of business, cannot follow his profession rationally, without being expert in most parts of practical mathematics.

That this class may obtain their end in the most effectual manner, it is necessary to throw the several parts to be learned into the shortest form. It is a great encouragement to proceed, when the end of the task is in view; nor is any thing more discouraging to a beginner, than to be told, that the science he is about to learn fills many volumes. The apparent length of the labour sets proficiency at so great a distance, that half a lifetime seems too little to acquire it; and, if the natural desire of knowledge be nipt in the bud by such an idea, it will be difficult afterwards to make any one apply diligently to the study.

But it is not enough to learn the several parts of practical mathematics at school; they must be remembered, or kept in view by some means or other, that the student

dent may be always prepared to perform his part with propriety.

The following Synopsis is composed with the design of answering both these ends. As a school-book, it will lessen the labour of teaching, and shorten the time of learning. As a memorandum, the young practitioner will find here many useful rules, delivered in such clear and easy terms, that he cannot mistake their meaning; and even the most learned will find some assistance to his memory.

I have revised and corrected the tables of logarithms, and of logarithmic sines and tangents with attention, and hope they are more correct than any small tables formerly published; and, although printed in the most concise form, will answer every purpose as well as Sherwin's tables.

In section 3, I have endeavoured to show, that most problems in the mensuration of heights and distances, and surveying of land, may be performed accurately enough by a right application of lineal measures, such as the surveying chain and poles; and therefore instruments for measuring angles, are not so very necessary as has been commonly supposed.

The favourable reception which the public has given to the former editions, has encouraged the Author to make this Fourth Edition still more complete than either of them, by several useful additions interspersed in the different sections; and it is hoped, that the book has now a very good claim to its title, and will be useful to all students, who desire to become expert in Practical Mathematics.



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*CHARACTER of this Book in the MONTHLY  
REVIEW.*

“ This Work is not offered to the Public as a complete treatise. Its object is to afford easy access to the study of Astronomy; and to enable such Students to solve its Problems, who are acquainted merely with Arithmetic, the Circles of the Sphere and Logarithms: — for such a purpose, the publication is well designed and properly executed.”

It may be added, that the Book was intended to assist those who study by themselves, and the Tables will be equally useful to Students in general.

## S E C T. I.

### GEOMETRICAL PROBLEMS.

#### DEFINITIONS.

1. **A** PLAIN angle is the inclination of two right lines to each other, which meeting do not lie in one straight line.

2. An angle is expressed by three letters, of which the letter at the angular point must be in the middle. Thus, the angle at the point B is called the angle ABC, or CBA. *Fig. 1.*

3. Every plain angle is measured by the arch of a circle described about the angular point as a center: Thus, the measure of the angle ABC is the arch *ac*.

4. Angles are equal to each other, when the arches which measure them are equal, the radius being the same in both, that is, when both the arches are described with the same extent of the compasses.

**PROBLEM 1.** *With a given right line AB, and at a given point A in it, to make an angle equal to a given plain angle DEF. Fig. 2.*

About the center E, with any radius *En*, describe the arc *mn*; and, about the center A, with the same radius *Af*, describe the arc *fg*, in which, take *fb* equal to *mn*, and through A and *b* draw the straight line AC: The angle BAC is equal to the given angle DEF.

A

DEFIN.

**DEFIN. 5.** Parallel right lines are every where equidistant from each other, or which, being infinitely produced both ways, would never meet.

**PROB. 2.** *Through a given point A, to draw a right line parallel to a given right line BC. Fig. 3.*

Set one foot of the compasses in the point A, and with the other find the nearest distance of BC, by describing the arc  $n$ ; then take any point  $e$  in the line BC, and about the center  $e$ , with the same radius, describe the arc  $m$ ; through A and  $m$  draw the right line DAE, which will be parallel to BC.

**DEFIN. 6.** When one right line, standing on another, makes the angles on each side of it equal between themselves, these angles are right ones; and the first line is perpendicular to the other.

*Note*—In some of the following problems, the point where two arches cut each other, is called a section.

**PROB. 3.** *To bisect a given right line AB by another CD cutting it at right angles. Fig. 4.*

With any radius greater than half of AB, from the points A and B, make the sections C and D; and draw the line CD, which will bisect AB at right angles in E.

**COR. 1.** The four angles about the point E are all right angles.

2. By this problem a given circle is quartered, and from the middle of any right line another is drawn perpendicular to it.

**PROB. 4.** *From a point A in a given right line BC, to draw another line perpendicular to it. Fig. 5.*

Take AE equal to AB; and from the points B and E, with

with any radius greater than  $AB$ , make the section  $D$ ; draw  $DA$ , which will be perpendicular to  $BC$ .

**PROB. 5.** *To make a right angle; or, from the end  $A$  of a given right line  $AB$ , to raise a perpendicular. Fig. 6.*

Take the point  $C$  nearer to  $A$  than to  $B$ ; about the center  $C$  with the radius  $CA$ , describe the circle  $EAD$ ; through the points  $E$  and  $C$  draw the line  $ED$ ; and join  $DA$ , which will be perpendicular to  $AB$ ; and  $BAD$  is a right angle.

Otherwise, Fig. 7.

About the center  $A$ , with any radius  $AE$ , describe the arc  $Enm$ ; set the same radius from  $E$  to  $n$  and from  $n$  to  $m$ ; from the points  $n$  and  $m$  make the section  $D$ ; and join  $AD$ ; which will be the perpendicular required to be drawn; and  $BAD$  is a right angle.

**PROB. 6.** *From a given point  $A$ , to let fall a perpendicular upon a given infinite straight line  $BC$ . Fig. 8.*

About the given point  $A$  describe a circle cutting  $BC$  in  $E$  and  $F$ ; from the points  $E$  and  $F$  make the section  $n$ ; and draw the line  $AD$  from  $A$  towards  $n$ ; and  $AD$  is the perpendicular required.

**DEFIN. 7.** A plain triangle is a figure contained by three straight lines.

**PROB. 7.** *To make a triangle whose sides shall be equal to three given right lines  $AB$ ,  $BC$ , and  $CA$ ; any two of these being greater than the third. Fig. 9.*

Draw the straight line  $AB$  of the given length; and from  $B$ , with a radius equal to  $BC$ , describe an arc; and from  $A$ , with a radius equal to  $AC$ , describe another

$A$  2 .

arc :

arc cutting the former in  $C$ ; join  $AC$ ,  $CB$ ; and  $ABC$  is the triangle required.

*Note 1.* If the given lines are all equal to one another, the triangle shall be equilateral.

2. If two of the given lines be equal, the triangle shall be isosceles.

3. If the given lines are unequal, the triangle shall be scalene. The same operation serves for making every kind.

**DEFIN. 8.** A square is a quadrilateral figure whose sides are equal, and its angles all right ones.

9. A rectangle is a quadrilateral figure, whose opposite sides are equal, and its angles all right ones. It is said to be contained under two lines, namely, the length and breadth.

**PROB. 8.** *To make a square on any given right line AB. Fig. 10.*

Make the right angle  $BAD$ ; take  $AD$  equal to  $AB$ ; and from the points  $D$  and  $B$ , with a radius equal to  $AB$ , make the section  $C$ ; join  $DC$  and  $BC$ , and  $ABCD$  is a square.

**PROB. 9.** *To make a rectangle of two given right lines AB and BC. Fig. 11.*

Make the right angle  $ABC$ ; take  $AB$  and  $BC$  of the given lengths; from  $A$ , with a radius equal to  $BC$ , describe an arc; and from  $C$ , with a radius equal to  $AB$ , describe another arc cutting the former in  $D$ ; join  $AD$  and  $DC$ ; and  $ABCD$  is the rectangle required.

**PROB.**

**PROB. 10.** *To divide a given right lined angle ABC into two equal parts, or to bisect it. . . . Fig. 12.*

About the angular point B, with any radius Bd, describe the arc de; and from the points d and e make the section F; join BF, and the angle ABC is bisected by the right line BF.

*N. B.* If the angles ABF and CBF are bisected, the given angle will be divided into four equal parts; and each of these being again bisected, it will be divided into eight equal parts; and thus the bisection may be carried as far as you please: But an arch or angle cannot be divided into three, five, or seven equal parts, by Euclid's propositions. This is to be done by trials.

## OF PROPORTION.

**DEFIN. 10.** When we compare any quantity with another of the same kind, in order to estimate the magnitude of the one in respect of the other; the comparison is made, either by considering what part or parts the one is of the other, or how oft the one contains the other.

11. The quantity compared is called the antecedent, and the quantity to which it is compared is called the consequent.

12. The ratio of any two quantities is the number expressing what part or parts the antecedent is of the consequent, or how often the antecedent contains the consequent. Thus, if any quantity A is compared to another B, the ratio is  $\frac{A}{B}$ . The ratio of 6 to 12 is  $\frac{6}{12}$ , and of 4 to 8 is  $\frac{4}{8}$ .

13. When

13. When the ratio between any two quantities is equal to the ratio between any other two, these quantities are proportional.

14. In three quantities, A, B, C; if the ratio of A to B be equal to the ratio of B to C, these three quantities are proportional; expressed thus, As A is to B, so is B to C, or  $A:B::B:C$ . Here A and C are called the extreme terms, and B is a mean proportional between A and C. Also C is a third proportional to A and B.

15. In four quantities A, B, C, D; if the ratio of A to B be equal to the ratio of C to D, the four quantities are proportional; or  $A:B::C:D$ .

Here A and D are the two extreme terms, and B and C are the means; also D is a fourth proportional to A, B, C.

#### *Examples in Numbers.*

1. In the numbers 4, 6, 9, the ratio of 4 to 6 is equal to the ratio of 6 to 9; that is,  $\frac{4}{6} = \frac{6}{9}$ ; therefore  $4:6::6:9$ , and 6 is a mean proportional between 4 and 9; also 9 is a third proportional to 4 and 6.

2. In the numbers, 16, 32, 12, 24, the ratio of 16 to 32 is equal to the ratio of 12 to 24; that is,  $\frac{16}{32} = \frac{12}{24}$ ; therefore  $16:32::12:24$ . Here the first and third terms, viz. 16 and 12, are antecedents, and the second and fourth terms, 32 and 24, are consequents.

In four proportional numbers, the product of the two extremes is equal to the product of the two means; *Eucl. B. 6. prop. 16.*; thus, if  $16:32::12:24$ , then  $16 \times 24 = 32 \times 12 = 384$ .

When four quantities are proportional, several variations may be made in their order and magnitude, which  
geometers



## P. R O B L E M S.

7

geometers call *Alternation, Inversion, Composition, Division, and Conversion.*

16. Alternate proportion is the comparing of the antecedent with the antecedent, and the consequent with the consequent.

*Exam.* Suppose four proportional quantities are represented by the proportional numbers  $18:6::21:7$ , these will be alternately proportional, viz. as  $18:21::6:7$ .

17. Inverse proportion is when the consequent is made an antecedent, and the antecedent a consequent; that is, if  $18:6::21:7$ , then, by inversion, as  $6:18::7:21$ .

18. Compounded proportion is when the sum of the antecedent and consequent is compared to the consequent. If  $18:6::21:7$ , then, by composition, as  $24:6::28:7$ .

19. Divided ratio is when the difference between the antecedent and consequent is compared to the consequent. If  $18:6::21:7$ , then, by division, as  $12:6::14:7$ .

20. Converse ratio is when the antecedent is compared to the difference between the antecedent and consequent. If  $18:6::21:7$ , then, by conversion, as  $18:12::21:14$ .

Four proportional quantities being given, to vary their order, so that any one of them may stand in the fourth place. This is done by inversion and alternation, as follows:

*Exam.* Let the four proportionals be  $12:3::16:4$

Then, by inversion, as  $4:16::3:12$

by alternation, as  $16:4::12:3$

and, by inversion, as  $3:12::4:16$

Hence, if four quantities be proportional, any three of them being given, the fourth may be found.

P R O B.

**PROB. 11.** *To find a mean proportional between two given right lines AB and BC. Fig. 13.*

Draw an indefinite line CE, on which take BA and BC of the given lengths; on AC describe the semicircle ADC; and from B draw BD perpendicular to AC; and BD is the mean proportional between AB and BC, or  $AB:BD::BD:BC$ .

**PROB. 12.** *To find a third proportional to two given right lines AB and BC. Fig. 14.*

Make any angle PAQ; and take AD equal to AB, and AE and DF each equal to BC; join DE; and through F draw FH parallel to DE; and EH is the third proportional required. For  $AD:DF::AE:EH$ ; that is,  $AB:BC::BC:EH$ .

**PROB. 13.** *To find a fourth proportional to three given right lines AB, BC, and CD. Fig. 15.*

Make any angle GEF; and take EH equal to AB, HK equal to BC, and EM equal to CD; join HM; and through K draw KL parallel to HM; and ML is the fourth proportional required. For  $EH:HK::EM:ML$ , that is,  $AB:BC::CD:ML$ .

**PROB. 14.** *To divide a given right line AB into any number of equal parts, suppose 10. Fig. 16.*

Draw AC, making any angle with AB, and through B draw BD parallel to AC; take ten equal parts on AC from A to m, without regarding whether Am be longer  
or

or shorter than  $AB$ ; take the same parts from  $B$  to  $e$  on the line  $BD$ ; join the opposite points, as in the figure; and  $AB$  will be divided into ten equal parts.

*N. B.* This operation divides, not only  $AB$ , but also  $Bm$  or  $Ae$ , into ten equal parts. For  $n1$  is  $\frac{1}{10}$ , and  $n2$  is  $\frac{2}{10}$  of  $Bm$ . By this method, a very short line may be divided into ten, or any other number of equal parts, as may be seen on the diagonal scale.

PROB. 15. *To make the diagonal scale of any length.*  
*Fig. 17.*

Make the right angle  $BAC$ ; take ten equal parts on  $AC$ , and through each division draw lines parallel to  $AB$ : Having determined the length of  $Ao$ , divide it into ten equal parts, and take the same parts from  $C$  to  $m$ ; then, through the ninth division, counting from  $o$  towards  $A$ , draw the line  $o\dot{C}$ ; and through the other divisions, draw lines parallel to it, and draw  $om$  parallel to  $AC$ ; these lines will divide each of the ten parts between  $o$  and  $A$  into ten equal parts.

Take the line  $oA$  in your compasses, and carry it along the line  $AB$ , as often as you please; and through these points, draw lines parallel to  $AC$  or  $om$ ; and the scale is made.

When the great parts  $Ao$ ,  $o1$ , &c. are accounted hundreds, the small divisions between  $o$  and  $A$  are tens, and the units are reckoned on the parallels, between  $Ao$  and  $Cm$ ; but, when the great parts are accounted tens, the small are units; and their parts on the parallels are decimals of an unit.

*To take any number from the scale.*

Take the hundreds between *o* and B, the tens between *o* and A, and carry back both feet of the compasses to the parallel whereon the units are marked.

A scale of equal parts, such as fig. 18. serves for taking numbers below an hundred, especially when a large plan is to be made. Its construction and use is evident by inspection.

The lengths of the sides of right lined figures may be taken from these scales, to a sufficient degree of accuracy for any purpose; but, in order to make or measure right lined angles, we must have recourse to the circle.

Every arc of a circle is measured by certain right lines, called Chords, Sines, Tangents, and Secants; and, for nautical purposes, a Rhumb line. *Vide Trigonometry.*

PROB. 16. *To construct the lines of chords, rhumbs, sines, tangents, and secants. Fig. 19.*

1. To make the line of Chords.

Having described the semicircle ADB, and drawn the right line DB, divide the quadrant DB into nine equal parts; and, about the center B, describe arcs through the several divisions, cutting the right line DB in 10, 20, 30, &c. and the line of chords is made for every ten degrees.

In the same manner, If each of these parts of the quadrant be divided into ten equal parts, and arcs described through the several points of division, the line of chords will be made.

*Note.*

*Note.* The chord of sixty degrees is equal to the radius of the circle.

2. To make the line of Rhumbs.

Draw the right line DA, and divide the quadrant DA into eight equal parts; and about the center D describe arcs cutting the right line DA in the points 1, 2, 3, &c. and DA will be a line of rhumbs, or points of the mariner's compass.

*N. B.* This is also a line of chords, each division being  $11^{\circ} 15'$ , and is used to describe the angle of a ship's course.

3. To make the line of Sines.

From each point of division in the quadrant BD, let fall perpendiculars on the radius CB; and these divisions being numbered 10, 20, &c. from C to B, will be a line of sines. The sine of  $90^{\circ}$  is equal to the radius of the circle.

4. To make the line of Tangents.

Through D draw DE parallel to CB; then a ruler laid to the center C, and each division in the quadrant DB will cut the line DE in the points 10, 20, &c.; and DE will be the line of tangents. The tangent of  $45^{\circ}$  is equal to the radius of the circle.

5. To make the line of Secants.

Produce CB to F; set one foot of the compasses in the center C, and with the other describe arcs through every division in the tangent line, cutting the line CF in the points 20, 30, &c.; and CF will be a line of secants.

The secant of every arc is greater than the radius of the circle; and therefore the secant line begins at the sine of  $90^\circ$ , and goes on without the circle.

The line of semitangents is made from the line of tangents; the semitangent of  $10^\circ$  being the whole tangent of  $5^\circ$ , &c.

The divisions on the lines above described, being transferred to right lines drawn on a scale of wood, or other matter, are of considerable use in practical mathematics.

The scale of equal parts, and the line of chords, serve for making and measuring right lined figures; and the lines of sines, tangents, and secants, are useful in spherical projections.

**PROB. 17.** *To make an angle of a proposed number of degrees, suppose  $55^\circ$  by the line of chords. Fig. 20.*

Draw the right line AB; about the center A, with a radius equal to the chord of  $60^\circ$ , describe the arc CD; then take the given number of degrees ( $55^\circ$ ) from the same line of chords, and set it from C to *m*, and through A and *m* draw the line AE; and EAB is the angle required.

**PROB. 18.** *To measure a right lined angle by the line of chords.*

With a radius equal to the chord of  $60^\circ$ , describe an arch about the angular point; take the arch in your compasses, and apply it to the line of chords; and thus the quantity of the angle will be known.

When

# PROBLEMS.

13

When an obtuse angle is to be made or measured, its measure being greater than  $90^\circ$ , must be taken from the scale at twice : Thus, suppose an angle of  $160^\circ$  was to be made ; having described an arc with the chord of  $60^\circ$ , take  $90^\circ$  and  $70^\circ$ , or  $80^\circ$  and  $80^\circ$  from the line of chords, and set them one after another on that arc.

DEFIN. 2<sup>d</sup>. A regular polygon, is a figure whose sides and angles are all equal, and is denominated by the number of its sides. One of five sides is called a Pentagon, of six sides, a Hexagon, &c.

22. The angle at the center of a regular polygon is contained between two right lines drawn from its center to the extremities of any of its sides, and is found by dividing  $360^\circ$  by the number of sides.

23. The angle at the circumference of a regular polygon is contained between any two adjacent sides of the polygon, and is found by subtracting the angle at the center from  $180^\circ$ .

A TABLE, shewing the Names, number of Sides, Angles at the Center, and at the Circumference of regular Polygons, from three to twelve Sides, inclusive.

Names.	Sides.	Ang. at the Center.	Ang at the Circumf.
Trigon	3	$120^\circ 00'$	$60^\circ 00'$
Square	4	$90^\circ 00'$	$90^\circ 00'$
Pentagon	5	$72^\circ 00'$	$108^\circ 00'$
Hexagon	6	$60^\circ 00'$	$120^\circ 00'$
Heptagon	7	$51^\circ 25\frac{5}{7}'$	$128^\circ 34\frac{2}{7}'$
Octagon	8	$45^\circ 00'$	$135^\circ 00'$
Nonagon	9	$40^\circ 00'$	$140^\circ 00'$
Decagon	10	$36^\circ 00'$	$144^\circ 00'$
Endecagon	11	$32^\circ 43\frac{7}{11}'$	$147^\circ 16\frac{4}{11}'$
Dodecagon	12	$30^\circ 00'$	$150^\circ 00'$

PROB.

**PROB. 19.** *To inscribe a regular polygon in a circle by the line of chords.*

Make an angle at the center of the circle equal to the angle at the center of the polygon required to be drawn; this will give one side of the polygon, which being applied round the circle, the lines joining the several points in the circumference will be the sides of the polygon.

*Exam.* Suppose a pentagon were to be inscribed in a given circle ABD. *Fig. 21.*

At the center, make the angle ACB of  $72^\circ$ , and join AB; then applying AB round the circumference, will give the points D, E, F; join BD, DE, EF, FA, and ABDEF is the pentagon required.

In the same manner, any other regular polygon may be inscribed in the circle.

*N. B.* The radius of the circle is equal to the side of a hexagon inscribed in it.

**PROB. 20.** *To make any regular polygon on a given right line AB. Fig. 21.*

At the points A and B, make the angles CAB, ABC, each equal to half of the angle at the circumference of the polygon; and from the point C with the radius CA or CB, describe a circle; and AB properly applied in the circumference will give the sides of the polygon required.

*Exam. of a pentagon.* Make the angles CAB and ABC each equal to  $54^\circ$ , the half of  $108^\circ$ ; and from the center C, with the radius CA, describe a circle, in which apply the right lines BD, DE, EF, and FA, each equal to AB; and ABDEF is a pentagon described upon the given line AB.

**PROB.**



## P R O B L E M S.

15

**PROB. 21.** *To draw a tangent to the circle from any point A, which is not within the circle. Fig. 22.*

1. If the given point A is in the circumference, draw a line from the center C to A, and through A draw BAD at right angles to AC; and BD is the tangent required.

2. If the point A is without the circumference, join CA, and on it describe the semicircle AEC, cutting the arch AE in E; then, through A and E, draw AD, which will touch the circle in the point E.

**PROB. 22.** *To describe a circle about a given triangle ABC; or, to describe a circle through any three given points A, B, C, which do not lie in the same right line. Fig. 23.*

Bisect the sides of the triangle, or the distances of the points AB and AC, at right angles, by the right lines DF and EF; and from the point F, where they meet, with a radius equal to FA, FB, or FC, describe the circle ABC.

**PROB. 23.** *To describe an oval figure representing an ellipse, whose longest diameter AB is given Fig. 24.*

Divide AB into three equal parts in the points C and D; and about the centers C and D, with the radius CD, describe two circles, intersecting each other in the points E and F; through the points C and D, draw the lines EN, EM, FO, and FP; and about the centers F and E, with the radius EN, describe the arches NM and PO; and AHBG is the oval figure required.

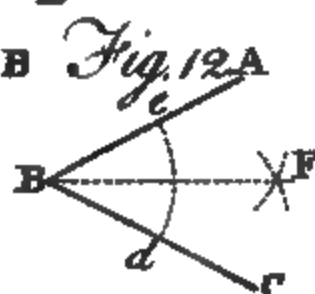
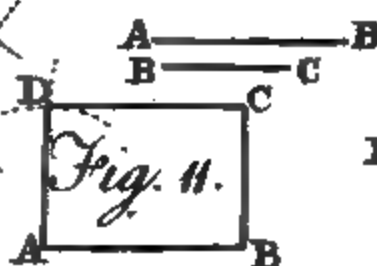
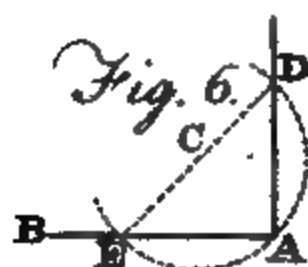
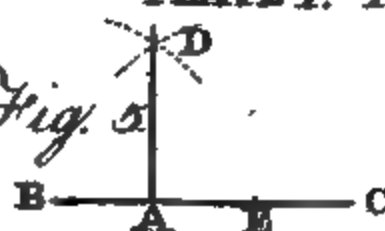
S E C T.

## S E C T. II.

# PLAIN TRIGONOMETRY.

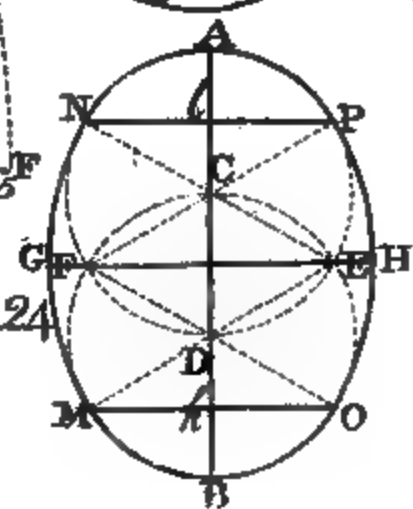
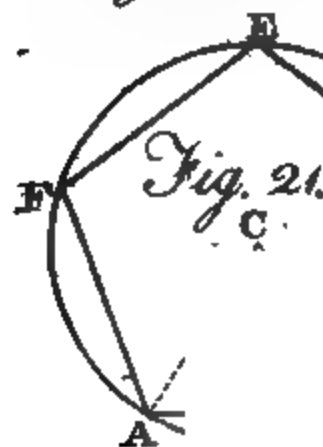
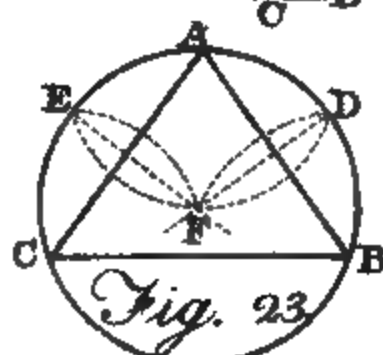
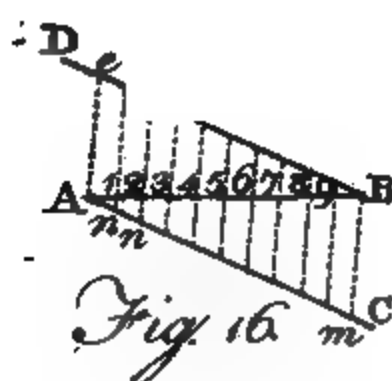
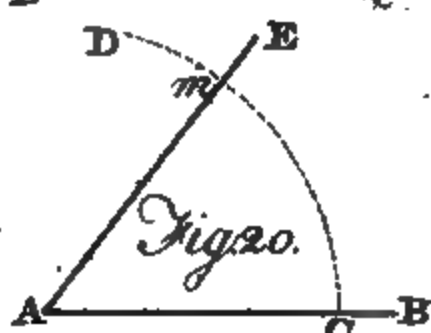
## D E F I N I T I O N S.

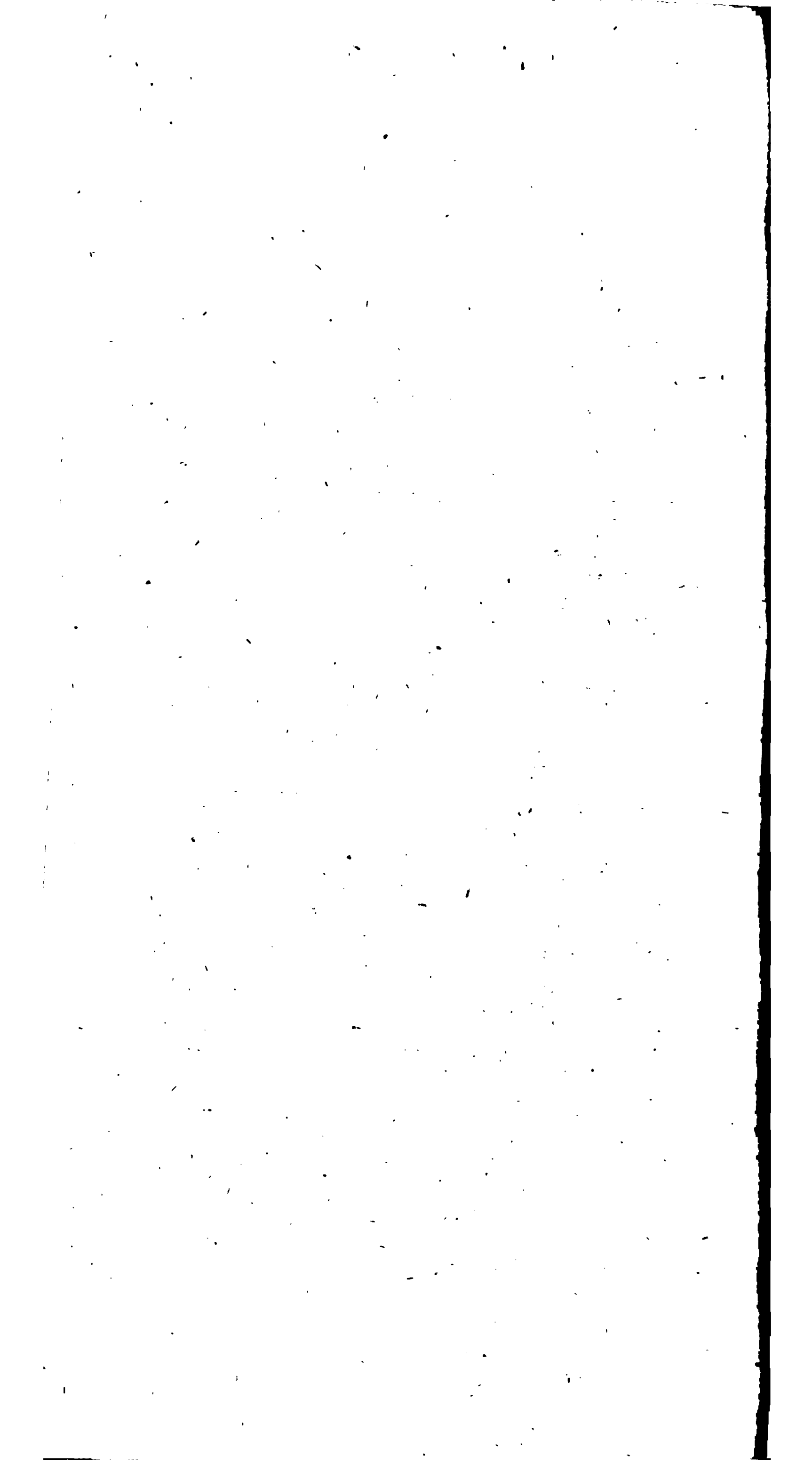
1. **A** PLAIN triangle hath six parts, *viz.* three sides and three angles ; any three of these being given, except the three angles, the rest may be found ; and the method of performing this in every possible case is called *Trigonometry*.
2. The sides of plain triangles are estimated by measures of length, such as, inches, feet, yards, miles, &c.
3. The angles of plain triangles, are measured by circular arcs described about the angular points ; thus the measure of the angle BCD is the arc BD ; and of DCH the arc DH. *Plate 2. Fig. 25.*
4. The circumference of every circle consists of 360 equal parts, called *Degrees* ; each degree of 60 minutes, and each minute of 60 seconds, &c. marked thus,  $1^{\circ}$ ,  $1'$   $1''$ , signifying, one degree, one minute, one second.
5. An angle is said to be of the same number of degrees, minutes, and seconds, with the arc that measures it.
6. The whole circumference of the circle is the measure of four right angles ; the measure of a right angle is a quadrant of the circle, or  $90^{\circ}$  ; of two right angles is  $180^{\circ}$  ; and the measure of an obtuse angle is greater than  $90^{\circ}$ , and of an acute angle, is less.
7. The



*Fig.*

*E*





7. The complement of an arc or angle, is its difference from a quadrant, or  $90^\circ$ ; thus the complement of the arc BD, or the angle BCD, is the arc DH, or the angle DCH; and the complement of the arc BK, or the angle BCK, is the arc HK, or the angle HCK.

8. The supplement of an arc or angle, is what it wants of a semicircle, or  $180^\circ$ ; thus the arc DHA, or the angle DCA, is the supplement of the arc DB, or of the angle DCB.

*N. B.* This is sometimes called the complement of an arc to a semicircle.

9. The diameter of a circle, is a right line drawn through the center, and terminated by the circumference, as AB.

10. The radius, is any right line drawn from the center, to the circumference, as CA, CB, or CD, &c.

11. The chord of an arc, is a right line joining its extremities; thus, DEO is the chord of the arc DBO, or of DHAO.

*N. B.* Every chord line, except the diameter, belongs to two arcs, the one less, and the other greater than a semicircle.

12. The sine of an arc or angle, is a right line drawn from one end of the arc, perpendicular to the radius drawn to the other end of it; or, it is half the chord of double the arc; thus, DE is the sine of the arc DB, or DHA; and DF is the sine of DH or DBV.

*N. B.* As every chord line, except the diameter, belongs to two arcs; so every sine, except the radius, belongs also to two arcs; the one less than a quadrant,

C,

and

and the other its supplement ; because every sine is half the chord of double the arc.

13. The tangent of an arc or angle, is a right line touching one end of the arc, and produced until it meets with the right line drawn from the center through the other end of the arc ; thus, BG is the tangent of the arc BD, and HI of HD.

14. The secant of an arc or angle, is a right line drawn from the center through one end of the arc, and produced until it meets with the tangent drawn from the other end of the arc ; thus CG, is the secant of the arc BD, and CI of DH.

15. The versed sine of an arc or angle, is that part of the diameter intercepted between the arc and its sine ; thus, BE is the versed sine of the arc BD, and AE is the versed sine of the arc DHA.

16. The co-sine, co-tangent, co-secant, and co-versed sine of an arc or angle, is the sine, tangent, secant, versed sine of the complement of that arc or angle.

Thus DE, BG, CG, EB, being respectively the sine, tangent, secant, and versed sine of the arc DB, which is the complement of DH ; therefore DE is called the co-sine, BG the co-tangent, CG the co-secant, and EB the co-versed sine of the arc DH, or of the angle HCD.

17. The chords and versed sines of arcs increase continually from the least quantity to  $180^\circ$ , or a semicircle ; but the sines, tangents, and secants increase only to  $90^\circ$ .

18. The sine of a right angle, or  $90^\circ$ , is equal to the radius ; but the tangent and secant of  $90^\circ$  are infinite ; because the tangent BG is parallel to the radius CH.

19. When

19. When the sine, tangent, or secant of an obtuse angle is sought in the tables, that of its supplement must be taken; thus for the sine, tangent, and secant of  $120^\circ$  take those of  $60^\circ$ .

20. Since the measure of a right-lined angle is the arc of a circle described about the angular point with any radius, we can have as many different arcs expressing the quantity of an angle as we please to assume different *radii*; each of these arcs hath a sine, tangent, secant, and versed sine; and therefore the sine, tangent, &c. of the same angle may be expressed by innumerable different right lines, all in proportion to each other as the *radii* assumed; for, let EAF be any right-lined acute angle, *Fig. 26.* produce the right lines EA and AF at pleasure; about the center A describe the several arcs, EB, HC, and LD, and draw the sines EF, HI, and Lr, and the tangents BG, CK, and DM; and then, by similar triangles, as

$$\begin{aligned} EF : HI :: AE : AH \\ GB : KC :: AB : AC \\ AG : AK :: AB : AC \end{aligned}$$

All trigonometrical calculations depend on this principle; as also, the construction of triangles, and the method of measuring the parts required by scale and compasses.

For, a radius being taken at pleasure, and the lengths of the sines, versed sines, tangents, and secants of all arcs from  $1'$  to  $90^\circ$ , calculated in proportion to it, and ranged in order in tables, or marked on a scale, affords a canon for calculating or measuring the angles and sides of any triangle; one side, and other two of its parts, being given.

21. The three angles of any plain triangle are equal to two right angles, or  $180^\circ$ .

COR. 1. If one angle of a plain triangle be given, the sum of the other two angles may be found, by subtracting the given angle from  $180^\circ$ .

2. If two angles of a plain triangle are given, the third may be found, by subtracting the sum of the two given angles from  $180^\circ$ .

3. The two acute angles of any right angled plain triangle, are equal to one right angle, or  $90^\circ$ .

4. If one of the acute angles of a right angled plain triangle be given, the other is found by subtracting the given angle from  $90^\circ$ .

22. In any right angled plain triangle ABC, *Fig. 27.* the line AC opposite to the right angle, is called the Hypothenuse; and AB and BC, which contain the right angle, are called Sides, or Legs.

23. Similar triangles are those which have their angles equal, each to its correspondent angle.

24. The sides about the equal angles of similar triangles are proportional. *Eucl. B. 6. prop. 4.*

THEOREM. If from any point in the side of a plain triangle, a right line be drawn parallel to one of the other sides, it will cut off a triangle similar to the whole. *Fig. A. plate 2.*

Let ABC be a triangle, and from any point E in the side AB draw EF parallel to EC; I say, the triangle AEF is similar to the triangle ABC.

For, since the line AB falls upon the two parallels EF and BC, the angle AEF is equal to the angle ABC; and, for the same reason, the angle AFE is equal to the  
angle



angle ACB; also the angle at A is common to the two triangles AEF, ABC; and therefore the triangle AEF is similar to the triangle ABC. In like manner, if FD be drawn parallel to AB, it may be demonstrated that the triangle FDC is similar to the triangle ABC. Hence, and from the 4th prop. of Euclid's 6th book, we have the following proportions, viz.

$$\begin{array}{ll} \text{AE} : \text{EF} :: \text{AB} : \text{BC} & \text{CD} : \text{DF} :: \text{CB} : \text{BA} \\ \text{EF} : \text{FA} :: \text{BC} : \text{CA} & \text{CF} : \text{FD} :: \text{CA} : \text{AB} \\ \text{FA} : \text{AE} :: \text{CA} : \text{AB} & \text{DC} : \text{CF} :: \text{BC} : \text{CA} \end{array}$$

25. By Radius, in Trigonometry, is understood, the radius of the tables of sines and tangents, commonly called the Trigonometrical Canon; and is generally supposed to consist of 100000000 equal parts.

### CONSTRUCTION of the TABLES of SINES and TANGENTS, &c.

1. The diameter of a circle being given, to find the chord of  $90^\circ$ .

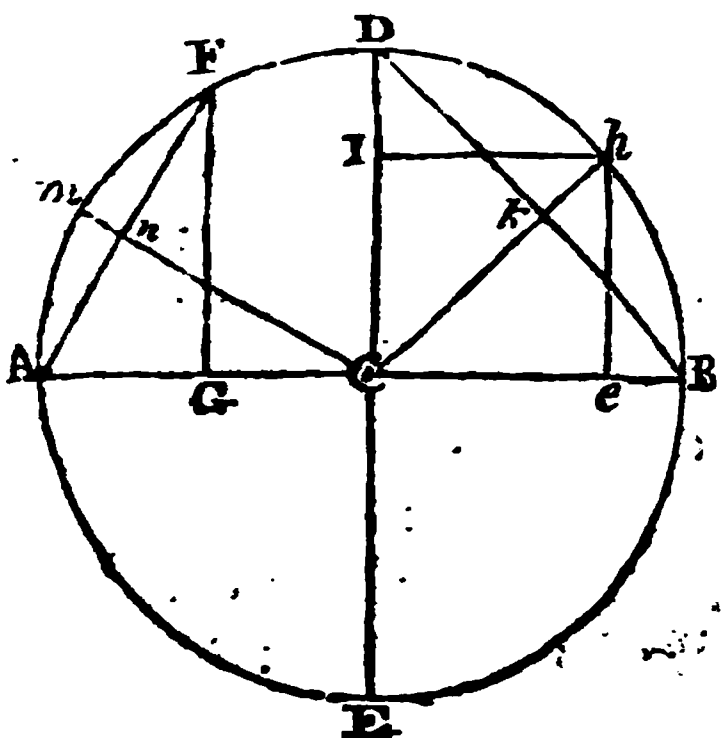
The diameter AB being given, the radius CB or CD will be known; and the chord of  $90^\circ$ , or a quadrant, viz.  $\text{BD} = \sqrt{\text{CB}^2 + \text{CD}^2}$  by the 47th Euclid, B. 1.

*Exam.* Let the diameter  $\text{AB} = 2$ , then the radius CB or  $\text{CD} = 1$ , and  $\text{BD} = \sqrt{\text{CB}^2 + \text{CD}^2} = \sqrt{2} = 1.414213562$ . Now, because the sine of an arch is half the chord of double the arch, we have the sine of  $45^\circ$ , viz. Bk or Dk = .707106781, which is half the chord of  $90^\circ$ .

The

## 22. PLAIN TRIGONOMETRY.

The co-sine of  $45^\circ$  is equal to its sine: and because the difference between the co-sine and the radius is the versed sine of any arch less than  $90^\circ$ , therefore, from the radius  $= 1$  subtract  $.707106781$  the remainder  $kb = .292893219$  is the versed sine of  $45^\circ$ .



Set the radius from A to F; join FA, and from F draw FG perpendicular to CA; also bisect the arch AF in  $m$ ; join Cm cutting the chord AF in  $n$ : then  $AF = 1$  is the chord of  $60^\circ$ , and FG is its sine:  $Am$  is an arch of  $30^\circ$ ,  $An$  is its sine and  $Cn$  is its co-sine. Now  $An = \frac{1}{2}$ , therefore the sine of  $30^\circ$  is  $.5000000$ , and its co-sine is  $Cn$ .

2. The radius of the circle and the sine of any arch being given, to find its co-sine.

*Exam.* The radius  $CA = 1$ ;  $Am$  an arch of  $30^\circ$ , its sine  $An = .5000000$ : it is required to find  $Cn$ , the co-sine of  $Am$ ?

By the 47th prop. of Euclid B. 1.  $CA^2 = Cn^2 + nA^2$ , and therefore  $Cn = \sqrt{CA^2 - An^2}$ .

From the square of  $CA = 1$ .

Subtract the square of  $An = .25$

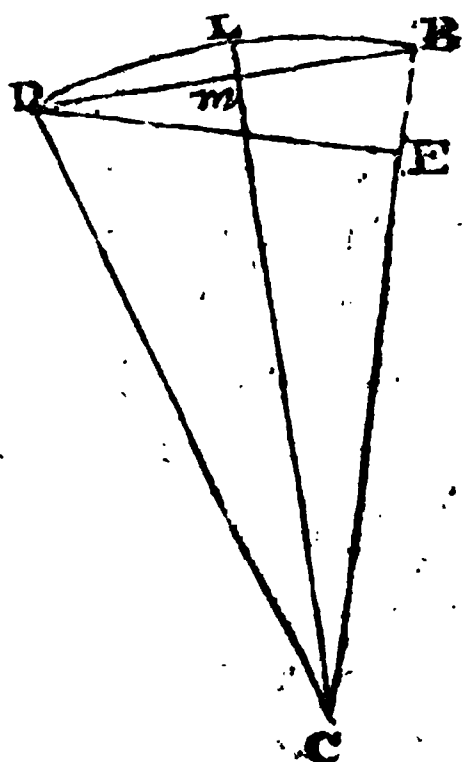
There remains the square of  $Cn = .75$  and  $\sqrt{75} = .8660254$ , the co-sine of  $30^\circ$ , or the sine of  $60^\circ$ .

3. The

3. The radius of the circle, and the sine of an arch being given, to find the sine of half that arch.

Let  $DB$  be an arch, and its sine  $DE$  given, to find  $Bm$ , the sine of half the arch  $DB$ .

The sine  $DE$  being given, the co-sine  $CE$  will be known. The co-sine being subtracted from the radius  $CB = 1$ , the remainder  $EB$  is the versed sine of the arch  $DB$ . Join  $DB$ ; then in the right angled triangle  $DEB$ , the sides  $DE$  and  $EB$  being known, we have  $DB = \sqrt{DE^2 + EB^2}$ , and half of the chord  $DB$ , viz.  $Bm$ , is the sine of half of the arch  $DB$ .



*Exam.* Let  $DB$  be an arch of  $30^\circ$ ; then its sine  $DE = .5000000$  and its versed sine  $EB = .1339746$ ; Required the sine of  $15^\circ$ ?

*Ans.*  $.25881904$

$$DE^2 = .2500000000000000$$

$$EB^2 = .01794919344516$$

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$$DB^2 = .26794919344516$$

The square root of  $.26794919344516 = .51763809$ , and the half of  $.51763809$  is  $.25881904$ , the sine of  $15^\circ$ .

4. The sine  $Bm$  of an arch  $BL$  being given, to find  $ED$  the sine of double that arch.

The sine  $Bm$  being given, the co-sine  $Cm$  may be found; the triangles  $CBm$ ,  $DBE$  are equiangular; for the

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the angles at  $m$  and  $E$  are right, and the angle  $EBD$  is common; therefore the sides are proportional, viz.

As  $CB : Cm :: BD : DE$ , that is,

As the radius

Is to the co-sine of the given arch  $BL$ ;

So is twice the sine of the given arch  $= 2Bm$  or  $DB$ ,

To  $DE$  the sine of double the arch  $BL$ .

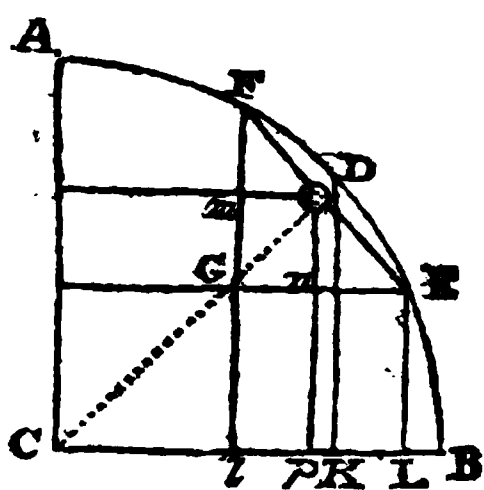
*Exam.* Let  $BL$  be an arch of  $15^\circ$ ; its sine  $= .25881904$ , it is required to find the sine of  $30^\circ$ , the double of  $15^\circ$ ?

The co-sine of  $15^\circ$  is  $.965925826$ , and twice the sine of  $15^\circ$  is  $.51763809$ . then,

As  $\text{Rad.} = 1 : .965925826 :: .51763809 : .499999989$ ,  
or  $.5000000$  nearly  $=$  the sine of  $30^\circ$ , which was to be found.

5. The sines of two arches  $BD$  and  $DF$  being given, to find  $FI$  the sine of their sum, and  $EL$  the sine of their difference.

Describe the quadrant  $ADB$ , and take  $BD$ , and  $DF$  of the given quantities: then  $BF$  is the sum of the two arches  $BD$ ,  $DF$ , and  $BE$  their difference. Draw the radii



$CD$ ,  $CB$ : take  $DE = DF$  and join  $FE$ : also draw  $FI$ ,  $DK$  and  $EL$  perpendiculars to  $CB$ : then  $CD$  being radius,  $FO$  is the sine of  $FD$ , and  $CO$  its co-sine. Through  $O$  draw  $OP$  parallel to  $DK$ , and through  $E$  and  $O$  draw  $EG$  and  $Om$  parallel to  $CB$ . The triangles,  $CDK$ ,  $COP$  and  $FOm$  are equiangular, because of the parallel lines, Therefore, as  $CD : DK :: CO : OP = Im$   
and as  $CD : CK :: FO : Fm = mG$  because  $FO = OE$ .

Now

Now  $OP + mF = FI$ , the sine of BF the sum of the arches BD, DF; and  $OP - mF = EL$ , the sine of BE the difference of the arches BD, DF.

*Coroll.* Because the difference of the arches BE, BD, BF, are equal to one another, the arch BD is an arithmetical mean between the arches BE and BF.

*Exam.* Suppose the mean arch  $BD = 45^\circ$ ; and  $DF = 15^\circ$ ; it is required to find the sine of their sum BF, and of their difference BE?

The radius  $CD = 1$ ; the sine of  $45^\circ$   $DK = .70710678$ , the co-sine of  $15^\circ$ ,  $CO = .96592582$ , and the sine of  $15^\circ$ ,  $FO = .2588190$ .

1 as  $CD : DK :: CO : OP$  that is,

as  $1 : .70710678 :: .96592582 : .683012696 = OP$

2 as  $CD : CK :: FO : Fm$ . In this exam.  $DK = CK$  that is, as  $1 : .70710678 :: .258819 : .183012669 = Fm$  or  $mG$ .

$$OP = .683012696$$

$$Fm = \pm .183012669$$

Sum is the sine of  $60^\circ = .866025365 = FI$

Diff. is the sine of  $30^\circ = .500000027 = EL$  true to the 8th place of decimals.

6. In the same figure, the radius is to double the co-sine of the mean arch BD, as the sine of DF the difference of the arches, is to the difference of the sines of the extreme arches BF and BE.

For  $CD : CK :: FO : Fm$ , and by doubling the consequents  $CD : 2CK :: Fm : 2 Fm$  or  $FG$ .

*Exam.* Let BD the mean arch  $= 20^\circ$ , the common difference  $10^\circ$ , which is also the least extreme; required the sine of  $30^\circ$ , the greater extreme?

D

The

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The mean arch is  $20^\circ$ ; its co-sine = .93969262, which being doubled is  $1.87938524 = 2CK$ , and the sine of the common difference  $10^\circ$  is  $.17364817 = FO$ .

Then as  $CD : 2CK :: FO : 2Fm$ , that is,

as  $1 : 1.87938524 :: .17364817 : .32635180 = FG$

Now,  $.32635180$  being added to the sine of  $10^\circ = .17364817$ , the sum,  $.4999999$  or  $.5000000$  nearly, is the sine of  $30^\circ$  the greater extreme: or the same number being subtracted from the sine of  $30^\circ$ , the remainder is the sine of  $10^\circ$ , the least extreme.

*Coroll. 1.* Hence, if the sines of all arches from the beginning of the quadrant to any point, differing from one another by a certain interval, such as  $1^\circ$ , are given, the sines of all other arches from that point to its double may be found: for example, suppose the sine of every arch consisting of whole degrees up to  $15^\circ$  is known; then by the last article, the sine of any arch from  $15^\circ$  to  $30^\circ$  may be found: For the radius is to the double co-sine of  $15^\circ$ , as the sine of  $1^\circ$  is to the difference of the sines of  $14^\circ$  and  $16^\circ$ , and the radius is to the double co-sine of  $15^\circ$ , as the sine of  $2^\circ$  is to the difference between the sines of  $13^\circ$  and  $17^\circ$ ; and as the sine of  $3^\circ$  is to the difference between the sines of  $12^\circ$  and  $18^\circ$ : and these differences being added to the sines of the less, will give the sines of the greater arches.

2. If the mean arch be  $30^\circ$ , as radius is to the double co-sine of  $30^\circ$ , so is the sine of  $1^\circ$  to the difference of the sines of  $29^\circ$  and  $31^\circ$ ; and so is the sine of  $2^\circ$ , to the difference of the sines of  $28^\circ$  and  $32^\circ$ : as also, so is the sine of  $3^\circ$  to the difference of the sines of  $27^\circ$  and  $33^\circ$ : But in this case double the co-sine of  $30^\circ$  is the square root of 3 =  $1.7320508$ , therefore, if the sine of any arch

below

below  $30^\circ$ , be multiplied by the square root of 3, the product will be the sine of an arch as much above  $30^\circ$ , as the given arch was below it. If, for example, the sine of  $20^\circ$  be multiplied by the square root of 3, the product will be the sine of  $40^\circ$ .

3. If the sines of all arches below  $60^\circ$ , are known, those above  $60^\circ$  may be found by addition: for since the radius is double the co-sine of  $60^\circ$ , as the sine of  $1^\circ$  is to the difference between the sines of  $59^\circ$  and  $61^\circ$ ; and the radius being 1, twice the co-sine of  $60^\circ$ , or twice the sine of  $30^\circ$  is also 1; and since the first and second terms of the proportion are equal, the third and fourth terms will also be equal; therefore the sine of  $1^\circ$  is the difference between the sines of  $59^\circ$  and  $61^\circ$ . For the same reason the sine of  $2^\circ$  is the difference between the sines of  $58^\circ$  and  $62^\circ$ , and the sine of  $3^\circ$  is the difference between the sines of  $57^\circ$  and  $63^\circ$ ; therefore add the sines of  $1^\circ$ ,  $2^\circ$ ,  $3^\circ$ ,  $4^\circ$  severally to the sines of  $59^\circ$ ,  $58^\circ$ ,  $57^\circ$  and  $56^\circ$ ; the sum shall be the sines of  $61^\circ$ ,  $62^\circ$ ,  $63^\circ$  and  $64^\circ$ , and so on to  $90^\circ$ .

*Exam.* Having the sines of  $45^\circ$  and  $15^\circ$  given, to find the sine of  $75^\circ$ .

To the sine of  $45^\circ = .70710678$

Add the sine of  $15^\circ = .25881904$

---

The sum is the sine of  $75^\circ = .96592582$

7. To find the sine and co-sine of 1 minute, the radius being 1. In very small arches, such as 1 minute, the length of the arch and of its sine are very nearly equal; therefore, the length of the arch may be taken for its sine.

If the diameter of a circle be 2, or the radius 1, the circumference will be 6.2831853071; divide this by the

D. 2

number.

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number of minutes in  $360^\circ = 21600'$ , and the quotient is the length of the arch of  $1' = .000290888208$ , which may be taken for the sine of  $1'$ , true to the 12th place of decimals.

The square of the sine of any arch being subtracted from the square of the radius, the remainder is the square of the co-sine of that arch.

In this example, the square of  $.0002908882$  is  $.00000008461594489924$ , which being subtracted from  $1$ , the square of the radius, there remains  $.999999915384$ , &c. and by extracting the square root, the co-sine of  $1'$ , or the sine of  $89^\circ 59'$  comes out  $.9999999577$ .

The sine and co-sine of  $1'$  being found, the sine of  $2'$  may be found by Article 4th, For the radius is to twice the co-sine of any arch, as its sine is to the sine of double that arch. That is,

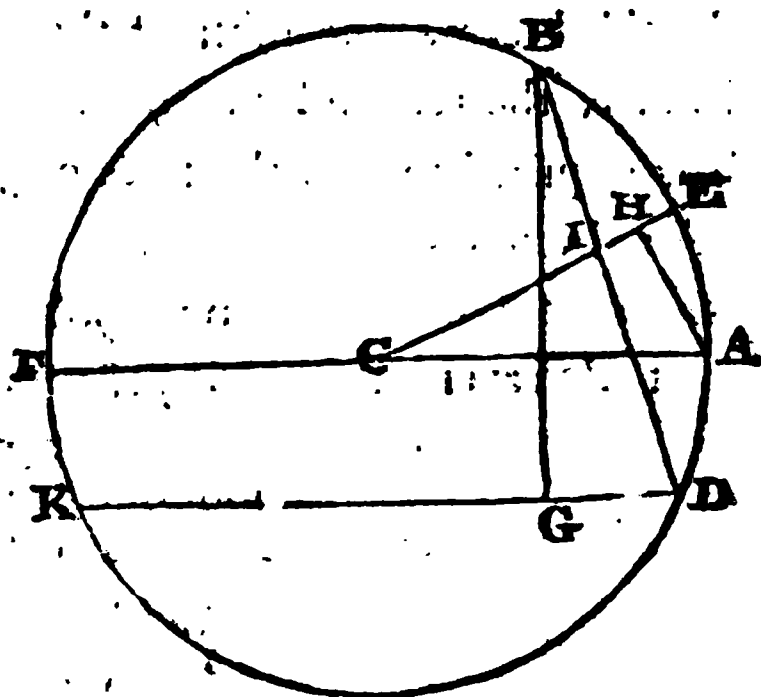
As  $1 : 1.9999999154 :: .0002908882 : .0005817753 =$   
Sine of  $2'$ .

8. Twice the rectangle under the sine of half the sum, and the co-sine of half the difference of two arches, is equal to the rectangle under the radius and the sum of their sines.

Let ABFK be a circle, its center C and diameter FA : let AB, AD be two arches. Draw the straight line DK parallel to AF, and BG perpendicular to DK : join BD and bisect it in I ; through I, draw the radius CE and through A, draw AH parallel to BD ; then BG is the sine of BAD the sum of the two arches, and BI is the sine of BE half their sum ; also CH is the co-sine of AE the half of their difference.

The





The triangles BGD, CHA are equiangular, for the angles at G and H are right, and the angles GDB, CAH are equal, by reason of the parallel lines BD and AH; therefore the sides are proportional, viz. as  $BD : BG :: CA : CH$  or as  $2BI : BG :: CA : CH$ , then  $2BI \times CH = BG \times CA$ . But  $2BI \times CH = BI \times 2CH$ , therefore  $BI \times 2CH = BG \times CA$ . Q.E.D.

From this proposition the following practical rule is derived:

9. The sine and co-sine of 1 minute being given, to find the sines of all arches from  $1'$  to  $90^\circ$ .

As the radius  $= 1$ , is to the double co-sine of 1 minute, so is the sine of  $1'$ , to the sum of the sines of  $2'$  and  $0'$ ; and so is the sine of  $2'$  to the sum of the sines of  $3'$  and  $1'$ ; and so is the sine of  $3'$  to the sum of the sines of  $4'$  and  $2'$ ; and so is the sine of  $4'$ , to the sum of the sines of  $5'$  and  $3'$ ; and so is the sine of  $5'$  to the sum of the sines of  $6'$  and  $4'$ , &c. and so on from minute to minute until the sine of every arch in the quadrant is found.

In the application of this rule; because the first term of the proportion is 1: multiply the double co-sine of 1 minute

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minute by the third term, and subtract the next less sine from the product, the remainder is the sine sought.

*Exam.* It is required to find the sines of 3' and 4'.

For the sine of 3'

double the cos. of 1' = 1.9999999154

sine of 2' reversed = 3677185000.

99999995

15999999

199999

139999

13999

1199

59

product = .00116355250

Subtr. the sine of 1' = .0002908882

remains the si. of 3' = .0008726643

For the sine of 4'

1.9999999154

Sine of 3' reversed 3466278000

159999993

13999999

399999

119999

11999

799

59

.00174532847

Subtr. S. of 2' .0005817763

Sine of 4' = .0011635521

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'It may be observed that the double co-sine of 1' being 1.9999999154 is very nearly = 2 and therefore when the sine of any arch is required true to 7 places, multiply the sine of the arch next less by 1' than that whose sine is sought by 2 and subtract the sine of the arch next less by 1' than that whose sine was multiplied, from the product, the remainder is the sine required.

*Exam.* It is required to find the sines of 3', 4', 5' and 6', &c. the sine and co-sine of 1 minute being known?

S. of 2' = .0005817763	S. 3' = .0008726644
Multiply by 2	2

product .0011635526	.0017453288
Subtr. S. of 1' = .0002908882	S. 2' = .0005817763

S. of 3' = .0008726644	S. of 4' = .0011635525
------------------------	------------------------

S. 4' = .0011635525	S. 5' = .0014544406
Multiply by 2	2

product .0023271050	.0029088812
Subtr. S. 3' = .0008726644	S. 4' = .0011635525

S. of 5' = .0014544406	S. of 6' = .0017453287
------------------------	------------------------

2. The sine of 22° 29' being .3824147 and the sine of 22° 30' = .38268343: required the sine of 22° 31'?

Sine of 22° 30' = .38268343

Multiply by 2

	.76536686
Subtract the S. of 22° 29'	.3824147

The S. of 22° 31' = .3829521

Fin

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Thus may the sine of any arch be calculated, or the truth of any sine in the table examined, if the sines of two arches differing by one minute next below that arch be known; than which, nothing more easy can be desired.

10. The radius and the length of an arch being given to find its sine.

The radius being 1; call the length of the given arch  $a$ ; and the sine of that arch is  $a - \frac{a^3}{6} + \frac{a^5}{120} - \frac{a^7}{5040} + \frac{a^9}{362880} - \frac{a^{11}}{39916800} &c.$

The first term of this series gives the sine of any arch less than 1 degree, true to 7 places; and the two first terms gives the sine of any arch below 5 degrees true to the same number of places; but in greater arches more terms of the series must be applied.

*Exam.* It is required to calculate the sine of 10 degrees?

Arch of 1' = .0002908882.

Multiply by  $600' = 10^\circ$

$a = .1745329200$   
Same reversed  $29235471.$

174532920

122173044

6981316

872664

52360

3490

1570

35

$a^2 = .0304617400$

$a^3 = .0052470244$

$$a^2 = .0304617400$$

$$a \text{ reversed } \underline{29235471}$$

$$304617400$$

$$213232180$$

$$12184696$$

$$1523087$$

$$91385$$

$$6092$$

$$2741$$

$$61$$

$$a^3 = .00531657642$$

$$\frac{a^3}{6} = .00088609607$$

$$a^3 \text{ reversed } \underline{47164030}$$

$$159497292$$

$$2126630$$

$$318994$$

$$5316$$

$$3711$$

$$212$$

$$a^5 = .000161952155$$

$$\frac{a^5}{120} = .000001349601$$

$$a = .174532920$$

$$\frac{a^3}{6} = -.000886096$$

$$.173646824$$

$$\frac{a^5}{120} = + .000001349$$

$$\text{The Sine of } 10^\circ = .173648173$$

E.

10. The

10. The same things being given, the co-sine of an arch

$$\text{is } 1 - \frac{a^2}{2} + \frac{a^4}{24} - \frac{a^6}{720} + \frac{a^8}{40320} - \frac{a^{10}}{3628800} \text{ \&c.}$$

*Exam.* It is required to calculate the co-sine of 10 degrees, or the sine of 80 degrees?

$\begin{array}{r} a^2 = .030461740 \\ \text{reversed} \quad 47164030. \\ \hline 9138522 \\ 121847 \\ 18277 \\ 304 \\ 213 \\ 12 \end{array}$	$\begin{array}{r} 1.000000000 \\ \frac{a^2}{2} = - .015230870 \\ \hline .984769130 \\ \frac{a^4}{24} = + .000038663 \\ \hline \end{array}$
$\frac{a^4}{24} = .0000386634$	
$\frac{a^6}{720} = .000000000$	
$\frac{a^8}{40320} = .000000000$	
$\frac{a^{10}}{3628800} = .000000000$	
$\text{Co-sine of } 10^\circ = .984807793$	
$\text{or the S. of } 80^\circ$	

*Note 1.* These series converge swiftly in arches near the beginning and end of the quadrant, but very slowly in those near the middle; and therefore the sines of the first and last  $10^\circ$  being calculated, the sines of the intermediate arches may be found more easily by the former rules.

2. The same series will serve for finding the sine of any arch however expressed; for example, suppose it were required to calculate the sine of one sixteenth part of the quadrant, or  $5^\circ 37' 30'' = 337.5$  minutes.

$$\text{Length of } 1' = .000290882$$

$$\text{Multiply by } 337.5'$$

$$\begin{array}{r} 14544410 \\ 20362174 \\ 8726646 \\ 8726646 \\ \hline s = .0981747650 \\ s^2 = .00963828497 \\ s^3 = .00094623638 \\ s^4 = .00000912008 \end{array}$$

$s =$

$$a = .0981747675$$

$$\frac{a^2}{6} = - .0001592060$$

$$.0980155615$$

$$\frac{a^3}{120} = + .0000000760$$

$$\text{Sine of } 5^\circ 37' \frac{1}{2} = .0980156375$$

The table of sines being made, the tangents, secants, and versed sines may be calculated by the following theorems :

1. The co-sine of any arch is to its sine as the radius is to the tangent of that arch. *Plate 2. Fig. 25.*

Let DB be an arch of a circle, DE its sine, CE its co-sine, and CB the radius. The two triangles CED, CBG are equiangular, because the angles at E and B are right, and the angle at C common to both ; therefore their sides are proportional, viz. as CE : ED :: CB : BG, the tangent of the arch BD.

*Exam.* Let BD be an arch of  $30^\circ$ , its sine DE = .5000000; its co-sine = .8660254 : required the tangent of  $30^\circ$  ?

<i>Cof.</i>	<i>Sine</i>	<i>R.</i>	<i>Tang.</i>
As .8660254	: .5000000	:: 1	: .57735027 = BG
.8660254	.5000000	(.57735027	= the tangent of $30^\circ$ .

2. The sine of  $15^\circ$  = .258819045 and its co-sine = .9659258 ; required the tangent of  $15^\circ$  ?

As .9659258	: .258819045	:: 1	: .26794917
.9659258	.258819045	(.26794917	= the tangent of $15^\circ$ .

2. The sine of any arch is to its co-sine as the radius is to the co-tangent of that arch.

In the same figure ; the triangles CDE, CIH are equiangular, for the angles at E and H are right, and the angle DCE is equal to the angle at I because CB is parallel to HI ; therefore

E 2

As

As  $DE : EC :: CH : HI$  = the co-tangent of the arch DB

*Exam.* Required the co-tangent of  $15^\circ$ ; its sine being .2588190 and co-sine .9659258?

As .2588190 : .9659258 :: 1 : .37320513 = the co-tangent of  $15^\circ$ .

3. The radius is a mean proportional between the co-sine of any arch and its secant, or as Cof. : R :: R : Sec.

For the triangles CED : CBG are similar, as has been shewn, therefore as EC : CD :: BC or CD : CG the secant of the arch BD.

*Exam.* Let BD be an arch of  $30^\circ$ , its co-sine is .8660254; required the secant of  $30^\circ$ ?

Cof.                      R.                      R.                      Sec.

As .8660254 : 1 :: 1 : 1.1547005 = CG.  
 .8660254) 1 0.00000000 (1.1547005 = the secant of  $30^\circ$ .

4. The versed sine of any arch less than  $90^\circ$  is the difference between its co-sine and the radius: thus the difference between the radius CB, and the co-sine CE is BE, the versed sine of the arch BD; and the versed sine of any arch greater than  $90^\circ$  is the sum of the radius and the co-sine of that arch.

Let AHD be an arch greater than  $90^\circ$ , its sine is DE and co-sine DF or CE. Now it is manifest that the versed sine of the arch AHD is AE, which is the sum of its co-sine EC and the radius AC. Hence, the co-sine of any arch being given, its versed sine may be found.

*Exam.* 1. The co-sine of  $20^\circ$  is .93969262: Required the versed sine?

From the radius                      1.00000000  
 Subtract                                      .93969262  
 The versed sine of  $20^\circ$  = .06030738

2. The



21 The co-sine of  $140^\circ$  is .76604443. What is the versed sine?

To the radius. 1.00000000

Add the co-sine. 0.76604443

The versed sine of  $140^\circ = 1.76604443$

These sines, tangents, secants and versed sines, here treated of, are called natural sines, tangents, &c. and the logarithms of these numbers are called Logarithmic sines, tangents, &c. Tables of either kind are called the trigonometrical Canon; for by them all the Cases of Trigonometry may be solved, as will appear farther on.

Plain triangles are either right angled, or oblique angled; and each kind hath several cases.

## RIGHT ANGLED TRIANGLES.

### CASE I. Fig. 27.

In the right angled plain triangle ABC, suppose the two sides AB and BC were given, and it were required to find the angles at A and C.

If the given sides are equal between themselves, each of the acute angles is half a right angle, or  $45^\circ$ ; but, if not, the angles may be found by this proportion:

As one of the given sides

Is to the other,

So is the radius

To the tangent of the angle adjacent to the first side.

For, produce AB, and AC to D and E, so that AD may be equal to the radius. About the center A, with the radii AB and AD, describe the arcs Be and DF, whose tangents are BC and DE; each of these arcs is the measure of the angle at A, and the triangles ABC and ADE

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$\triangle ADE$  are similar: Now, since  $AB$ ,  $BC$ , and  $AD$  are known;  $DE$ , or the tangent of the angle at  $A$  may be found thus, as  $AB:BC::AD:DE$ . That is,

As  $AB:BC::\text{Rad}:\text{Tan. } A$ . Hence,

As any side of a right angled plain triangle

Is to the other side containing the right angle,

So is the radius

To the tangent of the angle adjacent to the first side.

In the same manner it appears, that  $CB:BA::\text{Rad.}:$

$\text{Tan. } C$ ; and when either angle is found, subtract it from  $90^\circ$ , and the remainder is the other angle.

### CASE II. *Fig. 28.*

In the right angled plain triangle  $ABC$ , suppose the hypotenuse  $AC$  and the side  $AB$  were given, and it were required to find the angles at  $A$  and  $C$ .

As the hypotenuse  $AC$

Is to the side  $AB$ ,

So is the radius

To the sine of the angle at  $C$ .

Make  $CE$  equal to the radius; and about the center  $C$  describe the two arcs  $An$ ,  $Em$ , whose sines are  $AB$ ,  $ED$ ; either of which is the sine of the angle  $C$ , according to the radius assumed: And because the triangles  $ABC$ ,  $ECD$  are similar, and  $CA$ ,  $AB$ , and  $CE$ , are known;  $ED$ , or the sine of the angle at  $C$ , may be found thus,

As  $CA:AB::CE:ED$ ; that is,

As  $CA:AB::\text{Rad.}:\text{Sine } C$ .

In the same manner it may be demonstrated, that  $AC:CB::\text{Rad.}:\text{Sine } A$ . Therefore,

As

As the hypotenuse  
Is to any side,  
So is the radius  
To the sine of the angle opposite to that side.

Hence we have rules for finding any part of a right angled plain triangle, the hypotenuse, or a side, and any other two of its six parts, being given.

C A S E III. Fig. 27.

In the right angled triangle ABC, suppose the side AB, and the angles at A and C were given, to find the side BC, and the hypotenuse AC.

1. To find the side BC.

Since by Case 1.  $AB : BC :: \text{Rad.} : \text{Tan. A},$   
Therefore  $\text{Rad.} : \text{Tan. A} :: AB : BC.$

2. To find the hypotenuse AC.

Since by Case 2.  $AC : AB :: \text{Rad.} : \text{Sine C},$   
Therefore  $\text{Sine C.} : \text{Rad.} :: AB : AC.$

C A S E IV. Fig. 28.

In the right angled triangle ABC, suppose the hypotenuse AC, and the angles at A and C were given, and it were required to find the sides AB and BC.

1. To find AB.

Since by Case 2.  $AC : AB :: \text{Rad.} : \text{Sine C},$   
Therefore  $\text{Rad.} : \text{Sine C} :: AC : AB.$

2. To find BC.

By the same Case,  $\text{Rad.} : \text{Sine A} :: AC : CB.$

EXAM-

## 40: P. E. IN TRIGONOMETRY.

### EXAMPLES in right angled Triangles.

CASE 1. *Fig. 27.* In the right angled triangle ABC, there is given the side  $AB=429$ , and  $BC=316$ , to find the angles at A and C.

#### CONSTRUCTION.

Make the right angle ABC; take the numbers expressing the lengths of AB and BC from a scale of equal parts, and set them from B to A and C; then join AC, and the triangle is described.

Measure the angle at A by a line of chords, or protractor, and subtract it from  $90^\circ$ , the remainder is the angle at C.

Making AB Radius, we have.

As AB	429	2.6324573
Is to BC	316	2.4996871
So is the radius (tang. $45^\circ$ )		10.0000000
To tangent A	$36^\circ 22'$	9.8672298
Whose complement $53^\circ 38'$ is the angle C.		

EXAM. 2. *Fig. 29.* Suppose there are three cities, E, D, and C, so situate, that E is 542 miles directly west of D, and C 318 miles north of D; it is required to find the bearing of C from E? That is, in the right angled triangle CDE, there is given  $DE=542$ , and  $DC=318$ ; to find the angles at E and C.

ANSW.  $E=36^\circ 24'$   $C=59^\circ 36'$ .

3. When the sun shines, suppose a tower of 147 feet high, projects a shadow of 219 feet 6 inches on the horizontal plain, What is the altitude of the sun at that time? ANSW.  $33^\circ 48'$ .

CASE 2. *Fig. 28.* In the right angled triangle ABC, there is given the side  $AB=286$ , and the hypotenuse  $AC=463$ ; to find the angles at A and C.

CON-

# PLAIN TRIGONOMETRY. 41

## CONSTRUCTION.

Make the right angle  $ABC$ ; take the number expressing the length of  $AB$  from a scale of equal parts, and set it from  $B$  to  $A$ ; take also the number expressing the length of  $AC$  from the same scale, and setting one foot of the compasses in  $A$ , with the other cross the line  $BC$  in  $C$ ; draw the straight line  $AC$ , and the triangle is described.

Measure the angle  $ACB$  by a line of chords, and subtract it from  $90^\circ$ , the remainder is the angle  $CAB$ .

Making  $AC$  radius, we have

As $AC$ 463	2.6655810
Is to $AB$ 286	2.4563660
So is the radius (sine $90^\circ$ )	10.0000000
To sine $C$ $38^\circ 8'$	9.7907850

Whose complement  $51^\circ 52'$  is the angle at  $A$ .

*N. B.* When the proportion lies in tangents, the radius is the tangent of  $45^\circ$ ; and when in sines, radius is the sine of  $90^\circ$ .

**EXAM. 2.** *Fig. 30.* Suppose a ship sails between south and west from  $E$  to  $C$  500 miles, and thereby makes 300 miles of southing= $DE$ ; what was the angle of course  $CED$ ?

Here, in the right angled triangle  $CDE$ , there is given  $CE=500$ , and  $ED=300$ , to find the angles at  $C$  and  $E$ .

**ANSW.**  $C=36^\circ 52'$ , and the course is  $S. 53^\circ 8' W.$

3. Suppose the hypotenuse of a right angled triangle is 273 miles, and one of its sides is 194 miles; required the two acute angles? **ANSW.**  $45^\circ 17'$ , and  $44^\circ 43'$ .

**CASE 3.** *Fig. 27.* In the right angled triangle  $ABC$ , there is given the side  $AB=476$ , and the angle at  $A=47^\circ 9'$ ; to find the side  $BC$  and the hypotenuse  $AC$ .

**AN. W.**  $BC=513.1$   $AC=699.9$ .

**P**

**CON-**

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### CONSTRUCTION.

Make the angle at A of the given quantity by a line of chords or protractor; take the given length of AB from a scale, and set it from A to B; from B raise the perpendicular BC and the triangle is described.

Measure BC and AC by taking them severally in the compasses, and applying them to the scale from which AB was taken.

#### 1. Making AB radius.

As Rad. tan. $\tan=45^\circ$	10.0000000
So tan. $A=47^\circ 9'$	10.0326241
So is AB = 476	2.6776070
To BC = 513.1	2.7102311

#### 2. Making AC radius.

As Sine $C=42^\circ 51'$	9.8325609
To radius = S. $90^\circ$	10.0000000
So is AB = 476	2.6776070
To AC = 699.9	2.8450461

EXAM. 2. *Fig. 30.* Suppose a ship sails S. W. by S. until she hath made 319 miles of southing; how many miles hath she sailed, and how far is she west from the meridian of the place sailed from?

Here, in the right angled triangle CDE, there is given  $ED=319$ , and the angle at  $E=33^\circ 45'$ ; to find EC and CD.

ANSW. EC 383.6 m. and CD 213.1.

3. When the sun's altitude is  $30^\circ 20'$ , suppose the shadow of a tree on the horizontal plain is 69 feet 3 inches; what is the height of the tree; and, what is the distance from the top of the tree to the extremity of its shadow?

ANSW. The height of the tree is 40.5 feet, the other 80.23 feet.

CASE

CASE 4. Fig. 28. In the right angled triangle ABC, there is given the hypotenuse AC=357; and the angle at C=42° 13'; to find the sides AB and BC.

AnsW. AB=239.8, BC=264.3.

### CONSTRUCTION.

Make the angle CAB of the given quantity; take the number expressing the length of AC from a scale of equal parts, and set it from A to C; from C let fall the perpendicular CB, and the triangle is described.

Measure the sides AB and BC on the scale from which AC was taken,

Making AC radius.

1. As radius = S. 90°	10.0000000
To Sine C = 42° 13'	9.8273279
So is CA = 357	2.5526682

To AB = 239.8	2.3799961
---------------	-----------

2. As radius = S. 90°	10.0000000
To Sine A = 47° 47'	9.8695891
So is AC = 357	2.5526682

To CB = 264.3	2.4222573
---------------	-----------

EXAM. 2. Fig. 29. Suppose the slope side of a hill CE is 3625 links of a surveying chain, and the angle of ascent at E is 19° 30'; what is the length of the horizontal level ED, and the perpendicular height of the hill CD? AnsW. ED=3417 CD=1210.

When the two sides of a right angled plain triangle are given to find the hypotenuse; or, when the hypotenuse and one of the sides are given to find the other side; the problem may be solved by the 47th prop. of Euclid's, I B. as follows:

1. When the two sides are given to find the hypotenuse.

F 2

Square

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Square each of the sides, add them; and extract the square root of the sum; the root is the length of the hypotenuse.

EXAM. *Fig. 27.* Let  $AB=429$  and  $BC=316$ ; required the length of the hypotenuse  $AC$ ?

ANSW. 532.82.

To the square of  $AB=184041$

Add the square of  $BC=$  99856

The sum is the square of  $AC=283897$

And the square root of 283897 is  $532.82=AC$ .

2. The hypotenuse and one side being given to find the other side.

Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder; the root is the length of the side required.

EXAM. *Fig. 28.* Let the hypotenuse  $AC=463$ , and the side  $AB=286$ ; required the side  $BC$ ?

ANSW. 364.1

From the square of  $AC=214369$

Subtract the square of  $AB=$  81796

There remains the square of  $BC=132573$

And the square root of 132573 is  $364.1=BC$ .

Or, because the difference of the squares of any two straight lines is equal to the rectangle contained under their sum and difference; the last problem may be solved thus: multiply the sum of the hypotenuse and given side by their difference, the square root of the product is the length of the side required.



## PLAIN TRIGONOMETRY. 45

In the last Example,  $AC + AB = 749$ , and  $AC - AB = 177$ , then  $749 \times 177 = 132573 =$  the square of  $BC$ , as before; and  $BC = 364.1$ .

## OBLIQUE ANGLED TRIANGLES.

When one side of an oblique angled triangle, and all the angles are given, to find the other two sides; or, when two sides with an angle opposite to one of them are given, to find the other two angles and the third side; the solution depends on the following theorem.

Any side of a plain triangle is to any other side, as the sine of the angle opposite to the first side, is to the sine of the angle opposite to the other. *Fig. 31.* Let  $ABC$  be an oblique angled plain triangle, let fall the perpendicular  $AD$ ; then, by Case 2. of right angled triangles,

As  $BA : AD :: \text{Rad.} : \text{Sine } B,$

And as  $CA : AD :: \text{Rad.} : \text{Sine } C;$

---

Therefore  $BA \times \text{Sine } B = CA \times \text{Sine } C.$

Consequently, as  $BA : AC :: \text{Sine } C : \text{Sine } B.$

In the same manner, it might be demonstrated, that any other two sides are to each other as the sines of their opposite angles; and, on the contrary, the sine of an angle is to the sine of any other angle, as the side opposite to the first angle is to the side opposite to the other.

CASE

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CASE 1. *Fig. 31.* In the oblique angled triangle ABC, there is given the side AC=256, the angle at B=40° 34', and the angle at C=56°; to find the sides AB and BC.

### CONSTRUCTION.

Make the angle at C of the given quantity by a line of chords; take the number expressing the length of AC from a scale of equal parts, and set it from C to A; then at the point A, make the angle CAB of the given quantity, and the triangle is described. Measure AB and BC on the scale from which AC was taken.

Subtract the sum of the angles B and C from 180°, the remainder is the angle at A; and then,

$$\text{As Sine B: Sine C: CA: AB}=326.34,$$

$$\text{As Sine B: Sine A:: AC: CB}=391.06.$$

N. B. When two of the angles are equal, the sides opposite to them being also equal, the case will be solved by one proportion.

*Exam.* In the triangle ABC, suppose the side AC=256 and each of the angles at B and C=67° 39'; required the side BC?

Angle B = 67° 39'	From 180°
—— C = 67 39	Subtr. 135 18
Sum 135 18	Angle A = 44 42
As Sine B = 67° 39'	9.9656153
To Sine A = 44 42	9.8471991
So is AC = 256	2.4082400
To BC = 194.9	12 2554391
	2.2898238

EXAM. 2. *Fig. 32.* Suppose D and E are two stations from whence the object C can be seen, and the angles at D and

D and E measured with an instrument; and the distance of the stations DE measured with a chain, and it were required to find the distance of the object C from each station.

Let  $DE = 479$  feet, the angle at  $D = 36^\circ 50'$ , and the angle at  $E = 47^\circ 9'$ ; required the distances DC and CE?

ANSW.  $DC = 353.1$   $CE = 288.7$ .

3. The distance of two mountains lying east and west of each other is 29 miles: There is a third mountain to the northward of these, so situated, that the angle contained by a right line drawn to it from the eastern mountain, and the line joining the two first mountains is  $52^\circ 12'$ ; and the angle contained by a right line drawn to it from the western mountain, and the line joining the two first mountains is  $68^\circ 28'$ : Required the distance of the third mountain from each of the other two?

ANSW. 31.36m. and 26.64 miles.

CASE 2. *Figs. 31.* In the oblique angled triangle ABC, there is given the side  $AB = 324$ , and  $BC = 468$ , with the angle at  $A = 80^\circ 30'$ ; to find the angles at B and C; and the side AC.

### CONSTRUCTION.

Make the angle BAC of the given quantity; take AB from the scale, and set it from A to B; then take BC from the same scale, and setting one foot of the compasses in the point B, with the other cross the line AC in C and join BC, and the triangle is described.

Measure the angles at B and C by the line of chords; and measure AC on the same scale from which AB and BC were taken.

To find the angle at C.

As  $BC : BA :: \text{Sine } A : \text{Sine } C = 40^\circ 3'$ ; which add to the

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the angle at A, and subtract the sum from the  $180^\circ$ , the remainder is the angle at B.

To find the side AC

As Sine C : Sine B :: BA : AC = 395.45.

*N. B.* In this case, if the angle at A be right or obtuse, the angle at C being acute, will be found out by the proportion; but if the angle at A be acute, the angle at C may be either acute or obtuse, and therefore the species of this angle cannot be known by the fourth proportional; but must be determined before the solution is begun.

EXAM. 2. *Fig. 32.* In a triangular field CDE, suppose the angle at C was measured with an instrument, and found to be  $70^\circ 50'$ ; and the sides CE and ED being measured with a chain, CE was 490 links, and ED 532 links; required the angles at D and E, with the length of the side DC?

ANSW.  $D=60^\circ 27'$ ,  $DC=423.2$ .

3. Let two sides of an oblique angled plain triangle be 500 and 300, and let the angle opposite to the greatest side be  $70^\circ 30'$ ; required the other two angles and the third side? ANSW. The angles are  $34^\circ 26'$  and  $75^\circ 4'$  and the side 512.6.

THEOREM, *Fig. 33.* The half difference of any two quantities added to their half sum, produceth the greater quantity; and their half difference being taken from their half sum leaves the lesser quantity.

Let AB and BC represent two quantities, whose half sum is AF or FC, and half difference FB. It is manifest, that FB, the half difference, added to AF the half sum, is equal to AB, the greater quantity; and FB, the half difference, being taken from FC, the half sum, leaves BC the lesser quantity.

CASE

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**CASE 3.** *Fig. 34.* Two sides of an oblique angled triangle AB and BC, with the angle ABC contained between them, being given, to find the other angles at A and C, and the third side AC.

If the two given sides are equal between themselves, subtract the given angle from  $180^\circ$ , and half of the remainder is either of the other angles; and find the third side by Case 1.

If the given sides are unequal, the angles may be found by this Theorem.

In any plain triangle ABC,

As the sum of any two sides  $BC+BA$ .

Is to their difference  $BC-BA$ .

So is the tangent of half the sum of

the angles opposite to those sides  $\frac{A+C}{2}$

To the tangent of half their difference  $\frac{A-C}{2}$

Take BE and BF, each equal to AB; join FA and AE; and through C draw CD parallel to AE: Then F, A, E, are points in the circumference of a circle whose center is B, and FAE, FDC are right angles; FC is the sum of the sides AB and BC, and EC their difference.

The angle BAE is equal to the angle AEB, and their sum is equal to the sum of the two angles BAC and ACB; because the angle ABF is equal to either of these sums; therefore, the angle AEB, or its equal FCD, is the half sum of the angles BAC, ACB; and the angle EAC; or its equal ACD, is their half difference; be-  
G
cause,

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cause, this added to BAE, their half sum, is equal to BAC, the greater of the two.

Making CD the radius, FD is the tangent of the angle FCD, the half sum, and DA is the tangent of ACD, the half difference of the angles BAC and ACB; the triangles AFE, DFC are similar; therefore,

As FC:CE::FD:DA; that is,

As FC, the sum of AB and BC,

Is to EC their difference,

So is FD, the tangent of half the sum of the angles BAC and ACB,

To DA, the tangent of half their difference.

The half difference added to the half sum, gives the greater angle, and subtracted gives the lesser.

EXAM. 1. In the oblique angled triangle ABC, there is given the side AB=317, the side BC=429, with the contained angle at B=48° 40'; required the angles at A and C, and the side AC?

### CONSTRUCTION.

Draw the straight line BC, and at the point B make the angle ABC of the given quantity; take the numbers expressing the lengths of BA and BC from a scale of equal parts, and set them from B to A and C, then join AC and the triangle is described.

Measure the angles at A and C by the line of chords, and measure AC on the scale from which AB and BC were taken.

429	180° 00	
317	48 40	
746=BC+BA	228 40	
2)	131° 20'	=Sum of the ang. A and C.
112=BC—BA	65° 40'	=their half sum.

As

As  $BC+BA = 746$  To the half sum  $65^{\circ} 40'$   
 Is to  $BC-BA = 112$  add the half diff.  $18 22$   
 So is  $\tan. \frac{A+C}{2} = 65^{\circ} 40'$  \_\_\_\_\_

Greater ang.  $A=84 02$   
 To  $\tan. \frac{A-C}{2} = 18^{\circ} 22'$  From the half sum  $65^{\circ} 40'$   
 Subt. the half diff.  $18 22$  \_\_\_\_\_

The lesser ang.  $C=47^{\circ} 18'$

Having found the angles, the side AC is found by  
 Case 1. thus, as Sine C : Sine B :: BA : AC = 323.8.

EXAM. 2. *Fig. 32.* Suppose CDE a triangular field ;  
 the side DE being measured, is 564 links of a surveying  
 chain, and EC=368 links; also the angle at E being  
 measured with an instrument, is  $54^{\circ} 20'$ ; what are the  
 angles at C and D, and what is the length of the side  
 CD ?

ANSW.  $C=85^{\circ} 6'$ ,  $D=40^{\circ} 34'$ ,  $CD=459.8$ .

3. Suppose the distance of the earth from the sun is  
 81000000 miles, and the distance of the moon from the  
 earth is 240000 miles; and that the angle contained by  
 two right lines drawn from the center of the earth to the  
 centers of the sun and moon is  $89^{\circ} 6'$ ; what is the dis-  
 tance of the moon from the sun ?

ANSW. 80938000 miles.

CASE 4. When the three sides AB, BC, CA, of any  
 oblique angled plain triangle ABC are given; to find  
 the angles at A, B, and C.

If any two of the given sides are equal, the triangle  
 being isocles, its angles may be found thus. *Fig. 35.*

Suppose AB is equal to AC, let fall the perpendicular  
 AE, and the point E will fall in the middle of the side  
 BC;

BC; therefore, by Case 2. of right angled triangles, as  $AB:BE::Rad.:Sine\ BAE$ , whose complement is the angle at B or C; and its double is the angle BAC.

When the sides are unequal, *Fig. 36*, let fall the perpendicular AE, upon the base, or longest side BC; and, about the center A, with a radius equal to the shortest side AC, describe the circle FDG. Produce BA to G; then BG is the sum of the sides BA and AC, and BF their difference; also BE and EC are the segments of the base, and BD their difference. The rectangle under CB and BD is equal to the rectangle under GB and BF, by the property of the circle; *Euclid, B. 3, corrol. to prop. 36*. Therefore, as  $CB:BG::BF:BD$ ; that is,

As the base BC

Is to (BG) the sum of the other two sides  $BA + AC$ ,

So is (BF) the difference of these sides  $BA - AC$

To (BD) the difference of the segments of the base  $BE - EC$ .

Now, take the half of this difference, and also the half of BC, their sum is BE the greater segment, and their difference is EC the lesser segment. Then, in the right angled triangle ABE, there is given AB and BE; to find the angles, which is done by Case 2. of right angled triangles, viz.

As  $AB:BE::Rad.:Sine\ BAE$ , whose complement is the angle at B. For the same reason, in the right angled triangle AEC, as  $AC:CE::Rad.:Sine\ EAC$ , whose complement is the angle at C. And  $BAE + EAC =$  the angle BAC.

EXAM. In the oblique angled triangle ABC, there are given the three sides, viz.  $AB=264$ ,  $BC=384$ , and  $AC=196$ ; to find all the angles at A, B, and C.

CON-



CONSTRUCTION.

Draw the straight line BC; take the number expressing its length from a scale, and set it from B to C; then take the length of AB from the same scale, and setting one foot of the compasses in the point B, with the other describe an arch, and take the length of AC from the scale, then setting one foot of the compasses in the point C, with the other cross the former arch in A; draw the straight lines AB and AC, and the triangle is described.

Measure the angles at A, B, and C, by a line of chords, or with a protractor. Let fall the perpendicular AE.

264

196

460=BA+AC

264

196

68=BA-AC.

(1.) As BC

384

2)384

2)81.4

Is to BA+AC

460

192

40.7

So is BA-AC

68

40.7 add and subtract

To BE-EC

81.4 BE=232.7

EC=151.3

(2.) As AB : BE :: Rad. : Sine BAE=61° 49' whose complement 28° 11' is the angle at B.

(3.) As AC : CE :: Rad. : Sine EAC=50° 31', whose complement 39° 29'=the angle at C, and BAC=112° 20'.

EXAM. 2. Fig. 37. Let D and E represent two ports in the same latitude, whose distance is 480 miles; and suppose a ship from D sails 300 miles between north and east, and another sails from E 240 miles between north and

and west, and then meets with the first ship at C; required the course of each ship?

AnsW. The course of the first ship is N.  $65^{\circ} 49'$  E. and of the other N.  $59^{\circ} 43'$  W.

A TABLE of Proportions, for solving the several Cases of right angled Triangles. *Fig. 27 and 28.*

<i>Cases.</i>	<i>Given.</i>	<i>Sought.</i>	<i>Proportions.</i>	<i>Rad.</i>
1	AB, BC	A & C	AB:BC::Rad. Tan. A whose complement is C	AB
2	AC, AB	A & C	CA:AB::Rad. : Sine C whose complement is A.	AC
3	AB & A	BC AC	Rad. : Tan. A::AB:BC Sine C : Rad. :: AB:AC	AB AC
4	AC & C	AB BC	Rad.: Sine C :: CA : AB Rad.: Sine A :: CA : CB	AC

The solution of the 1st Case admits of no variation. In the 2d Case, if AB be made radius, the proportion may be, as AB : AC :: Rad. : Secant A, whose comp. is C. In the 3d Case, if AC be made radius, the solution may be as Sine C : Sine A :: AB:BC; and, if AB be made radius, as Rad.:Secant A::AB:AC. The solution of the 4th Case may be, as Sec. A:R::AC:AB; and, as Sec. C:R::AC:CB, each side being radius.

These proportions are true, but unnecessary; for, since every case may be solved by a right application of sines or tangents, there is no reason for introducing secants.

A TABLE

# PLAIN TRIGONOMETRY. 55.

**A** TABLE of PROPORTIONS for solving the several Cases of Oblique angled Triangles. Fig. 31. 34. and 36.

Cases	Given.	Sought.	Proportions.
1.	AC and B, C.	AB BC	Sine B : Sine C :: CA : AB. Sine B : Sine A :: AC : CB.
2.	AB, BC, and A.	C, B, and AC.	CB : BA :: Sine A : Sine C. Sine C : Sine B : BA : AC.
3.	AB, BC, and B.	A, C, and AC.	$BC+BA : BC-BA :: \text{Tan.} \frac{A+C}{2} : \text{Tan.} \frac{A-C}{2}$ Add and subtract the half sum and half diff. and you have the angles A and C, and then, Sine C : Sine B :: BA : AC.
4.	AB, BC, and CA.	B C and A.	BC : BA+AC :: BA-AC : BE-EC. Hence, BE and EC will be known. AB : BE :: Rad. : Sine BAE, whose complement is B. AC : CE :: Rad. : Sine CAE, whose complement is C. And the sum of BAE and CAE is the angle BAC.

The

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The 4th case may be solved by the following Theorem:

In any plain triangle, the rectangle contained by half the sum of the sides, and the excess of that half sum above the side opposite to the angle sought, is to the rectangle contained by the differences between the half sum of the sides, and the two sides containing the angle sought; as the square of the radius is to the square of the tangent of half the angle sought.

EXAMPLE 1. of Case 4, resumed. *Fig. 36.*

1. To find the angle BAC.

AB=264	422	422
BC=384	264	196
CA=196	<hr/>	<hr/>
	158	226

2)844 sum

422 half

Subt. 384

38

As 422 x 38

$$\left\{ \begin{array}{r} 2.6253125 \\ 1.5797836 \\ \hline 4.2050961 \end{array} \right.$$

Is to 158 x 226

So is squ<sup>r</sup> Rad.

$$\left\{ \begin{array}{r} 2.1986571 \\ 2.3541084 \\ 20.0000000 \\ \hline 24.5527655 \end{array} \right.$$

$$\begin{array}{r} 24.5527655 \\ 4.2050961 \\ \hline \end{array}$$

To squ<sup>r</sup> tan  $\frac{A}{2}$

$$2)20.3476694$$

$$\begin{array}{r} 56^{\circ} 10' 20'' \\ 56 \quad 10 \quad 20 \\ \hline \end{array}$$

$$10.1738347$$

$$112^{\circ} 20' 40'' = \text{BAC}$$

2. To

2. To find the angle at C.

$$\text{As } 422 \times 158 \quad \left\{ \begin{array}{l} 2.6253125 \\ 2.1986571 \end{array} \right.$$

$$\hline 4.8239696$$

$$\text{Is to } 38 \times 226 \quad \left\{ \begin{array}{l} 1.5797836 \\ 2.9541084 \end{array} \right.$$

$$\text{So is } \text{sq}^{\text{r}} \text{ rad.} \quad 20.0000000$$

$$\hline 23.9338920$$

$$\hline 4.8239696$$

$$\text{To } \text{sq}^{\text{r}} \tan. \frac{C}{2} \quad 2) 19.1099224$$

$$\hline 19^{\circ} 44' 34'' \quad 9.5549612$$

$$\hline 19 \quad 44 \quad 34$$

$$\hline \text{ACB} = 39 \quad 29 \quad 8$$

3. To find the angle at B.

$$\text{As } 422 \times 226 \quad \left\{ \begin{array}{l} 2.6253125 \\ 2.9541084 \end{array} \right.$$

$$\hline 4.9794209$$

$$\text{Is to } 38 \times 158 \quad \left\{ \begin{array}{l} 1.5797836 \\ 2.1986571 \end{array} \right.$$

$$\text{So is } \text{sq}^{\text{r}} \text{ rad.} \quad 20.0000000$$

$$\hline 23.7784407$$

$$\hline 4.9794209$$

$$\text{To } \text{sq}^{\text{r}} \tan. \frac{B}{2} \quad 2) 18.7990198$$

$$\hline 14^{\circ} 5' 6''$$

$$\hline 14 \quad 5 \quad 6$$

$$\hline \text{ABC} = 28 \quad 10 \quad 12$$

H

Proof.

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Proof.

$$\begin{array}{r}
 A = 112^{\circ} \ 20' \ 40'' \\
 B = \quad 28 \quad 10 \quad 12 \\
 C = \quad 39 \quad 29 \quad 8 \\
 \hline
 180^{\circ} \quad 0 \quad 0
 \end{array}$$

We have made, as usual, eight Cases in Plain Trigonometry, viz. four in right angled triangles, and four in oblique; but these may be reduced to three: for,

1. Since the sides of any plain triangle are in proportion to each other as the sines of the angles opposite to these sides; by this we can solve the 2d, 3d, and 4th Cases of right angled triangles, as also the 1st and 2d Cases of oblique angled triangles.

2. Since the sum of any two sides of a plain triangle is to their difference, as the tangent of half the sum of the opposite angles is to the tangent of half their difference; hence we can solve the 1st Case of right angled triangles, and the third Case of oblique angled triangles.

3. When the three sides of a plain triangle are given to find the angles; if the triangle be right angled, the acute angles may be found by Theorem 1st above; and if it be oblique angled, the angles may be found by the Theorems, p. 37.

Some authors comprehend the whole of Plain Trigonometry, in three Cases as follows:

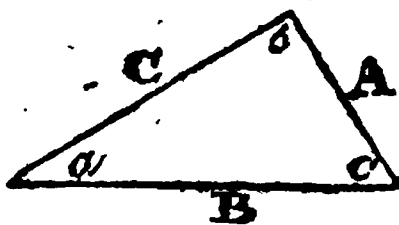
### CASE I.

The three sides of any plain triangle being given, to find the angles.

Let  $A, B, C$ , represent the three sides, and  $a, b, c$ , the opposite angles. Put  $s$  = the sine,  $s'$  = the co-sine,  $t$  = the tangent, and  $t'$  = the co-tangent of any angle.

Also

Also put  $P = \frac{1}{2} A + B + C$ ,  $L =$   
the logarithm of any number and  
 $L' =$  the arithmetical complement of  
a logarithm. Then



the  $s \frac{1}{2} a = \sqrt{\frac{P-C \times P-B}{B \times C}}$  or by Logarithms

the  $s \frac{1}{2} a = \frac{1}{2} L P - C + L P - B + L' B + L' C$

the  $s \frac{1}{2} b = \sqrt{\frac{P-A \times P-C}{A \times C}}$  and by Logarithms

$s \frac{1}{2} b = \frac{1}{2} L P - A + L P - C + L' A + L' C$

The  $s \frac{1}{2} c = \sqrt{\frac{P-A \times P-B}{A \times B}}$  and by Logarithms

$s \frac{1}{2} c = \frac{1}{2} L P - A + L P - B + L' A + L' B$

*Exam.* Let the side  $A = 49$ ,  $B = 96$  and  $C = 66$ : it  
is required to find the angles at  $a$ ,  $b$ , and  $c$ ?

1. For the angle  $a$ .

49	$P = 105.5$	105.5
96	$C = - 66.$	$B = - 96.$
66		
<hr/>		
	$P - C = 39.5$	$P - B = 9.5$
2)211		

$P = 105.5$

$L P - C = 39.5$	1.3965971
$L P - B = 9.5$	0.9777236
$L' B = 96$	8.0177288
$L' C = 66$	8.1804561

$\sim 2)18.7725056$

$s \frac{1}{2} a = 14^{\circ} 5' 6''$  9.3862528

or the angle  $a = 28^{\circ} 10' 12''$

H 2

2. For

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2. For the angle  $b$ .

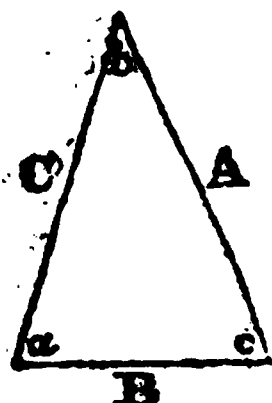
$P = 105.5$	$L.P - A = 56.5$	1.7520484
$A = -49.$	$L.P - C = 39.5$	1.5965971
	$L' C = .66.$	8.1804561
$P - A = 56.5$	$L' A = 49$	8.3098039
		<u>2) 19.8389055</u>
	$s \frac{1}{2} b = 56^\circ 10' 20''$	9.9194527
	or the angle $b = 112^\circ 20' 40''$	

3. For the angle  $c$ .

$L.P - B = 9.5$	0.9777236
$L.P - A = 56.5$	1.7520484
$L' A = 49.$	8.3098039
$L' B = 96.$	8.0177288
	<u>2) 19.0573047</u>
	$s \frac{1}{2} c = 19^\circ 44' 33''$
	9.5286523
	or the angle $c = 39^\circ 29' 6''$

In an isosceles triangle, when the side  $A$  is equal to the side  $C$ , the angle  $b$  contained between the equal sides may be found by this Theorem :

$$s \frac{1}{2} b = \frac{B}{A} \times \frac{r}{2} = \frac{B r}{2 A} = \frac{B}{2 A}$$



because the radius  $r=1$  ; and by Logarithms  $s \frac{1}{2} b = L B + L' 2 A$ .

*Exam.* Let  $A = 80$   $C = 80$  and  $B = 60$  ; required the angles  $b$ ,  $a$ , and  $c$  ?

$L B = 60$	1.7781513
$L' 2 A = 160$	7.7958800
	<u>9.5740313</u>
$s \frac{1}{2} b = 22^\circ 1' 27''$	
the angle $b = 44^\circ 2' 54''$	
from $180^\circ$	
Subtr. $b = 44^\circ 2' 54''$	
	<u>2) 135 57 6</u>
	$67^\circ 58' 33'' = a \text{ or } c$

When

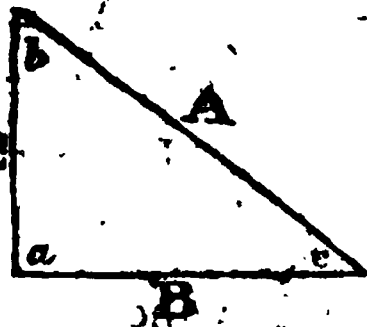


When the three sides of a right angled plain triangle are given, to find the angles, the Theorem becomes more simple. For example, suppose the angle  $a$  is a right angle, then

$$s.c = \frac{C}{A}, \text{ and } s.b = \frac{B}{A}; \text{ or by logarithms.}$$

$$s.c = L.C + L'A \text{ and } s.b = L.B + L'A$$

*Exam.* Let the hypotenuse  $A = 50$ . the side  $B = 40$  and the side  $C = 30$ ; required the angles at  $c$  and  $b$ ?



For the angle  $c$ .

$$L.C = 30 \quad 1.4771213$$

$$L'A = 50 \quad .8.3010300$$

$$\text{angle } c = 36^\circ 52' 11'' \quad 9.7781513$$

For the angle  $b$ .

$$L.B = 40 \quad 1.6020600$$

$$L'A = 50 \quad .8.3010300$$

$$\text{angle } b = 53^\circ 7' 48'' \quad 9.9030900$$

CASE 2. When one side and all the angle of any plain triangle are given; or when two sides and an angle opposite to one of them are given, to find the other parts of the triangle.

Because in any plain triangle the sides are to each other as the sines of their opposite angles; the ratio between any side, and the sine of its opposite angle is equal to the ratio between any other side, and the sine of its opposite angle, and therefore

$$\frac{A}{s.a} = \frac{B}{s.b} = \frac{C}{s.c}. \quad \text{From}$$

these equations any of their terms may be found. For example, when the side  $A$  and the angles are given, the

$$\text{side } B = \frac{s.b}{s.a} \times A; \text{ and the side } C = \frac{s.c}{s.a} \times A. \text{ or by}$$

Logarithms.

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**Logarithms.** The side  $B = LA + Ls b + L'a$ , and the side  $C = LA + Ls c + L's a$ .

*Exam.* Let the side  $A = 32$ , the angle  $a = 17^\circ 44'$ , the angle  $b = 96^\circ 12'$  and the angle  $c = 66^\circ 4'$ : required the sides  $B$  and  $C$ ?

For the side  $B$

$LA = 32$	1.5051500
$Ls b = 96^\circ 12'$	9.9974523
$L's a = 17 44$	0.5162883
side $B = 104.4$	2.0188906

For the side  $C$

$LA = 32$	1.5051500
$Ls c = 66^\circ 4'$	9.9609548
$L's a = 17 44$	0.5162883
side $C = 96.$	1.9823931

And if the sides  $B$  and  $C$  and the angle  $b$  were given, to find the angles at  $c$  and  $a$  with the side  $A$ ?

$s c = \frac{C}{B} \times s.b$ , or by logarithms  $sc = LC + Ls.b + L'B$

The side  $A = \frac{sc}{sa} \times C$ . or by logarithms,  $A = LC + Lsa + L'sc$ .

*Exam.* Let the side  $B = 43$ , the side  $C = 32$ , and the angle  $b = 84^\circ 2'$ ; required the angles  $c$  and  $a$ , with the side  $A$ ?

For the angle  $c$ .

$L C = 32.$	1.5051500
$Lr. b = 84^\circ 2'$	9.9976408
$L' B = 43$	8.3665315
	9.8693223

$s. c = 47^\circ 44' 40''$

hence the angle  $a = 48^\circ 13' 20''$

For

# PLAIN TRIGONOMETRY. 63

For the side A.

$$\begin{array}{r} L\ C = 32 \quad 1.5051500 \\ L\ s. a = 48^\circ\ 13'\ 20'' \quad 9.8725842 \\ L' sc = 47\ 44\ 40 \quad 9.1306785 \\ \hline 1.5084127 \end{array}$$

$$\text{Side } A = 32.24$$

**CASE 3.** When two sides of a plain triangle, and the angle contained between them are given, to find the other two angles and the third side.

Suppose  $c$  = the given angle;  $A$  and  $B$  the two given sides. Put  $S = 90^\circ - \frac{1}{2} c$ ,  $d$  = the  $\frac{1}{2}$  difference of the angles  $a$  and  $b$ , then tangent  $d = \frac{A-B}{A+B} \times \tan. S$ . having found  $d$ ,  $S+d = a$ , and  $S-d = b$ .

$$\text{The side } C = \sqrt{\frac{4AB \times \sin \frac{1}{2} c}{\cos r} + A - B^2} \quad \text{See the Fig. on p. 59.}$$

*Exam.* Suppose the side  $A = 141$ ,  $B = 92$ , and the contained angle  $c = 54^\circ\ 20'$  and it were required to find the angles at  $a$  and  $b$ , and the side  $c$ .

For the angles  $a$  and  $b$ .

$$\begin{array}{r} 90^\circ \quad 141 \\ \frac{1}{2} c = 27\ 10 \quad 92 \\ \hline S = 62\ 50 \quad A-B=49 \\ \hline A+B=233 \end{array}$$

By logarithms,  $\tan. d = L A - B + L \tan. S + L A + B$ .

$$\begin{array}{r} L\ A-B = 49 \quad 1.6901961 \\ L\ \tan. S = 62^\circ\ 50' \quad 10.2897176 \\ L' \tan. A+B=233 \quad 7.6326441 \\ \hline \tan. d = 22^\circ\ 17' \quad 9.6125578 \end{array}$$

$S. =$

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$$S. = 62^{\circ} 50'$$

$$d = 22 \quad 17$$

$$\text{angle } a = 85 \quad 7$$

$$\text{angle } b = 40 \quad 33$$

For the side C. by logs  $L 2 A + L 2 B + 2 L s \frac{1}{2} c = Q$ ;  
and  $2 L A - B = L R$ ; lastly,  $L C = \frac{1}{2} L Q + R$ .

$$L 2 A = 282 \quad 2.4502491$$

$$L 2 B = 184 \quad 2.2648178$$

$$L s \frac{1}{2} c = 27^{\circ} 10 \quad 9.6595173$$

$$\text{Ditto } 27 \quad 10 \quad 9.6595173$$

$$Q = 10816.86 \quad 4.0341015$$

$$L A - B = 49 \quad 1.6901961$$

$$R = 2401 \quad -3.3803912$$

$$Q = 10816.68$$

$$\sqrt{13217.68} = 114.968$$

$$\text{The side } C = 114.968$$

By this Theorem the third side C is found directly from the data, without finding the angles a and b.

In this Case; when the given angle c is a right angle, the other two angles and the hypotenuse may be found

thus; the  $t. a = \frac{A}{B} \times r$ , the  $t. b = \frac{B}{A} \times r$ , wherein the radius r being = 1 may be left out. The hypotenuse

$$c = \sqrt{A^2 + B^2}$$

And by logarithms,  $L a = L A + L B$ ;  $L b = L B - L A$  and  $C = \frac{1}{2} L A^2 + B^2$

*Exam* In a right angled triangle, let the side A=213, the side B=199, containing the right angle c: required the angles a and b, and the hypotenuse C?

For

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For the angles  $a$  and  $b$ .

$$\begin{array}{r} \text{L } A = 213 \quad 2.3283796 \\ \text{L' } B = 199 \quad 7.7011469 \\ \hline \end{array}$$

$$\begin{array}{r} \text{angle } a = 46^\circ 57' \quad 10.0295265 \\ b = 43^\circ 3' \end{array}$$

For the hypotenuse  $C$ .

$$\begin{array}{r} A^2 = 45369 \\ B^2 = 39601 \\ \hline 84970 \quad 4.9292656 \\ \frac{1}{2} \quad 2.4646328 \\ C = 291.49 \end{array}$$

## TRIGONOMETRICAL PROBLEMS.

**PROB. I.** One side of a right angled triangle, and the sum of the other side and hypotenuse being given ; to find the angles, the hypotenuse, and the other side severally. *Fig. 38.*

**EXAM.** In the right angled triangle  $ABC$ , suppose the side  $AB=70$ , and the sum of  $BC$  and  $AC=200$ ; it is required to find the angles  $ACB$ ,  $CAB$ , with the hypotenuse  $AC$ , and the side  $BC$ ?

**ANSW.**  $ACB=38^\circ 36'$ ,  $CAB=51^\circ 24'$ ,  $AC=112.32$ ,  $BC=87.68$ .

**CONSTRUCTION.** Make the right angle  $ABD$ ; make  $AB=70$ , and  $BD=200$ ; then find the point  $C$  equally distant from  $A$  and  $D$ , and join  $AD$  and  $AC$ .

**SOLUTION.** In the right angled triangle  $ABD$ , we have  $AB$  and  $BD$  given ; to find the angle  $BAD$ ; from which subtract its complement, viz. the angle  $D=DAC$ , and there will remain the angle  $CAB$ , whose comple-

I ment

ment is the angle  $ACB$ ; and then, in the right angled triangle  $ABC$ , we have all the angles with the side  $AB$ ; to find  $AC$  or  $CB$ ; either of which being found, subtract it from 200, and the remainder is the other side.

PROB. 2. One side of an oblique angled triangle, the angle opposite to that side, and the sum of the other two sides, being given; to find the other two angles and sides severally. *Fig. 39.*

EXAM. In the oblique angled triangle  $ABC$ , suppose the side  $BC=532$ , and the angle  $BAC=110^{\circ} 30'$ ; also the sum of the sides  $BA$  and  $AC=637$ ; it is required to find the angles  $ACB$  and  $ABC$ , with the sides  $BA$  and  $AC$ ?

ANSW.  $ACB=45^{\circ} 5'$ ,  $ABC=24^{\circ} 25'$ ,  $AC=234.7$ ,  $AB=402.3$ .

CONSTRUCTION. Make the angle at  $D$  of  $55^{\circ} 15'$  equal to half  $BAC$ ; take 637 from the scale, and set it from  $D$  to  $B$ ; then take 532 from the same scale, and setting one foot of the compasses in  $B$ , with the other cross the line  $DC$  in  $C$ ; and join  $BC$ ; find the point  $A$  equally distant from  $D$  and  $C$ ; and join  $AC$ . Now, since  $AD$  is equal to  $AC$ , the angle  $ACD$  is equal to the angle  $ADC$ , and each of them half of the angle  $BAC$ .

SOLUTION. In the triangle  $BCD$ , we have  $DB$ ,  $BC$ , and the angle at  $D$ ; to find the angle  $BCD$ .

Subtract the angle  $ACD$  from  $BCD$ , the remainder is the angle  $ACB$ ; and, by taking the sum of  $BAC$  and  $ACB$  from  $180^{\circ}$ , we have the angle  $ABC$ . Then, in the triangle  $ABC$ , we have all the angles, and the side  $BC$ ; to find either  $AB$  or  $AC$ .

PROB. 3. One side of an oblique angled triangle, the angle opposite to that side, and the difference between the

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the other two sides, being given; to find the other angles: and sides severally. *Fig. 40.*

**EXAM.** In the oblique angled triangle  $ABC$ , let the side  $BC=250$ , and the angle  $BAC=96^{\circ} 50'$ , also the difference between the sides  $AB$  and  $AC$ , viz.  $BD=106$ ; required the angles  $ACB$  and  $ABC$ , with the sides  $AB$  and  $AC$ ?

**ANSW.**  $ACB=57^{\circ} 55'$ ,  $ABC=25^{\circ} 15'$ ,  $AB=213.4$ ,  $AC=107.4$ .

**CONSTRUCTION.** Subtract the angle  $BAC$  from  $180^{\circ}$  and divide the remainder by 2, the quotient is the angle  $ADC$ ; which being subtracted from  $180^{\circ}$ , the remainder is the angle  $BDC$ .

Make the angle  $BDC$ . Take 106 from the scale, and set it from  $D$  to  $B$ ; then take 250 from the scale; and setting one foot in  $B$ , with the other cross the line  $DC$  in  $C$ , and join  $BC$ ; produce  $BD$ , in which find the point  $A$  equally distant from  $D$  and  $C$ ; and join  $AC$ .

**SOLUTION.** In the triangle  $BDC$ , we have  $BD$ ,  $BC$ , and the angle  $BDC$ ; to find the angle  $BCD$ .

Having found the angle  $BCD$ , add it to  $ACD$ ; the sum is the angle  $ACB$ .

Subtract the sum of the angles  $BAC$  and  $ACB$  from  $180^{\circ}$ , the remainder is the angle  $ABC$ .

Then, in the triangle  $ABC$ , we have all the angles, and the side  $BC$ ; to find  $AC$  or  $AB$ ; either of which being found, the other will be known by applying the difference  $BD$  to the side found.

### *Description of GUNTER'S SCALE.*

The Logarithms were invented by Lord Napier, and published at Edinburgh in the year 1614; and, about

ten years after, Mr. Edmund Gunter, Professor of Astronomy in Gresham College, London, applied the logarithms to lines on a two foot scale, whereby all questions in trigonometry, and such parts of practical mathematics as depend on that science, may be readily solved without calculation, by applying a pair of compasses to the divisions on the scale.

Gunter's Scale contains,

1. A line marked S. R. (sine rhumbs) on which are the logarithm sines of the degrees to each point and quarter point of the mariner's compass.

2. A line marked T. R. (tangent rhumbs) being the logarithm tangents of the said points and quarters.

3. A line marked Numb. (numbers) on which the logarithms of numbers are laid down.

4. A line marked Sin. containing the logarithm sines.

5. A line of logarithm versed sines marked V. S.

6. A line of logarithm tangents marked Tan.

7. A meridional line marked Mer.

8. A line of equal parts marked E. P.

The two first and two last lines are used only in navigation; the rest in trigonometry.

The figures on the several lines have always the same values, except on the line of numbers; there, the figures represent either the series of numbers from 1 to 100, or from 1 to 1000, and consequently have different values, according to the series they represent.

Because the logarithms of proportional numbers have equal differences, these logarithms on a scale will be at equal distances; therefore, if four numbers are proportional, the distance between the first and second terms will be equal to the distance between the third and fourth



## MENSURATION.

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fourth on the scale. Hence, the rule for working any proportion on Gunter's Scale is, Set one foot of the compasses to the first term of the proportion, and extend the other to the second; then, keeping that extent in the compasses, set one foot to the third term, and the other will fall upon the fourth term or answer.

### S E C T. III.

*The Application of PLAIN TRIGONOMETRY to the  
MENSURATION of HEIGHTS and DISTANCES.*

**T**O perform problems of this kind, the first thing to be done is, to measure the length of any straight line on the ground; and, for this purpose, some measure of a determinate length must be chosen, such as a yard, fathom, pole, the surveying chain, or some part of it.

Measures should be adapted to the quantities to be measured. Small distances may be measured by a foot rule, or yard, or by a tape divided into inches, and rolled up in a box; but large distances require larger measures, such as long poles of some known length, or the surveying chain.

The Scots surveying chain is a measure of  $74\frac{2}{3}$  English feet, (commonly made only 74 feet), consisting of 100 links, each link being 8.928 inches. The English surveying chain is 66 feet in length, consisting of 100 links, each link being 7.92 inches.

*To measure an accessible distance on the ground, as from  
A to B. Fig. 56.*

Apply one end of your measure to A, directing the other towards B, and at the extremity E put a mark,  
or

or pin; then carry the measure into the position EG, placing a mark at G; and again carry the measure into the position GK, and so on until you come to B. The number of pins will show the number of measures from A to B; and hence the length of the line AB will be known.

When a large distance, such as the side of a field, is to be measured with the surveying chain, ten pins, or arrows of wood, or iron, must be made use of to number the chains; thus, he who leads the chain takes the ten arrows at setting out, and, at the end of the first chain sticks one in the ground; and, when they have advanced forward another chain, he sticks another in the ground, and the man at the other end of the chain takes up the first arrow, and so on; the man who leads the chain putting down one at the end of every chain, and the man who follows the chain taking them up as he advances, until he have all the ten; and then they must be carried to the man who leads the chain: And thus the number of chains in any distance will be known.

Besides the surveying chain, a pole of wood ten links long, the lengths of the links being marked on it, and one of them divided into ten equal parts, is necessary to measure a few links and parts of a link accurately; and thus the distance will be known in links, and 10ths of a link, or 1000 parts of the whole chain.

Although the whole chain is commonly made use of, it is difficult to measure a line accurately with it, on account of its great length. A half chain, or one of 50 links, can be held more straight, and managed more easily, and consequently less liable to errors than the whole chain.

The

The common method of measuring an accessible distance is abundantly simple, and seems easy enough to execute; and yet, even in this simple operation, it is very hard to avoid errors entirely.

The surveying chain is very well contrived for the purpose of measuring land; and, when applied with care, the area of any field may be found; but, in measuring distances from whence calculations are to be made, in order to find out larger distances, greater exactness is necessary than can be expected from the surveying chain: For it requires a great deal of care to keep the chain always truly straight; and even supposing this done, we cannot be sure that the chain does not lengthen by constant stretching; and, if this be true, small errors may arise from different degrees of tension; nor is it possible to avoid these errors with any flexible measuring line; and, therefore, when the true length of any straight line on the ground is required, we must chuse a measure which will neither contract nor lengthen in the application, such as a pole of wood, or a rod of iron of some known length.

When a distance is to be measured by a pole, the operation is performed in the same manner as by the chain. A small wire, or some such mark, should be stuck in the ground at the end of the pole, that, when it is moved forward, the hindermost end in every successive position may lie at the same point where the foremost end lay in the last position; and, for this purpose, a notch should be made in the end of the pole to receive the pin; and then the pole being carried forward, and its end laid close to the pin in every advance, the distance may be measured very accurately.

Otherwise,

Otherwise, take two straight poles of any convenient length, such as 10 feet, having their ends well smoothed, and take supporters for them, so contrived, that they may be easily raised or depressed with screws, or by some other means. Then, having placed the poles upon their supporters in a line at one extremity of the distance to be measured, and made them level by a plummet; lift the hindermost pole with its supporter, and carry it forward to the extremity of the other; place it there in a level-line, adjusting it as before; then take up the hindermost of the two poles as they now lie, and carry it forward with its supporter, and so on, always bringing the poles to a true level, and taking care that their ends touch each other exactly. By this method, a distance may be measured with great accuracy.

The length of any accessible right line may be found by a careful application of some known measure; but, in many problems, the lengths of right lines are required, which are either wholly, or partly, inaccessible, and sometimes invifible, such as the diameters and perpendicular heights of mountains. In order to solve such problems, we must not only be able to measure straight lines, but to find out their positions, or to measure the quantity of any right lined angle accurately. This may be done by the Theodolite, Graphometer, Quadrant, &c. But the same thing may be performed as well by measuring certain right lines.

*There are two kinds of Angles to be measured, namely, Horizontal and Vertical.*

A horizontal angle is contained between two right lines which lie in the plane of the horizon, such as the  
two

two sides of a field, or any two imaginary straight lines drawn from a given point in the horizontal plane. For example, suppose any point on the surface of the earth is taken, and from that point one straight line runs north, and another north-east; the angle contained by these lines is called a Horizontal Angle.

A vertical angle is contained by two straight lines drawn in a plane perpendicular to the plane of the horizon. For example, suppose a straight line is drawn from any point on the surface of the earth in the horizontal level, and, from the same point, another straight line is drawn to the top of a high building, a mountain, a star, or to any object which is above the plane of the horizon, the angle contained by these two straight lines, or its complement, is called a Vertical Angle.

*To measure a Horizontal Angle on the Ground, as ABC, by any lineal measure.*

This may be done by taking a radius at pleasure in any of the containing sides, and measuring either the chord, sine, tangent, or secant of the angle.

1<sup>st</sup>, By measuring the chord; *Fig. 41.* Measure any distance, such as a chain from B to A, and there stick an arrow in the ground. Measure the same distance from B to C, and there also place an arrow. Then measure the distance from A to C, and the three sides of the isosceles triangle ABC are known. Let fall the perpendicular Be, and the point e will be in the middle of AC; therefore Ae is known; and, as  $BA : Ae :: \text{Rad.} : \text{Sine } ABe$ ; which doubled, is the angle ABC.

This method is general, and will answer equally well whatever the length of AB or BC is; but, if AB and BC

K

are

are each one chain, or any measure consisting of 100, or 1000 equal parts, the angle ABC may be found from a table of sines, without calculation; thus, Measure AC in 10th parts of a link, or 1000th parts of the chain; then AB being 1000, and the radius of a table of sines also 1000, Ae being a number of the same parts, will be the natural sine of the angle ABe; and therefore, this number being found in a table of natural sines, the quantity of the angle ABe will be known. If you have not a table of natural sines, take the logarithm of Ae, and find it among the logarithm sines, neglecting the indices of both, and you have the angle ABE, which being doubled, is the angle ABC.

If there be no mistake in measuring the right lines, and if AC lies in, or near, the true level, the angle will be found perfectly true. An error in AC will affect the angle more than an equal error in AB or BC; but even here, if the error does not exceed  $\frac{1}{10}$  of a link, or  $\frac{1}{1000}$  part of the radius, the angle will not be above two minutes wrong, (unless the angle be large), which is a degree of exactness far exceeding that of the common instruments.

EXAM. 1. The radius being one chain, suppose the length of the chord AC is  $67\frac{3}{8}$  links, or .673 1000th parts of the chain; the half of this is .3365, which is the natural sine of  $19^{\circ} 40'$ ; and therefore the angle ABC is  $39^{\circ} 20'$ .

2. Let the chord AC be  $127\frac{7}{8}$  links, or 1277 of such parts as the radius is 1000; the half of this number is .6385, which is the natural sine of  $39^{\circ} 41'$ ; and therefore the angle ABC is  $79^{\circ} 22'$ .

The radius being one chain, an error of  $\frac{1}{10}$  of a link, or  $\frac{1}{1000}$  of an inch in the length of AC, would cause an

error

error of three minutes in an angle of about  $80^{\circ}$ ; but such an error could scarcely happen, if due care be taken in measuring AC.

2dly, To find out the quantity of a horizontal angle by measuring its tangent; *Fig. 29.* Let CED represent an angle on the ground. Measure any distance, such as one chain from E to D in one of the sides containing the angle for a radius; then from the point D mark out a perpendicular to ED, and place a pin, or arrow in C, at its extremity, in the other side containing the angle. Measure CD, and reduce it to 1000th parts of ED; it is the natural tangent of the angle CED; and hence the angle will be known from the tables as before.

EXAM. Suppose the radius ED is one chain, or 1000 equal parts of any measure, and that the tangent DC is found to be  $43\frac{8}{10}$  links, or .438 parts of ED; this number is the natural tangent of  $23^{\circ} 39'$ ; and so much is the angle CED.

3dly, The quantity of a horizontal angle may be found by measuring its secant; for, (in the same figure,) having measured the radius ED, and found the point C, where the perpendicular drawn from D meets with the other side; EC is the natural secant of the angle CED; and EC being measured in 1000th parts of the radius, and found in a table of secants, the angle CED will be known. But it is to be observed, that unless the angle be above  $45^{\circ}$ , there is greater uncertainty in measuring the secant than the tangent; and I would never recommend this method, except in large angles, and when the tangent could not be measured on account of some obstacle in the ground.

Upon the whole, the best method of finding the quantity of any horizontal angle, is, to measure either its chord or its tangent. Acute angles can be measured more accurately by their chords than obtuse angles; and, therefore, when the angle is large, I would divide it into two parts, equal or unequal; and, by measuring the chord of each part separately, the quantity of the whole will be known.

There is less chance of error in measuring the tangent of an angle than in measuring its chord, especially if the angle be between  $30^{\circ}$  and  $80^{\circ}$ , on account of the rapid increase of the tangents; for, if the angle be about  $60^{\circ}$ , and the radius one chain, an error of one inch in the tangent will make the angle only one minute wrong; and so great an error could scarcely be committed by a careful person.

Acute angles only can be measured directly by their tangents; but the method will apply to obtuse angles, by dividing them into parts, and measuring the tangent of each part separately; and, in this case, the radius should be measured in a straight line, dividing the angle into two parts, as near equal as can be guessed by the eye; and, on the extremity of this radius, set off a perpendicular to it towards each side containing the angle, these will be the tangents of the parts; and being measured, the quantity of the whole angle will be known. This should be done in any angle which is about or above  $80^{\circ}$ .



*At a given point on the ground, to mark out a straight line perpendicular to another given straight line passing through that point.*

Take a smooth board of any form, and thereon draw two right lines cutting each other at right angles, and place two sights on each line a few inches distant from each other. Contrive some means to fix the board horizontally on the top of a staff. Then, having fixed the staff in the ground at the given point, with the board on the top of it; turn the whole, until through one pair of sights you see along the given right line, and then the other pair of sights will be in the direction of the perpendicular required. In that direction, place a pole at any convenient distance, and the right line between your staff and the pole is perpendicular to the given right line.

*Note.* Sights are small pieces of wood, or brass, having small holes, or slits, in them, to view distant objects through. They are fixed perpendicular to the plane of any instrument, and serve to direct the view in a right line. Such as have cross hairs in them are reckoned the best.

#### *A DESCRIPTION of Two POLES for Measuring VER- TICAL ANGLES.*

Provide two straight poles of wood, one about 10 or 12 feet long, divided into feet, inches, and roth parts of an inch. Provide also a stool about 12 or 15 inches high, having three feet, the whole of a considerable weight to stand firm, and having a hole in the middle of it to receive the end of the pole easily; and let the end of the pole that goes into the stool be put through the  
middle

middle of a board of hard wood, which board must be firmly fixed to the pole, and so near the end of it, that, when the pole is put into the stool, the board may be only two or three inches distant from the surface of the stool, and parallel to it. Make three or four screws pass through the board and the stool, to fix the pole firmly to the stool, and also to set it truly perpendicular to the plane of the horizon. In order to ascertain this, a plummet may be hung from some point of the pole, or it may be adjusted by a separate plummet. Also, let there be two pieces of wood or brass put upon the pole to slide up or down, each having a fiducial edge to show the true point where they rest on the pole.

Make also another pole about four or five feet long, having a sharp end to stick in the ground, or else a three-footed stool to fix it in like the other; but there is no need of screws here; if the pole stand firm any how, the end will be answered. At, or near the top of this pole, fix a small spirit-level with hair sights, and to turn round an axis, which axis must be perpendicular to the pole. By this means, the sights will be easily directed towards the top of any high object, and also turned into the level line. And thus the two poles are prepared for use.

*To Measure any Vertical Angle, not exceeding  $64^{\circ}$ , by the two Poles above described.*

Let AEB be the vertical angle to be measured; A being the top of a high object, and FB the level ground.  
*Fig. 43.*

At any convenient point C, place the longest pole CD, and adjust it truly perpendicular to the plane of the horizon.

zon. Measure some convenient distance from C to F, and there place the shortest pole FE with the spirit-level. Turn the level on its axis until A, the top of the high object, is seen through the sights; then move the sliding mark D on the long pole, until the fiducial edge rest on the long pole precisely in a right line between E and A, which will be known by seeing A and D through the sights. Then turn the sights into the level line, and move the other sliding piece until its fiducial edge rest at G, where the level line meets the long pole.

Hence GD, the distance between the sliding pieces on the long pole, will be known; and EG, the distance between the poles, is known. Therefore,

As the distance between the poles EG

Is to the space between the sliding pieces GD,

So is radius

To the tangent of the angle AEB; which was required.

If the poles, level, and sliding marks be made, adjusted, and applied as above, there is no danger of any error in the angle.

The poles may be placed at any distance; but, if the ground will admit, it will be best to place them at some determined distance, such as 10 or 20 feet, or, rather at 100 or 1000 equal parts of some known measure, as 100 inches, 100 half feet, or one surveying chain of 100 links; and then the distance of the poles being the same number with the radius of the tables, if GD be reduced to 1000th parts of EG, and found in the table of tangents, the quantity of the angle will be known without calculation.

EXAM.

EXAM. 1. Let the distance of the poles EG be 10 feet, and the distance between the level point G and the sliding piece D on the long pole be  $17\frac{1}{4}$  inches ; it is required to find the quantity of the angle AEB?

Here  $17\frac{1}{4}$  inches is 1.4791 feet ; and, because the radius is 10 feet, 1.4791 feet in 100000th parts of the radius is .14791, which, in the tables, is the natural tangent of  $8^{\circ} 25'$ , the angle sought.

2. Let the distance of the poles be 20 feet, and GD on the longest pole 7 feet 9.75 inches ; required the angle AEB ?

7 feet 9.75 inches is 7.8125 feet ; to make this 1000000 parts of the radius, divide 20 feet and 7.8125 feet both by 20, and GD is .390625, which is the natural tangent of  $21^{\circ} 20'$ , the quantity of the angle AEB.

The long pole may have only one sliding piece upon it ; and, when that is the case, having placed the poles at a proper distance, turn the sights into the level line, and move the slider until the fiducial edge rests at G in the level, and note down the height of it ; then turn the sights towards the high object, and move the sliding piece until you see the fiducial edge of it, and the top of the high object in a right line, and note down the height ; then the difference of the heights of G and D will be known, and consequently the angle may be found as before.

The longest pole may have a sliding piece in it, to be extended upwards when there is occasion ; and, when that is necessary, the slider may be moved up only so far, that the top of the pole and of the high object may appear through the sights in a straight line ; and then the  
sliding

Sliding mark will be of no other use but to find the point G on the long pole in the level-line with E, the top of the shortest pole.

The long pole may be divided only into feet and inches; for the odd parts being taken with a pair of compasses, and applied to a scale, the distance between the sliding marks will be known in 100th parts of an inch; and this will be sufficiently exact for any purpose.

When the angle to be measured is small, as most of those are which serve to find the altitude and horizontal level of rising grounds, hills, mountains, &c. the distance of the poles may be 20 feet or more; but, when a large angle is to be measured, as in taking the altitude of the sun, or any star, the distance of the poles must be less.

When the angle is less than  $45^\circ$ , the distance of the poles may be 10 feet at least; but, if the angle be above  $45^\circ$ , either the distance of the poles must be less than 10 feet, or the longest pole must be lengthened by a sliding piece, or by fixing another pole on the top of the former. The distance of the poles should never be much less than 10 feet, nor much greater than 20 feet.

If the angle be under  $12^\circ$ , and the distance of the poles 10 feet, the 100th part of an inch on the long pole is the tangent of about 20 seconds; and, if the distance of the poles be 20 feet, the 100th part of an inch is the tangent of about 8 seconds; but, if the angle be  $45^\circ$ , or above, the 100th part of an inch is the tangent of 10, or 4 seconds, according as the distance of the poles is 10 or 20 feet.

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*To measure a verticle angle by the Poles on sloping ground: that is, when the ground slopes from the longest towards the shortest Pole. Fig. 1 and 2. Plate 5.*

If the top of the shortest pole be in a level line with the under part of the longest, measure from C to D, and from the square of CD subtract the square of the shortest pole ED; the square root of the remainder is the level distance of the poles EC.

If the level line meets the longest pole in some point n, subtract Cn from ED, and from the square of CD subtract the square of the remainder Dn, and the square root of the last remainder is EC, the level distance of the poles. By repeating this operation, the level distance between any two stations may be found, although measured on sloping ground. The rest of the operation is the same as before.

The instruments (now in use) for measuring angles, consist of a circle, or some part of it, divided into degrees; an index, moveable about the center, and sights of some kind or other for viewing distant objects; with a contrivance to fix it on a fulcrum; or foot.

*To measure a horizontal angle ABC, Fig. 41. with any circular instrument.*

Place poles at A and C. Set the instrument on its foot, so that its center be at the angular point B, and truly level. Set the index to the beginning of the degrees; then turn about the instrument until through the sights you see the pole at A; fix the instrument in this position, and turn the index about, until through the sights you see the pole at C; and the number of degrees passed

## MENSURATION.

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passed over by the index is the measure of the angle ABC.

*To measure any vertical angle with a circular instrument.*

Set the plane, or face of the instrument, truly perpendicular to the plane of the horizon, and the index truly level; then turn the index until through the sights you see the top of the high object; and the number of degrees passed over by the index is the measure of the vertical angle.

*Heights and Distances*, which are the subject of the following problems, are either accessible or inaccessible. Terrestrial objects, such as trees, towers, or other high buildings, hills, mountains, &c. are accessible in certain situations, and inaccessible in others. Celestial objects, such as the sun, moon, and stars, are always inaccessible, on account of their distance.

Having shown how any angle, either horizontal or vertical, may be measured, I shall, in what follows, only point out the angles to be measured in the field, and leave the practitioner to his own judgment with respect to the means and manner of performance.

**PROB. I.** *To measure the height of an accessible object, such as a growing tree, a tower, or other high wall AB.*  
*Fig. 42.*

From B, the root of the height, measure any convenient distance to E, and there measure the angle AEB; then, in the right angled triangle ABE, the side EB and the angles are known; to find the side AB.

L. 2.

EXAM.

EXAM. Let the distance  $EB=366$  links, and the angle  $AEB=37^{\circ} 30'$ ; required  $AB$ ?

Answer. 280.84.

There are several other methods of measuring an accessible height, which may be applied with advantage on some occasions; such as,

1<sup>st</sup>, By its shadow. When the sun shines, fix a pole in the ground perpendicular to the plane of the horizon; measure the shadow of the pole, and of the object whose height is required, and also the height of the pole; and then,

As the length of the shadow of the pole,  
Is to the length of the shadow of the high object;  
So is the height of the pole,  
To the height of the object.

EXAM. Let the length of the shadow of the pole be 47 inches, and its height  $39\frac{1}{2}$  inches, and the shadow of a tree 79 feet 6 inches; what is the height of the tree? Ans. 66 8 feet.

N. B. This is the shortest and easiest method of measuring the heights of growing trees, or other accessible objects, when great exactness is not required.

2<sup>dly</sup>, By a mirror, Fig. 44. Place a plain mirror at E, and move back to D, chusing your station so, that your eye being at F, you may see the top of the high object A, at some particular point of the mirror E.

Then, because the angle of incidence AEB is equal to the angle of reflection FED, and the angles at D and B are right ones; the triangles FDE, ABE, are similar.

Measure



Measure BE, ED, and DF; and then, as  $ED:DF::EB:BA$ .

*Note.* In order to have a steady view, it will be proper to use a pole DF, and apply your eye to F.

EXAM. Let  $EB=248$  feet,  $ED=8$  feet, and the height of the eye  $DF=5$  feet; required AB? *Ans.* 155.

It may sometimes happen, that a man would wish to measure a height and a distance when he hath no instruments to measure angles. In such cases, an equilateral triangle made of a smooth board, having sights placed on its three angles at A, B, and C, and a plummet hung from C; also two of its sides bisected at D and E by indented lines, and a sight placed occasionally at D or E. If such a wooden triangle be made, each side about 12 inches long, and some means be thought of to fix it on the top of a staff, either in a horizontal or vertical position, many problems may be solved by it. *Fig. 3. plate 5.*

In order to apply the triangle to the solution of problems, it must be known, that half the side AD is to the perpendicular DC, as 1000 is to 1732; and that the side AC is to the perpendicular CD, as 1000 is to 866.

*To measure the altitude of an accessible object, such as a rock, tower, tree, &c. by an equilateral triangle. Fig. 42.*

Find the station E by trials, where the base of the triangle being placed in the level line, by the plummet, you can see the top of the high object A through the sights, placed on the side of the triangle; then measure from your station E to the high object, and EB will be known. And as 1000 is to 1732, so is EB to BA, the altitude required.

EXAM.

**EXAM.** Let  $EB=96$  feet; required the height of the tower? *Ans.* 166.27 feet.

**PROB. 2.** *To measure the height of any inaccessible object AB, as also its distance CB. Fig. 45.*

Chuse two convenient stations C and D, from whence the highest point A can be seen, and measure the two angles ACB, ADB, and the distance of the stations C and D.

Then, because the angle ACB is equal to both the angles CAD and ADC, subtract the angle at D from the angle ACB, the remainder is the angle DAC. Now, in the oblique angled triangle ADC, all the angles, and the side DC, are given; to find AC. And then, in the right angled triangle ABC, the angles, and the hypotenuse AC, are given; to find AB and BC.

**EXAM.** Let the angle  $ACB=50^{\circ} 39'$ , the angle at  $D=33^{\circ} 30'$ , and the distance of the stations  $CD=360$  feet; required AB and BC? *Ans.*  $AB=521$ , and  $BC=427.2$ .

*To measure an inaccessible altitude AB by the equilateral triangle. Fig. 45.*

Find the station C by trials, where the base of the triangle being placed in the level-line, you can see the top of the high object through the sights on the side of the triangle; then find the station D, where the triangle being placed as before, you can see the top of the high object through the sights, which are placed, one at the extremity of the base, and the other in the middle of the opposite side at E. Measure CD, the distance of the stations,

stations, and  $CD$  is equal to  $CA$ ; which is therefore known. Then, as 1000 is to 8.6, so is  $CD$ , or  $CA$  to  $AB$ , the altitude; and, as 1732 is to 1000, so is  $AB$  to  $BC$ , the distance of the high object from the station  $C$ .

The only difficulty in applying the equilateral triangle, is to find the stations exactly, which a little practice will render easy: But, unless you can come so near the high object, that the angle  $ACB$  is  $60^\circ$ , the triangle cannot be applied.

**PROB. 3.** *To measure the height of a tower  $AE$ , standing on a high rock or mountain  $EB$ . Fig. 46.*

Take two stations  $C$  and  $D$ , from whence the points  $E$  and  $A$  can be seen, and find the quantities of the angles  $ADB$ ,  $ACB$ , and  $ECB$ ; and measure  $DC$ . Then calculate  $AC$ ,  $AB$ , and  $BC$  as in the last problem. Lastly, In the right angled triangle  $ECB$ , there is given  $CB$ , and the angle  $ECB$ ; to find  $BE$ ; which being subtracted from  $AB$ , the remainder is  $AE$ .

**EXAM.** Let the angle  $ECB = 50^\circ 10'$ ,  $ACB = 67^\circ 50'$ , and the angle at  $D = 43^\circ 40'$ , also the distance of the stations  $CD = 134$  feet; required  $AE$  and  $EB$ .

*Ans.*  $AE = 107.07$ , and  $EB = 102.22$ .

This problem may be solved by the equilateral triangle in the same manner as the last.

**PROB. 4.** *To find the perpendicular height of a hill  $AB$ , and reduce the slope side  $AC$  to the horizontal level  $BC$ . Fig. 47.*

If  $AC$  be such an ascent as can be measured with a chain; and, if the point  $A$  can be seen from  $C$ ; then  
measure

measure the angle  $ACB$ , and the slope side  $AC$ ; and, in the right angled triangle  $ABC$ , the hypotenuse  $AC$ , and the angle at  $C$  being known,  $AB$  and  $BC$  may be found, by Case 4th of right angled triangles. But, if  $AC$  cannot be measured with a chain, on account of cliffs, or other obstacles, it is an inaccessible height, and must be measured by the two last problems.

EXAM. Let  $AC=2496$  links, and the angle of ascent  $ACB=21^{\circ} 49'$ ; required  $AB$  and  $BC$ ? *Ans.*  $AB=927.6$ ,  $BC=2317.2$  links.

PROB. 5. *To measure the altitude of the Sun, Moon, or any Star.*

When the sun is visible, he will either appear in the horizon, where he hath no altitude, or in the zenith, (as in the torrid zone at noon day) where his altitude is  $90^{\circ}$ ; or somewhere between the zenith and horizon, and then the number of degrees between the sun's center and horizon at any time of the day, is called his *altitude*: Or, the sun's altitude is the angle contained between one right line drawn from some point in the plane of the horizon, and another right line drawn from the same point to the center of the sun. This angle is measured the same way as any other vertical angle; but here there is need for a greater degree of accuracy, than in measuring other angles; for, unless the sun's altitude be taken true to half a minute, the observation can be of no use in finding the latitude of the place, the hour of the day, or the sun's azimuth, which are the usual *quæsitæ* depending on the sun's altitude.

To perform the problem without an instrument to measure an angle, chuse some high wall, such as the end of a house; take a pole nearly equal to the height of your eye, and, by trial, find out a station, where the pole being set perpendicular to the plane of the horizon, your eye applied to the top of the pole, may see the top of the wall, and the upper limb of the sun in one right line. Then find a point in the wall on a level with the top of the pole. Measure the distance of the pole from the wall, and the height of the wall above the pole: And then,

As the distance of the pole from the wall,  
Is to the height of the wall above the pole;  
So is radius,

To the tangent of the sun's altitude.

From which subtract 10' for the sun's semidiameter; the remainder is the apparent altitude of the sun's center.

This problem may be solved by the two poles, described in pages 77, 78, 79, *et seq.*

**PROB. 6.** *To measure the distance between any two objects A and B, the point A being accessible. Fig. 48.*

Chuse some convenient station C, from whence the objects A and B can be seen, and place poles at A and C. Measure the angles BAC, ACB, and the distance of the stations AC.

The sum of the angles BAC, ACB, being subtracted from  $180^\circ$ , the remainder is the angle at B; then, in the triangle ABC, the angles and the side AC being known, the side AB may be found by calc. 1<sup>st</sup>, of oblique angled triangles.

M.

EXAM.

**EXAM.** Let the angle  $BAC=69^{\circ} 24'$ , and the angle  $ACB=75^{\circ} 24'$ , and the distance of the stations  $AC=496$  feet; required the distance  $AB$ ? *Ans.* 832.68.

*N. B.* When it is not convenient to measure the angles  $BAC$ , and  $ACB$  directly, their supplements  $DAC$ ,  $ACF$ , may be measured.

2. The distance of the stations  $A$  and  $C$  must bear some reasonable proportion to  $AB$ . It ought not to be less than  $\frac{1}{5}$  or  $\frac{1}{10}$  of it, if possible.

This is the general method of measuring a distance; but there are other methods which may answer very well in particular cases, especially when a small distance is to be measured, such as the breadth of a river.

*To measure a distance, accessible at one extremity, by four poles. Fig. 48.*

Place poles at  $A$  and  $C$ , so that  $BAC$  may be a right angle. Measure  $AC$ , and take  $AD$  in the same right line with  $AB$ , equal to one chain, or any convenient length, and place a pole at  $D$ . Place another pole at  $E$ , making  $ADE$  a right angle; then  $DE$  will be parallel to  $AC$ . Take  $DE$  equal to  $AC$ , and measure forward in the same right line with  $DE$ , until you come to the point  $F$ , where you see the pole at  $C$ , and the point  $B$  in the same right line; and then by similar triangles, as  $FE:EC::CA:AB$ .

**EXAM.** Let  $AC=426$  links,  $AD$  or  $CE=100$  links, and  $EF=57$  links; required the distance  $AB$ ? *Ans.* 747.3 links.

2dly, *The same thing may be done by an equilateral triangle. Fig. 49.*

Place the triangle horizontally at A, so that along one side of it you can see the point B; and, in the direction of the other side, place a pole at C; then measure from A to C, (the point C being found by trial), where the triangle being placed horizontally, you can see the point A along one side of it, and the point B along the other. Then ABC is an equilateral triangle; and so AB is equal to the measured distance AC.

If the perpendicular distance between the line AC and the point B be required: Having placed the triangle at the point A as before, and marked out the line AC by a pole, measure along that line until you find the point E, where the base of the triangle being placed in the direction of AC, you can see the point B through the sight. E in the middle of the base, and that on the opposite angle; and then, as 1000 is to 1732, so is AE to EB, the distance required.

3dly, *By measuring the sides of a triangle equal to the triangle ABC. Fig. 50.*

Place poles at A and C, so that BAC be a right angle; measure AC, and in the same right line measure CD equal to AC; and from D, set off DE at right angles to DC. Measure along DE until you come to the point E, in a right line with C and B; then ED is equal to AB, which therefore is known. This method is good enough in small distances. But, when the distance AB is large, take CD equal to some part of AC, such as  $\frac{1}{4}$ ,  $\frac{1}{5}$ , or  $\frac{1}{10}$ ; and then DE being measured, and multiplied by 4, 5, or

10, the product will be equal to AB. . This method will apply to any distance within your view; and, if you measure EC, you may find the distance CB by the same kind of operation. In this manner, the diagonals, or sides of fields, may be found out, when obstacles hinder the actual mensuration.

PROB. 7. *To measure an inaccessible distance AB from two stations C and D, from whence its extremities can be seen. Fig. 51.*

Measure the angles ACD, BCD, CDB, and CDA, with the distance of the stations CD. Then, in the triangle ACD having the two angles ACD, ADC, and the side CD, we can find the side AC; and, in the triangle BCD, having the two angles BCD, BDC, and the side CD, we can find BC. Lastly, in the triangle ABC, having the two sides AC and CB, with the contained angle ACB, we can find the angles CAB, ABC, and the side AB.

EXAM. Let the angles  $ACD=100^{\circ} 30'$ ,  $BCD=38^{\circ}$ ,  $CDB=108^{\circ}$ ,  $CDA=42^{\circ} 30'$ , and  $ACB=62^{\circ} 30'$ , and let the distance of the stations  $CD=398$  links; required AB?  
*Ans.* 615 links.

*To measure an inaccessible distance AB by the equilateral triangle. Fig. 4, plate 5.*

Place the equilateral triangle horizontally at E, so that you can see the point B along one side of it; and, in the direction of the other side, mark out the line EDCF by poles, and leave a pole at E. Then measure from E until you come to the point D, where EDB is a right angle,



angle, and mark down the length of ED. Measure forward in the same right line until you come to the point C, where ACD is a right angle, and mark the length of DC. Measure on in the same line until you come to the point F, (which must be found by trials), where the triangle being placed horizontally, one side of it in the direction of FCD, you can see the point A along the other side of it, and mark down the length of CF. Then, if FC be equal to ED, the distance CD is equal to AB; which therefore is known; but, if FC be not equal to ED, calculate DB and CA thus; as 1000 is to 1732, so is ED to DB; and so is FC to CA. Take the difference between AC and BD, and the square of this difference added to the square of CD, gives the square of AB, which, by extracting the square root, will be known.

This method of measuring an accessible distance is both easy and accurate, and may be applied with success to the distances of mountains, and the sides of fields, where the ground does not admit of applying the measure to each side; but is not so proper for large distances.

The equilateral triangle has been proposed as the most simple instrument, which can be made in a short time, with very little trouble; but, if we use a square, and divide two of its sides into some number of equal parts, we may solve problems more conveniently.

#### DESCRIPTION of the GEOMETRICAL SQUARE.

*Fig. 5. plate 5.*

The *Geometrical Square* may be made of wood, brass, or any other matter, either solid throughout, or consisting of four plain rulers joined together at right angles.

A is

A is the center, from which hangs a plummet; each of the sides BE, DE, is divided into 100 or 1000 equal parts; C and F are two sights fixed on the side AD; there is also an index AL, which turns round the center A; and thereon are two sights K and L. The side DE is called the upright side, and BE the reclining side of the square.

*To measure an accessible height AB by the Geometrical Square. Fig. 6, plate 5.*

Let BR be a horizontal plane, on which the high object AB stands perpendicular; and let BP be the distance of the observer=96 feet; and let the height of the observer's eye be 5 feet.

Set the square perpendicular to the horizon, so that the eye being at D, and looking through the sights, sees A the top of the high object; and the plummet hanging freely, suppose the thread cuts off 80 equal parts from the side, viz. DN. Then the triangles LDN, ADC being similar, as  $ND:DL::DC:CA$ ; that is, as  $80:100::96:120=CA$ .

But, if the plum-line fall on the angle opposite to the center, as at the station E; then because LD is equal to DP, DC is equal to CA; for the triangles LDP, DCA are similar; therefore, CA will be known by measuring DC.

If the distance of the observer BF be greater than the altitude, such as 300 feet, then the plum-line will cut the side of the square in N: Suppose  $NE=40$  equal parts, the triangles LNE, DCA, are similar; for the angles LEN, DCA, are right, and the angle LNE

$\angle ENE$  is equal to the angle  $DAC$ , and consequently the angle  $ELN$  is equal to the angle  $ADC$ ; therefore, as  $LE:EN::DC:CA$ ; that is, as  $100:40::300:120$ ; to which add 5 feet, the height of the observer's eye, and  $AB=125$  feet.

*To measure a distance AF, accessible at one extremity, by the Geometrical Square. Fig. 7. plate 5.*

Place the square horizontally on a support at A, so that through the sights on the side of the square you see the point F; turn the index, until it coincides with the other side of the square, and in the direction of that side place a pole at B, any convenient distance from A. Leave a pole at A, and measure AB. Then place the square horizontally at B, so that through the fixed sights you see the pole at A; and turn the index until through the sights on it you see the point F. Suppose the index cuts the right side of the square in G, then the triangles  $BDG$ ,  $BAF$  are similar; and, as  $BD:DG::BA:AF$ .

But, if a large distance, as AH, is to be measured, the square being placed at B, the index will cut the reclined side of the square, suppose at L; then the triangles  $BLI$ ,  $BAH$ , are similar, for the angles at A and I are right; the angle  $BLI$  is equal to the angle  $ABH$ , and the angle  $IBL$  to the angle  $AHB$ ; therefore, as  $LI:IB::BA:AH$ .

If the square be well made, and properly applied, most problems may be solved by it; but, because it requires the same trouble and expence to fix the square on a support, and to set it level or perpendicular to the horizon as any circular instrument, without any peculiar advantage, it is now almost out of use.

PROB.

**PROB. 8.** *To measure the distances of any number of inaccessible objects from each other, by means of two stations, from whence the objects can be seen.*

The method of performing this is in all respects the same as in problem 7th, p. 92; for, it is only taking a greater number of angles at each station, and solving a greater number of triangles.

**EXAM.** Suppose A, B, C, *Fig. 52.* are three inaccessible objects, whose distances from each other are required, and E and D two stations, whose distance is known, and from whence the objects A, B, and C can be seen. The angles being measured, suppose them to be,

*At the station D.*

$$ADB = 29^{\circ} 15'$$

$$BDC = 39^{\circ} 30'$$

$$CDE = 51^{\circ} 45'$$

$$ADE = 120^{\circ} 30'$$

*At the station E.*

$$CEB = 40^{\circ} 30'$$

$$BEA = 25^{\circ} 30'$$

$$AED = 46^{\circ} 45'$$

$$CED = 120^{\circ} 45'$$

And the distance of the stations  $DE = 740$  links; required AB, BC, and AC? *Ans.*  $AB = 1243$ ,  $BC = 1702$ , and  $AC = 2822$ .

**PROB. 9.** *To measure the distance of any inaccessible high object from a given station, and its altitude above the level of that station.*

Find out the distance by problem 6th; then measure the angle of ascent at the given station, and, by means of this, and the distance before found, calculate the height.

**EXAM.** Let BE, *Fig. 53.* be any high object, and A the given station from whence it can be seen; chuse another station C; then measure the angle  $BAC = 75^{\circ} 12'$ , and the angle  $ACB = 64^{\circ} 29'$ , and let the distance of the stations

stations  $AC=425$  links. At A measure the angle of ascent  $EAB=11^{\circ} 19'$ ; required the distance AB, and height BE? *Ans.*  $AB=592.7$ , and  $BE=118.6$ .

**PROB. 10.** *The distances of three objects A, B, C, in the same plane being given, and the angles observed at a fourth place as at the station S; to find the distances of these three objects from that station. Plate 5. Fig. 8, 9, &c.*

**CASE 1.** If the station S be taken without the triangle ABC in one of its sides produced. *Fig. 8. plate 5.*

Find the angle ACB by the 4th Case of oblique triangles, and then the angle ACS will be known: Now, in the triangle ASC, the angles ACS, CSA, and consequently the angle CAS, with the side AC being given, the sides SA, SC, and consequently SB, may be found by Case 1st of oblique triangles.

**CASE 2.** If the station S be in one side of the triangle ABC, as in AC. *Fig. 9.*

Find the angle ACB as before; and, in the triangle SBC, the angles BCS, CSB, and the side BC being known, SB, SC, and consequently SA, may be found. To find the point S, make the angle CAD equal to the observed angle CSB, and through B draw BS parallel to AD.

**CASE 3.** If the three objects lie in a right line, as A, C, B. *Fig. 10.*

1. Suppose a circle described through A, B, and the station S. From S draw the right line SC, which will meet the circle in the point D; join BD, DA, AS, and SB. Then, in the triangle ABD, all the angles and the side AB are known; for the angle ABD is equal to the  

N
observed

observed angle  $ASC$ , because they stand on the same arc  $AD$ ; and the angle  $BAD$  is equal to the angle  $CSB$  for the same reason: Hence the side  $AD$  may be found.

2. Now, in the triangle  $ACD$ , the sides  $CA$ ,  $AD$ , and the contained angle  $CAD$ , are known; to find the angle  $ACD$ , the supplement whereof is the angle  $SCA$ ; and then in the two triangles  $ASC$ ,  $BSC$ , all the angles, and a side in each triangle are known, from which  $SA$ ,  $SC$ , and  $SB$ , may be found.

**CASE 4.** If the station  $S$  be without the triangle  $ABC$ , but not in any of its sides produced; describe a circle through the points  $A$ ,  $B$ ,  $S$ , as before. *Fig. 11.*

1. Since the angle  $ASC$  is equal to the angle  $ABD$ , and  $BSC$  equal to  $BAD$ ; in the triangle  $ABD$  all the angles and the side  $AB$  being known,  $AD$  may be found.

2. All the sides of the triangle  $ABC$  being known, by these find the angle  $CAB$ ; hence the angle  $CAD$  will be known.

3. In the triangle  $ACD$ , the sides  $CA$ ,  $AD$ , and the contained angle  $CAD$  are given; by which find the angles  $ACD$ ,  $ADC$ , and then the angle  $ACS$  will be known.

4. In the triangle  $ACS$ , all the angles, and the side  $AC$  are given, to find the sides  $SA$  and  $SC$ .

5. In the triangle  $ABS$ , all the angles, and the side  $AB$  are given, to find  $SB$ .

**CASE 5.** If the station  $S$  be within the triangle  $ABC$ . *Fig. 12.*

1. A circle being described through  $A$ ,  $S$ ,  $B$ , as before, the supplement of the angle  $CSB$  is equal to the angle

angle BSD ; that is, BAD ; and the supplement of the angle ASC is equal to the angle ASD, or ABD ; hence the angle ADB will be known.

2. In the triangle ABD, all the angles, and the side AB are known ; from which the side AD may be found.

3. In the triangle ABC, calculate the two angles BAC, ACB, from the three given sides, and the angle CAD will be known.

4. In the triangle ACD, the sides CA, AD, with the contained angle CAD, are given ; and by these the angle ACS may be found ; which being subtracted from the angle ACB, there will remain the angle SCB.

5. In the triangle ACS, all the angles, and the side AC being known, the sides AS and SC may be found ; and, in the triangle CSB, the side CS and the angles being known, SB may be found.

CASE 6. If the station S be so situated with respect to the objects A, B, C, that the point D in the circumference of the circle is not in a right line with S and C, as in the three last cases ; suppose a circle described through the station S, and two of the given objects as B and C ; then make the angle CBD equal to the observed angle CSA, and make the angle BCD equal to the supplement of the angle ASB ; draw ADS and SC. *Fig. 13.*

1. Since the angle BDC is equal to the observed angle BSC ; in the triangle BCD, all the angles, and the side BC, are known ; by which the side BD may be found.

2. Calculate the angle ABC, and subtract it from the angle CBD, the remainder is the angle ABD ; and in the triangle ABD the two sides AB, BD, and the angle

N. 2

ABD.

$ABD$  being known; the angles  $BAD$ ,  $ADB$ , may be found: Hence the angle  $SDB$ , which is equal to the angle  $SCB$ , will be known:

3. In the triangle  $ABS$ , the angles and the side  $AB$  being known, the sides  $AS$  and  $SB$  may be found.

4. In the triangle  $SCB$ , the angles, with the side  $BC$ , are known; from which the side  $SC$  may be found.

Q. E. I.

**PROB. II.** *Let  $B$  and  $C$  be two objects whose distance  $BC$  is known; and let  $A$  and  $E$  be two stations from whence these objects can be seen, and the angles  $BAC$ ,  $BAE$ ,  $AEB$ , and  $AEC$  measured; it is required to find the distances  $AB$ ,  $AC$ ,  $AE$ , and  $EC$ ? Plate 5. Fig. 14.*

Since in each of the triangles  $ABE$  and  $ACE$  two angles are known by observation, the third angle in each triangle will be known.

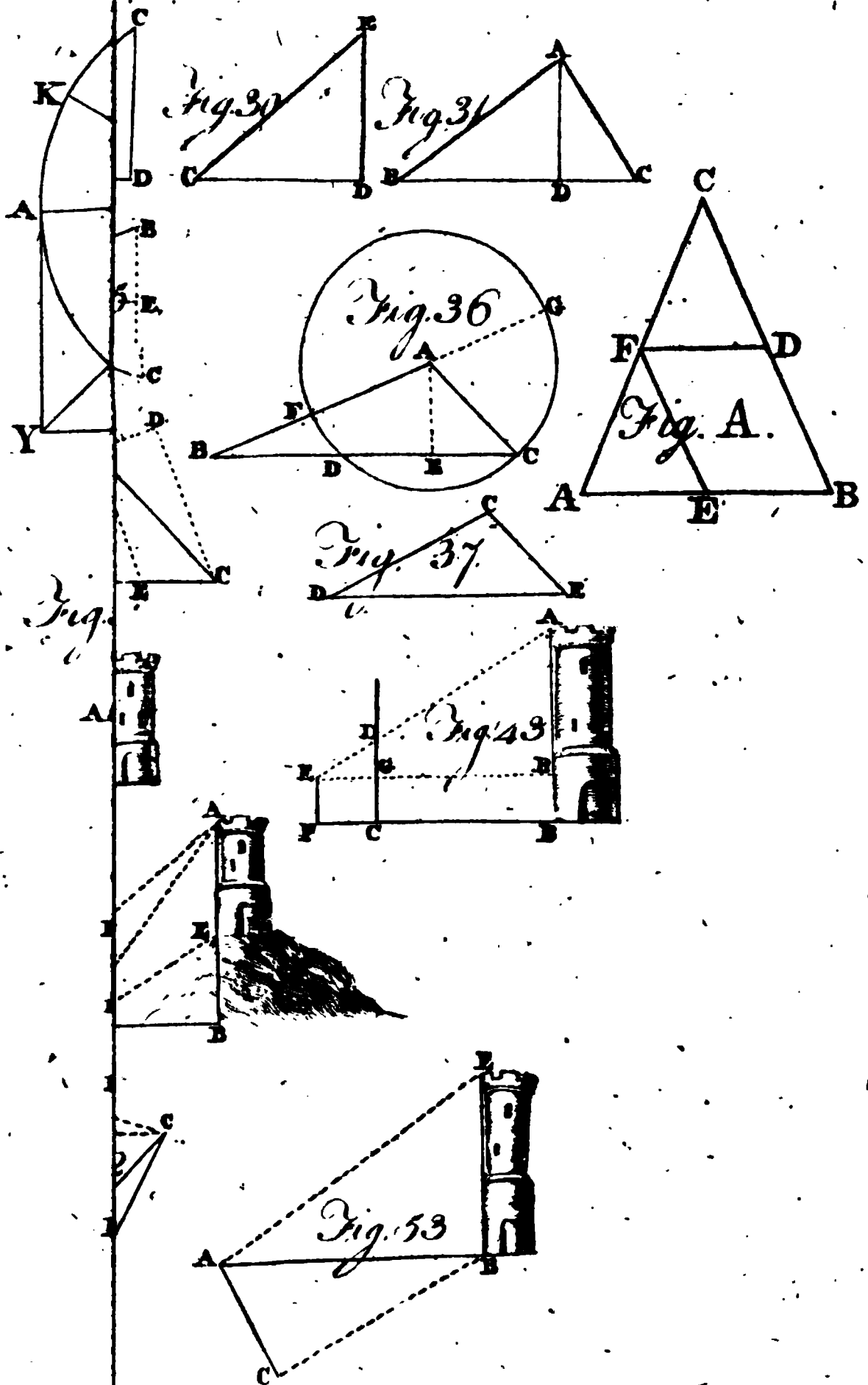
Take a right line  $GH$  at pleasure, Fig. 15. and thereon constitute the triangles  $GHD$ ,  $GHF$ , equiangular to the triangles  $EAB$ ,  $EAC$ ; join  $DF$ . Then take  $BC$  of the given length, and upon it constitute the triangles  $BCA$ ,  $BCE$ , equiangular to the triangles  $DFH$ ,  $DFG$ ; and the figure  $ABCE$  is similar to the figure  $HDFG$ .

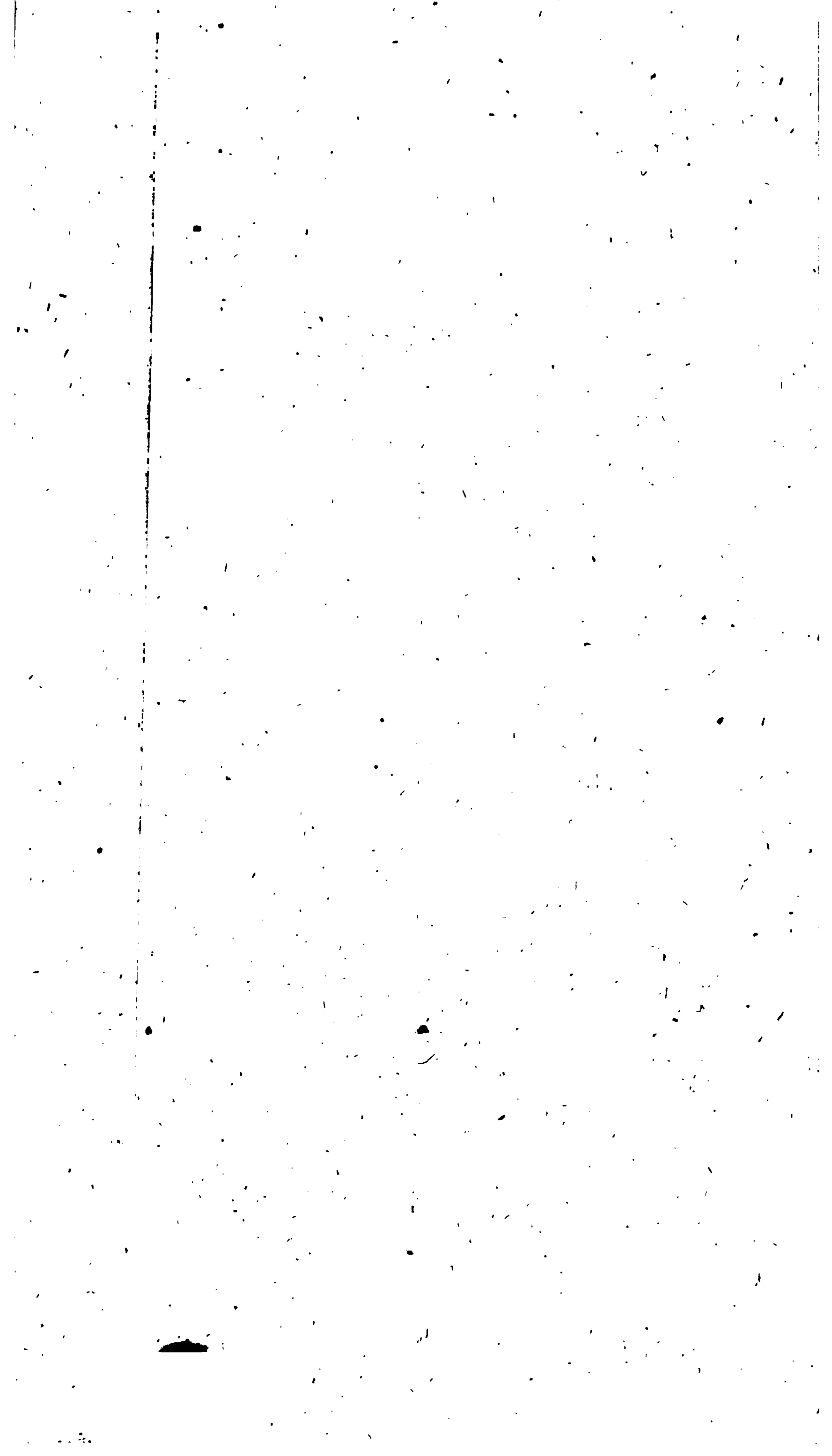
Calculate  $HD$ ,  $HF$ , and  $FD$ , as in Problem 7th, and then, as  $DF$  is to  $BC$ , so is  $DH$  to  $BA$ ; and so is  $HF$  to  $AC$ , and  $HG$  to  $AE$ ; and, in the triangle  $ACE$ , there is sufficient data to find  $CE$ .

MIS-



PLATE 2 Page no.





MISCELLANEOUS PROBLEMS.

1. Suppose a man stands on the Terraqueous Globe, the height of his eye being 6 feet; and his sensible horizon free of all obstructions to the sight; it is required to find the extent of his view in every direction? Fig. 54.

Let CA be the semidiameter of the earth = 21024960 feet, and CB represent the same, increased by the given height of the eye, and consequently = 21024966: Let BA be the visual ray meeting the curve surface of the earth in A. Now, supposing the earth to be a perfect globe, the triangle ABC is right angled at A, and the curve line EA will be nearly equal to AB, the tangent.

The square of AB is equal to the difference of the squares of BC and CA.

The square of CB = 442049195301156

The square of CA = 442048943001600

The difference is  $\overline{AB}^2 = 252299556$  the square root of which is 15883 feet = 3 miles 43 feet; the extent of a man's view on a plain horizon, the height of his eye being 6 feet. Hence it is evident, that a man whose eye is 6 feet high, standing on a plain, such as a frozen sea, can see another man of the same height, at the distance of 6 miles 86 feet.

By a like calculation, if the height of the eye be 12 feet, the extent of the view will be 4 miles 447 yards; and such a spectator would see an object of the same height at the distance of  $5\frac{1}{2}$  miles.

IF

If the height of the eye be 24 feet, the extent of the view will be 6 miles 29 yards; and such a spectator would see another object of the same height at the distance of 12 miles 58 yards.

If the height of the eye be 48 feet, as on the mast of a ship at sea, the extent of the view would be 8 miles 895 yards; and a spectator in this position would see an object of the same height with his eye, at the distance of 17 miles 30 yards.

*2. To measure the distance of any object by an explosion, such as the firing of a cannon.*

It has been found by many experiments, that the motion of sound is about 1142 feet in one second of time; and hence large distances may be measured pretty exactly, thus: Let there be two men, one placed at each extremity of the distance to be measured, and let one of them fire off a great gun, and the moment that the other sees the flash, let him look on a watch which shows the seconds until he hears the report. Then multiply 1142 by the number of seconds, and the product is the distance in feet.

If a watch be not at hand, set a pendulum of  $39\frac{1}{2}$  inches long a-going, and count the vibrations, or number of times it crosses the perpendicular, and this will show the number of seconds.

By this method, we can measure the distance of thunder from the place where we are, by counting the seconds between the time of seeing the lightning and hearing the report.

3. *To measure the velocity of running waters.*

Put a piece of cork, or other light matter, into the water, and place a mark in the bank where the cork was put in. Then follow the cork as it goes down the stream for one minute, or any other space of time; and place another mark in the bank at the place where the cork is at the end of the time. Measure the distance between the marks, and then, as the time the cork was in the water is to one hour, so is the space between the marks to the velocity of the water in an hour.

4. *To measure the velocity of the wind.*

When the sun shines, observe when the shadow of a cloud is at some particular place on the ground, and then count the seconds as it goes along until it come to some other place which you can remember. Measure the distance between the two places, and then say, as the number of seconds is to one hour, so is the distance measured to the velocity of the wind in one hour.

*N.B.* The greatest velocity of the wind is near 60 miles *per* hour.

O.F. LEVELLING.

Those who understand the mensuration of heights and distances, will not find much difficulty in levelling, as it consists in finding the height or depression of any part of the earth's surface above or below any other part. I shall, therefore, only explain some things whereon the practice depends, and conclude with an example.

The earth is a round body, and, in the business of levelling, may be considered as a perfect globe.

All heavy bodies, such as earth, water, and air, gravitate towards the center of the earth.

A level

A level is either true or apparent.

A true level is any part of the earth's surface, every point of which is equally distant from the center, such as a body of smooth water. Any line drawn in such a surface, is called a *true level line*, and is therefore an arc of a great circle, described on the terraqueous globe.

An apparent level, is properly the sensible horizon, or a plane touching the earth's surface in one point, viz. in the place where the spectator stands. Any right line drawn in this plane, is the line of apparent level, and is a tangent to the curve, which is a line of true level passing through that place. Hence, the lines of true and apparent level coincide only in one point; and in every other point will be distant from one another, more or less, according to their extent. But, as the earth is a great globe, the lines of true and apparent level will lie very near each other in small distances.

PROB. *To find the distance between the apparent and true level at the distance of one mile from the point of contact. Fig. 54.*

In the right angled plain triangle ABC, there is given the side CA, the semidiameter of the earth = 3982 miles, and the side AB = 1 mile; to find the hypotenuse CB.

The square of CA = 15856324

The square of AB = 1

Sum of the squares of CA and AB = 15856325

The square root of which is 3982.0001255 = CB.

Therefore, the apparent level at the distance of one mile from the observer's station, is higher than the true level by .0001255 of a mile, or very near 8 inches.

A TABLE,

**A TABLE**, showing the number of Feet, Inches, and Decimals of an Inch, which are to be subtracted from the apparent, to find the true level for the several distances therein mentioned.

Miles.	F.	inches.	Miles.	F.	inches.	Miles.	F.	inches.
$\frac{1}{4}$	0	0.49	9	53	8.4	20	265	2.3
$\frac{1}{2}$	0	2.	10	66	3.48	21	292	4.5
$\frac{3}{4}$	0	4.47	11	80	2.56	22	320	10.6
1	0	7.95	12	95	5.6	23	350	8.6
2	2	7.8	13	112	0.5	24	381	10.5
3	5	11.6	14	129	11.33	25	414	4.37
4	10	7.3	15	149	2.	26	448	2.
5	16	6.9	16	169	8.68	27	483	3.77
6	23	10.57	17	191	7.2	28	519	9.3
7	32	5.60	18	214	9.6	29	557	6.8
8	42	5.10	19	239	4.	30	596	8.2

To find what must be subtracted for any distance not found in the table. Multiply 6629832 by the square of the distance in miles. The product is the number of feet; and decimals of a foot, to be subtracted from the apparent level to find the true.

*Note.* 6629832 feet is the quantity to be subtracted from the apparent level, at the distance of one mile

The instrument commonly made use of in levelling, is a little glass tube filled with spirits of wine, all to a very little space, which contains a small quantity of air; and, when the tube is perfectly level, the bubble of air rests in the middle of it. To this level are fitted sights of some kind or other. A little telescope, with cross hairs in its focus, is best.

In levelling, two poles of ten or twelve feet length, divided into inches, and tenths of an inch, with pieces of  

O

paper

paper or pasteboard fitted to them, to slide up or down, are necessary. On the pasteboard should be drawn a horizontal black line, which may be seen through the sights of the levelling instrument, in order to adjust the height or depression exactly.

**EXAM.** Let *ABCD*, *Fig. 55.* represent an uneven ground; it is required to find how much the point *D* is higher than *A*?

Chuse a station *B* between *C* and *A*, and there place your levelling instrument. Place poles in *A* and *C*, and directing the sights to the pole in *A*, move the pasteboard up or down, until the horizontal black line appear in the level. Then direct the sights to the pole in *C*, moving the pasteboard as before, until the horizontal black line appear through the sights. Measure the heights of the horizontal black lines on the pasteboards above the ground, and their difference is the height of *C* above *A*. In the same manner, take some station *E* between *C* and *D*, and find the height of *D* above *C*; the sum of these two is the height of *D* above *A*. Proceed in the same manner when more stations are necessary, remembering to reduce the apparent to the true level, when the distance is above 400 yards.

*N. B.* The several stations may be chosen according to conveniency. It is not necessary that they be all in the same right line.

**SECT.**



## S E C T. IV.

## MENSURATION OF SURFACES.

**A**S every length or distance is measured by a line of some determinate length, expressed in inches, feet, yards, or decimal parts of itself, as the surveying chain : So every superficies, or magnitude of two dimensions, is estimated by some known square, as a square inch, foot, yard, chain, or mile.

If we grant the common postulate, that magnitudes may be generated by motion, *viz.* a line by the motion of a point, and a surface by the motion of a line, we shall easily discover those properties of right lined figures, on which the mensuration of surfaces depend.

Let AB, *Fig. 56.* be a right line of 5, and AD of 3 equal parts, such as feet, inches, or any other measure, containing the right angle BAD ; and let AD be supposed to move along the line AB until it comes to the several positions EF, GH, KL, MN, and BC, and by this motion generate the right angled parallelograms AF, EH, GL, KN, and MC. It is evident at sight, that each of these parallelograms contains three equal squares, that is, as many as AD contains equal parts ; and that the number of parallelograms is equal to the number of equal parts in AB. Therefore, the whole rectangle ABCD contains 3 times 5, or 15 equal squares, whose common side is one of the equal parts in AB or AD. Or the rectangle contains a number of squares equal to the product of the length multiplied by the breadth.

O 2

Hence,

Hence, the reason of the common rule for finding the area of a rectangle is abundantly evident.

And, because effects are always proportional to their causes; it may be inferred, that parallelograms generated by equal right lines are equal, in whatever position the generating right lines proceed, provided that these lines move equal distances from their first position.

*Fig 57.* Let the right line BC generate the parallelograms ABCD, EBCF, by moving from its first position BC, until it rest in AD or EF; these parallelograms must be equal, as being produced by the same cause, acting in the same circumstances.

Hence, the rule for finding the area of any parallelogram, is,

1. Multiply the base by the perpendicular height; the product is the area in squares of the same name with the dimensions taken.

Draw the diagonal DB, *Fig. 56.* and the triangle DAB is equal to the triangle DCB; for, the parallelogram being cut in two by the line DB, the two triangles properly applied will coincide. Hence, every triangle is half of its circumscribing parallelogram; and the rule for finding the area of any triangle is, Multiply the base by the perpendicular height, and half the product is the area. Or,

2. Multiply the base by half the perpendicular, or the perpendicular by half the base; the product is the area.

The mensuration of all plain surfaces depends on these few principles. For every surface bounded by right lines, is either a parallelogram or a triangle, or a figure which can be reduced to triangles, by drawing  
lines

lines within it ; and therefore, the rules for finding the area of the parallelogram and the triangle, will serve for finding the area of any right lined figure whatever.

And the areas of figures, whose boundaries are curve lines, such as the circle, are computed upon the same principles. The particular rules given for computing them, are derived from the supposition, that they consist of a great number of very small rectilineal figures, which, taken all together, approach infinitely near to the true area ; and therefore the mensuration of all plain surfaces depend on, and arise from, the rules for measuring the parallelogram and triangle.

Plain surfaces, which men have occasion to measure, are those of timber and stone ; as in buying and selling of wood and marble, or computing the extent of artificers work ; such as masons, brick-layers, carpenters, and painters work ; or some part of the earth's surfaces, as in buying, selling, and letting of land.

In timber, stone, and artificers work, the measures are commonly taken in feet, inches, and parts of an inch, by a foot-rule, yard, or tape rolled up in a box. In surveying of land, the measures are taken in links, of the surveying chain, already described.

I. TABLE of English Lineal measures.

12 Lines	} make	1 Inch
12 Inches		1 Foot
3 Feet		1 Yard
2 Yards, or 6 feet		1 Fathom
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet		1 Pole, or perch
4 Poles, or 66 feet		1 Chain
10 Chains, or 40 poles		1 Furlong
8 Furlongs, or 80 chains,		1 Mile
3 Miles		1 League.

A com-

A common pace is  $2\frac{1}{2}$  feet. A geometrical pace is 5 feet. 5280 feet, or 1760 yards, is one English mile.

## 2. TABLE of Scots Lineal Measures.

37 English inches	} make {	1 Ell
6 Ells, or $18\frac{1}{2}$ feet		1 Fall, or rood
4 Falls, or 74 feet		1 Chain
10 Chains, or 40 falls		1 Furlong
8 Furlongs, or 80 chains		1 Scots mile

*Note.* 6 ells is a rood in mason-work.

If the Scots surveying chain be  $74\frac{2}{3}$  English feet, as it ought to be, the Scots mile will be 5952 feet, or longer than the English mile by 672 feet.

## 3. TABLE of English Square Measures.

144 Square lines	} make {	1 Square inch
144 Square inches		1 Square foot
9 Square feet		1 Square yard

In Scotland, 36 square ells is one rood of mason-work.

## 4. ENGLISH Land Measures.

9 Square feet	} make {	1 Square yard
$30\frac{1}{4}$ Square yards		1 Square pole, or perch
40 Square poles		1 Square rood
4 Square roods		1 Square acre.

## 5. SCOTS Land Measures.

9.5 Square feet	} make {	1 Square ell
36 Square ells		1 Square fall
40 Square falls		1 Square rood
4 Square roods		1 Square acre

*Note.* The feet mentioned in this Table are English feet.

In

In computing the areas of plain surfaces, the most general and best method, is, to reduce the fractional parts to decimals ; and, by working properly, the area will be found in squares and decimal parts, of the same name with the dimensions. But several artificers chuse rather to take the dimensions in feet, inches, and 12th parts of an inch, or lines ; and, by multiplying these, get the area in square feet, and 12th parts of a square foot, (which are improperly called inches), and the lower denominations are 12th parts, each of the next above it. This operation is commonly called Cross Multiplication, and depends on the following rules :

1. Feet multiplied by feet produce square feet.
2. Feet multiplied by inches, and the product divided by 12, give square feet ; and the remainder is 12th parts of a square foot, here called inches.
3. Feet multiplied by lines, and the product divided by 144, give square feet ; the remainder divided by 12, gives 12ths of a square foot ; and the second remainder 12ths of the former, here called lines.
4. Inches multiplied by inches, and the product divided by 12, give 12ths of a square foot ; the remainder is lines.
5. Inches multiplied by lines, and the product divided by 144, give 12ths of a square foot ; the remainder divided by 12, gives lines, and the second remainder, 12ths of these.
6. Lines multiplied by lines, and the product divided by 12, give 12ths of a line ; the remainder is 12ths of the former.

**EXAM.**

EXAM. Multiply 29 feet 11 inches by 13 feet 10 inches.

F.	I.		
29	11	$13 \times 11 = 143$	$10 \times 11 = 110$
13	10	$29 \times 10 = 290$	and $\frac{110}{12} = 9.2$
<hr/>			
87		$12)433$	
29		<hr/>	
36	1	36	1
	9 2		
<hr/>			

Prod. 413 10 2

1. Multiply 29 by 13, the product is square feet.
2. Multiply 13 by 11, and 29 by 10, and divide the sum of the products 433 by 12, the quotient is 36 square feet, and  $\frac{1}{12}$  of a square foot.
3. Multiply 11 by 10, and divide the product by 12, the quotient is 9 inches 2 lines. The several articles being properly placed and added, the whole product is 413 square feet  $\frac{10}{12}$  of a square foot, and  $\frac{2}{12}$  of  $\frac{1}{12}$  of a square foot, which, in the tradesmens language, is 413 square feet, 10 inches, 2 lines.

EXAM. 2. Multiply 27 feet, 9 inches, 4 lines, by 19 feet, 8 inches, 11 lines.

F.	I.	L.	
27	9	4	$19 \times 9 = 171$
19	8	11	$27 \times 8 = 216$
<hr/>			
143			$12)387$
27			<hr/>
32	3		32
2	7	1	3
	6	0	
		10 11	
		3 8	
<hr/>			

Prod. 548 5 0 2 8

F. I. L.

$$\begin{array}{r} 144)373(2 \quad 7 \quad 1 \\ \underline{288} \end{array}$$

$$12)85$$

$$7 \quad 1$$

$$8 \times 9 = 72 \text{ and } \frac{7}{12} = 6 \text{ inc.}$$

$$8 \times 4 = 32$$

$$9 \times 11 = 99$$

$$12)131$$

$$10 \quad 11$$

$$11 \times 4 = 44 \text{ and } \frac{4}{12} = 3 \quad 8$$

3. Multiply 39 feet, 10 inches, 9 lines, by 19 feet, 9 lines. Product, 760 feet 6 2 0 9.

4. Multiply 31 feet 7 inches, by 9 feet 4 inches 6 lines. Product, 296 feet 1 1 6.

5. Multiply 28 feet 10 inches, 9 lines, by 19 feet, 2 inches, 3 lines. Product, 554 feet 5 3 2 3.

6. Multiply 49 feet 2 inches, by 39 feet 5 inches. Product, 1937 feet 11 10.

7. Multiply 12 yards, 2 feet, 11 inches, by 13 feet, 9 inches, 5 lines. Product, 59 yards 5 feet, 5 5 7.

Otherwise. Reduce each term to inches, and multiply the one by the other, the product will be square inches; which may be reduced to square feet or yards by the table. If there be lines in both or either term, reduce them to decimals of an inch.

EXAM. 1. Multiply 25 feet 6 inches, by 18 feet 9 inches.

25 feet 6 inches = 306 inches, and 18 feet 9 inches = 225 inches.  $306 \times 225 = 68850$  square inches, which, being divided by 144, the quotient is 478 square feet 18 inches.

P

2. Mul-

2. Multiply 31 feet 7 inches, by 9 feet, 4 inches, 6 lines.

$379 \times 112.5 = 42637.5$  square inches, which being reduced, is 296 square feet 13.5 square inches.

1. *To measure any right angled quadrilateral figure, and compute its area.*

Measure its length and breadth, and multiply the one by the other; the product is the area in squares of the same denomination with the measures of the sides.

What is the area of a square, each of its sides being 63 yards, 1 foot, 9 inches?

*Ans.* 4042 square yards, 7 feet, 81 inches.

Y.	F.	I.
63	1	9
<hr style="width: 10%; margin: 0 auto;"/>		
3		
<hr style="width: 10%; margin: 0 auto;"/>		
190		
<hr style="width: 10%; margin: 0 auto;"/>		
12		
<hr style="width: 10%; margin: 0 auto;"/>		
2289 inches.		
<hr style="width: 10%; margin: 0 auto;"/>		
2289		
<hr style="width: 10%; margin: 0 auto;"/>		
20601		
<hr style="width: 10%; margin: 0 auto;"/>		
18312		
<hr style="width: 10%; margin: 0 auto;"/>		
4578		
<hr style="width: 10%; margin: 0 auto;"/>		
4578		
<hr style="width: 10%; margin: 0 auto;"/>		
44)5239521(36385.		
<hr style="width: 10%; margin: 0 auto;"/>		
Rem. 81	4042	7 81
	yds.	ft. in.

In this and such like questions, the best method of calculation is to reduce the given number, or numbers, to the lowest denomination in either, and then multiply according to the rule; the product is the area in squares of that denomination, which may be brought to a higher denomination by the Tables.

#### *More EXAMPLES of SQUARES.*

1. What is the area of a square, each of its sides being 12 feet?

2. What is the area of a square whose side is 16 feet 10 inches?

*Ans.* 283 square feet 52 inches.

3. Re-



3. Required the area of a square, its side being 12 yards 2 feet 7 inches ?

*Ans.* 165 square yards, 3 feet, 97 inches.

4. Required the area of a square pavement, each side of it being 21 feet, 9 inches, and 10 lines ?

*Ans.* 476 square feet  $12\frac{25}{10}$  inches.

OF RECTANGLES.

1. The length of a grass-plot is 75 yards 2 feet, and its breadth 49 yards  $1\frac{1}{2}$  feet ; what is its area ?

*Ans.* 3 roods, 3 poles, 25 yards.

Y.	F.	Y.	F.	
75	2	49	$1\frac{1}{2}$	148.5 :
3		3		227
<hr/>		<hr/>		<hr/>
227		148.5		10395
				2970
				<hr/>
				9)33709.5
				40
				30.25) 3745.9
				(123
				<hr/>
				Rem <sup>r</sup> . 25.15
				3 3 25

2. The floor of a room is 25 feet 9 inches long, and 18 feet 11 inches wide ; what is its area ?

*Ans.* 487. feet 15.

3. The circumference of a room is 90 feet 11 inches, and its height 9 feet 8 inches ; what is the area of its walls in square yards ? Multiply the circumference by the height.

*Ans.* 97 yards, 5 feet, 124 inches.

4. The length of a dry stone dyke is 5280 feet 9 inches, and its height 4 feet 7 inches ; how many roods of

of mason-work are in it? and, what does the building come to, at 1cs.  $7\frac{1}{4}$ d. *per* rood?

*Ans.* 74 roods, 25 yards, &c. and the price comes to L. 39 : 13 :  $8\frac{1}{2}$ .

The following questions occur frequently in business :

1. What length of a deal, whose breadth is  $9\frac{1}{4}$  inches, will make a square foot?

Divide 144 by  $9\frac{1}{4}$ , the quotient 15.56 inches, is the answer.

2. The length of a pavement is 47 feet 9 inches, and its breadth 13 feet 11 inches; how many stones, each 17 inches long, and  $10\frac{1}{2}$  inches broad, will serve to pave it? and, what will the expence amount to, at  $7\frac{3}{4}$ d. *per* square foot?

Find the area of the pavement, as also of one stone in inches, and divide the former by the latter; the quotient is the number of stones.

*Ans.* The number of stones is 536; and the whole expence comes to L. 21 : 9 : 2.

3. The length of a house is 50 feet 9 inches, and its breadth 23 feet 5 inches, within the walls; how many deals, each 18 feet 10 inches long, and  $9\frac{1}{4}$  inches wide, will floor it? *Ans.*  $81\frac{3\frac{1}{4} \times 97}{87}$  or 82 deals nearly.

4. Suppose the length of a house is 40 feet 9 inches, and the sloping height of the roof above the walls 19 feet 5 inches; how many slates will cover the roof, supposing each slate to cover  $17\frac{1}{2}$  square inches?

*Ans.*  $13021\frac{1}{3}$ .

or

*Of Oblique Angled PARALLELOGRAMS.*

2. *To measure any oblique angled parallelogram, and compute its area.*

Measure one side, and the perpendicular falling from the opposite angle upon that side, and then multiply the one by the other, the product is the area.

EXAM. 1. What is the area of an oblique angled parallelogram, its base being 49 feet 3 inches, and the perpendicular falling from the opposite angle 36 feet 6 inches; also its acute angle, being  $70^{\circ}$ ?

*Ans.* 199 yards, 6 feet, 90 inches.

2. How many square yards are there in an oblique angled parallelogram, its base being 37 feet 10 inches, and the perpendicular height 28 feet 9 inches; also its acute angle being  $63^{\circ} 30'$ ?

*Ans.* 120 yards, 7 feet, 102 inches.

OF TRIANGLES.

3. *To measure any triangular plain surface, and compute its area.*

Measure one side for a base, and also the perpendicular which falls from the opposite angle upon that base; and then multiply the base by half of the perpendicular; or, multiply the perpendicular by half of the base; the product is the area.

EXAM. 1. What is the area of the triangle ABC, (Fig. 31.) its base BC being 49 feet 6 inches, and the perpendicular AD 42 feet 9 inches; also the segment BD=36 feet?

*Ans.* 117 square yards, 5 feet 9 inches.

2. What is the area of the upper part of the gavel of a house, the breadth of the house being 23 feet 9 inches,  
and

and the perpendicular height of the gavel above the side-walls being 14 feet 6 inches?

*Ans.* 19 square yards, 1 foot, 27 inches.

4. To find the area of a plain triangle, its three sides being given.

Add the three sides, and take half of the sum; from the half sum subtract each side; then multiply the half sum by the three remainders continually, and extract the square root of the product; the root is the area of the triangle.

*Demonstration, Fig. 31.*

Put  $b=AB$ ,  $a=BC$ , and  $c=AC$ . From  $A$  let fall the perpendicular  $AD$ , and put  $x=BD$ ; then  $a-x=CD$ .

$$b^2 - x^2 = c^2 - (a-x)^2 = AD^2$$

that is  $b^2 - x^2 = c^2 - a^2 + 2ax - x^2$

$$2ax = b^2 - c^2 + a^2 \text{ and } x = \frac{b^2 - c^2 + a^2}{2a} \text{ Substi-}$$

tute the square of this for  $x^2$  in the first step, and we

$$\text{have } b^2 - \frac{b^4 + 2b^2c^2 - 2b^2a^2 - c^4 + 2a^2c^2 - a^4}{4a^2} = AD^2$$

$$\text{that is } \frac{2a^2b^2 + 2b^2c^2 + 2a^2c^2 - b^4 - c^4 - a^4}{4a^2} = AD^2$$

$$\text{therefore } \frac{\sqrt{2a^2b^2 + 2b^2c^2 + 2a^2c^2 - b^4 - c^4 - a^4}}{2a} = AD$$

This multiplied by  $\frac{a+b+c}{2}$ , gives the area of the triangle, viz.

$$\frac{\sqrt{2a^2b^2 + 2b^2c^2 + 2a^2c^2 - b^4 - c^4 - a^4}}{4}$$

$$\sqrt{\frac{a+b+c}{2} \times \frac{a+b-c}{2} \times \frac{a+c-b}{2} \times \frac{b+c-a}{2}}$$

which is the words of the above rule expressed in symbols.

EXAM.

## O F S U R F A C E S .

49

**Exam.** Let the three sides of a triangle be 186, 276, and 146 feet; required the area?

$186 + 276 + 146 = 608$ , whose half is 304.

From 304 subtract each side, the three remainders are 118, 28, and 158.

$304 \times 118 \times 28 \times 158 = 158697728$ , whose square root is 12597.52 square feet, the answer.

By logarithms. Add the logarithms of the half sum, and three differences together, and take half of the sum; the number answering to this is the area of the triangle.

Numb.	Log.
304	2.4828736
118	2.0718820
28	1.4471580
158	2.1986571

2)8.2005707

*Ans.* 12597.52    4.1002853

2. What is the area of a triangular field whose three sides are 1900, 1620, and 1230 links of the Scots Surveying chain?

*Ans.* 987540 square links.

3. Required the area of a triangular field, its three sides being 136, 247, and 296 yards in English land measure?

*Ans.* 16673 square-yards, or 3 acres, 1 rood, 31 poles.

## O F T R A P E Z I U M S .

A trapezium is a quadrilateral figure, whose opposite sides and angles are unequal.

When two opposite sides of a trapezium are parallel, as ABCF, (*Fig. 57.*) its area may be found thus: Multi-  
ply

ply the half sum of its parallel sides by the perpendicular distance between them, the product is the area.

EXAM. Let  $AF=42$  yards,  $BC=34$  yards 2 feet, and  $CD=30$  yards, 1 foot, 4 inches; what is the area of the trapezium  $ABCF$ ?

*Ans.* 1167 yards, 48 inches.

5. To find the area of any trapezium.

Divide it into two triangles by a diagonal line, to which, draw perpendiculars from the opposite angles; measure the diagonal and the two perpendiculars, and multiply the diagonal, by the sum of the perpendiculars, and half of the product is the area of the trapezium.

*Note.* This rule is short and easy, and when it can be applied, is the best for practice.

EXAM. *Fig. 58.* Let the diagonal  $DB=236$  feet, the perpendicular  $Ce=137$  feet, and the perpendicular  $Af=95$  feet; what is the area of the trapezium  $ABCD$ ?

*Ans.* 27376 square feet.

EXAM. 2. Let the diagonal  $DB=349$  feet 9 inches; the perpendicular  $Ce=152$  feet 6 inches, and the perpendicular  $Af=112$  feet 2 inches; also  $Bc=94$  feet, and  $Bf=192$  feet. Required the area of the trapezium  $ABCD$ ? *Fig. 58.*

<i>F.</i>	<i>I.</i>	<i>F.</i>	<i>I.</i>
152	6	112	2
12		12	
<hr/>		<hr/>	
1830		1346	
1346			
<hr/>			
2)3176			
<hr/>			
1588			

# OF SURFACES.

321

$  \begin{array}{r}  F. \quad I. \\  349 \quad 9 \\  12 \\  \hline  4197 \\  1588 \\  \hline  33576 \\  33576 \\  20985 \\  4197 \\  \hline  144)6664836 \quad (46283 \quad 84 \\  \text{Rem. } 84  \end{array}  $	$  \begin{array}{l}  \text{Ans. } 5142 \text{ yds. } 5 \text{ ft. } 84 \text{ in.} \\  \\  5142 \quad 5 \quad 84 \\  \text{yd.} \quad \text{ft.} \quad \text{in.}  \end{array}  $
--	---

2. What is the area of a trapezium, whose diagonal is 119 yards, 2 feet, 6 inches; and the perpendicular, 50 yards, distant from one end of the diagonal, is 62 yards; also the perpendicular, 36 yards distant from the other end of the diagonal, is 84 yards, 2 feet, 3 inches?

*Ans.* 8792 yards, 6 feet, 135 inches.

Or, the area of any trapezium may be found thus: Measure the four sides and the diagonal, and find the area of each triangle from its three sides; the sum of these is the area of the trapezium.

EXAM. What is the area of a trapezium, whose sides are as follow, viz. AB=46, BC=37, CD=79, and DA=63 feet; also the diagonal AC=73 feet? *Fig.* 58.

*Ans.* 2823.52 square feet.

6. To find the area of an irregular polygon, or multi-lateral figure. Draw diagonals, and let fall perpendiculars on them from the opposite angles. Measure the diagonals and perpendiculars, and by these find the area of each triangle; the sum of these areas is the answer. Or,

Q.

Measure -

Measure all the sides and diagonals, and find the area of each triangle from its three sides; the sum of these is the answer.

This is the most proper method in surveying of land.

EXAM. *Fig. 59.* Let the diagonal  $AC=245$  feet,  $AD=208$  feet, the perpendicular  $Ef=110$  feet,  $Dg=130$  feet, and  $Bh=97$  feet; what is the area of the irregular polygon  $ABCDE$ ?

*Ans.* 39247.5 square feet.

2. Let  $AB=174$ ,  $BC=150$ ,  $CD=152$ ,  $DE=148$ , and  $EA=160$  feet; also the diagonals  $AC=246$ , and  $AD=220$  feet; it is required to find the area of the irregular polygon  $ABCDE$ ?

*Ans.* 41234 square feet.

7. To find the area of any regular polygon, such as a heptagon, hexagon, &c. Every regular polygon is equal to a plain triangle, whose base is equal to the sum of the sides of the polygon, and height equal to the perpendicular falling from the centre on any of its sides. Therefore, measure one of its sides, and the perpendicular falling from the centre on that side; and multiply half the sum of the sides by the perpendicular, the product is the area of the polygon.

EXAM. *Fig. 60.* Let the side of the hexagon  $AB=329$  feet, and the perpendicular falling from the center on that side  $CD=284$  feet; what is its area?

*Ans.* 280308 square feet.

2. What is the area of a pentagon, each of its sides being 45 feet 9 inches; and the perpendicular falling from the center on any side  $=31$  feet 8 inches?

*Ans.* 3621.875 square feet.

3. What



3. What is the area of an octagon, whose side is 13 yards, 1 foot, 3 inches; and the perpendicular falling from the center on any of its sides is 16 yards 6 inches?

*Ans.* 867 square yards, 5 feet, 72 inches.

OF THE CIRCLE, *Fig. 61.*

In all questions concerning the circle, it is considered as a regular polygon of a great number of sides, which, taken altogether, are equal to the circumference.

The length of a right line, nearly equal to the circumference of a circle, whose diameter is known, is found in this manner: A regular polygon of a great number of sides is supposed to be inscribed within the circle, and another of as many sides described about it. The circumference of the circle lying between both, is greater than the one, and less than the other; but the difference is very small; and therefore, the sum of the sides of each polygon being calculated, the arithmetical mean between them is taken for the true circumference.

By this method, it has been found, that the diameter being 1, the circumference will be 3.1415926535, or 3.1416, and its area .7853981633, or .7854.

To exemplify this in the plainest manner.

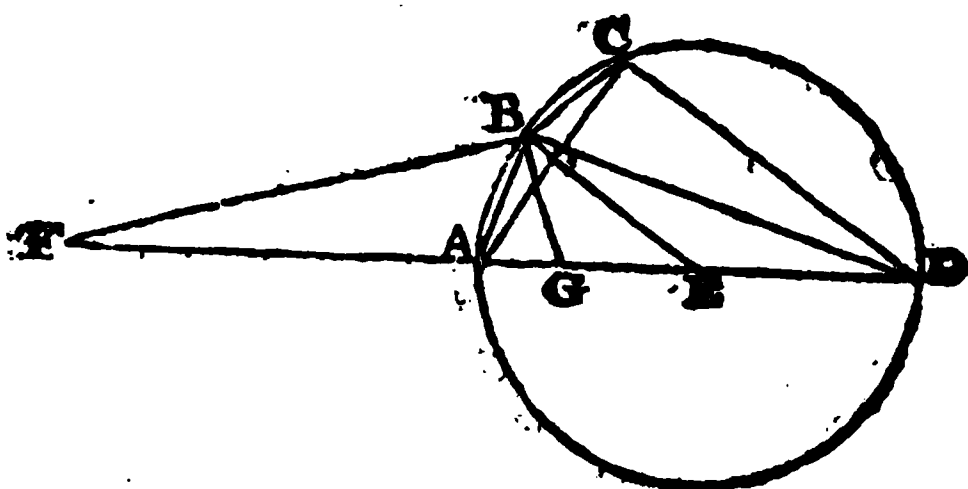
DEFINITION. If any point B be taken in the circumference of a circle ACD, and from that point straight lines BA, BD, be drawn to the extremities of the diameter; then AB is the chord of the arch AB, and BD is the supplemental chord.

The diameter of a circle being given to find the circumference. Let AD be the diameter, E the center, and AB, BC two equal arches: join AB, BC and CD;

Q 2

produce

produce DA to F, so that  $AF=DC$ , and draw the straight lines FB, BD: then DB is a mean proportional between ED and DF.



**DEMONSTRATION.** The triangle ABF is equal to the triangle CBD, for AF is equal to CD, AB to BC, and the angle FAB is equal to the angle BCD; the angle CDB is equal to the angle at F, and the angle CBD to the angle ABF. But the angle CDB is equal to the angle BDE or EBD, and therefore the triangle EDB is equiangular and similar to the triangle BDF. therefore as  $ED : DB :: DB : DF=AD+DB$ , then  $\overline{DB^2} = ED \times DF$  or  $\overline{DB^2} = \overline{ED \times AD + DB}$ .

If the radius  $ED = 1$ , then  $\overline{DB^2} = AD + DB$ , and  $DB = \sqrt{AD + DB}$ .

If DG be taken equal to DC; join BG, and the triangles EAB, BAG are similar; and  $EA : AB :: AB : AG = AD - DG$ .

Hence the chord of any arch is a mean proportional between the radius and the difference between the diameter and the supplemental chord of double the arch.

If the diameter  $AD = 2$ , the radius  $EA = 1$  then  $\sqrt{2 - CD} = BD$  the supplemental chord of half the arch AC: also  $\sqrt{2 - CD} = AB$  the chord of half the arch AC.

To

To find the circumference of a circle, its diameter being 2.

If AC be the side of a hexagon inscribed in the circle; then  $AC = \text{the radius} = 1$ , and the supplemental chord  $CD = \sqrt{3}$ ; for  $CD = \sqrt{DA^2 - AC^2} = \sqrt{4 - 1} = \sqrt{3} = a$ : also the supplemental chord of the arch AB, the 12th part of the circumference  $= \sqrt{2 + \sqrt{3}} = b$ . And by calculating the supplemental chord of the half of every succeeding arch, we shall have one as small as we please, and from thence the chord of this small arch, which will be the side of the polygon of as many sides as the denominator of the fraction contains units; then multiply this side by the number of sides, the product shall be the perimeter of the polygon, which is nearly the circumference of the circle.

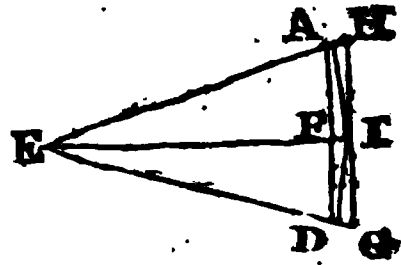
The steps of the operation are in the following Table:

<i>Parts of the</i>		
CIRCUMFERENCE.	SUPPLEMENTAL CHORDS.	
$\frac{1}{6}$	$\sqrt{3}$	$1.732050807568 = a$
$\frac{1}{12}$	$\sqrt{2 + a}$	$1.931851652557 = b$
$\frac{1}{24}$	$\sqrt{2 + b}$	$1.982889722742 = c$
$\frac{1}{48}$	$\sqrt{2 + c}$	$1.995717846472 = d$
$\frac{1}{96}$	$\sqrt{2 + d}$	$1.998929174951 = e$
$\frac{1}{192}$	$\sqrt{2 + e}$	$1.999732275818 = f$
$\frac{1}{384}$	$\sqrt{2 + f}$	$1.999933067834 = g$
$\frac{1}{768}$	$\sqrt{2 + g}$	$1.999983266888 = h$
$\frac{1}{1536}$	$\sqrt{2 + h}$	$1.999995816717 = k$

Add 2 to the supplemental chord  $k$  and subtract the sum  $3.999995816717$  from 4, the square of the diameter, and there remains  $.000004183283$ , which is the square of the side of a polygon of 3072 sides inscribed in

in the circle, and the square root of .000004183283 is .0020453071 the side of the interior polygon.

To find the side of a polygon of 3072 sides circumscribed about the circle. Let AID be the arch; EI the radius, AD the side of the inscribed polygon, and HG the side of the circumscribed polygon. The perpendicular  $EF = \sqrt{EA^2 - AF^2}$ .



From  $EA^2 = 1$ .

Subtr  $AF^2 = .0000010458207$

$FE^2 = .9999989541793$  then  $EF = .999999477$

As  $EF : EI :: AD : HG$  = the side of the exterior polygon; that is,  $.999999477 : 1 :: .0020453071 : .0020453082 = HG$ .

Interior side .0020453071

N<sup>o</sup> of sides 3072

40906142

143171497

613592130

perim. of the inter. 6.2831834112

— of the exter. 6.2831867904

2) 12.5663702016

6.2831851 = the circumf. of the circle, the diameter being 2; true to the 7th decimal place.

Exterior side .0020453082

N<sup>o</sup> of sides 3072

40906164

143171574

613592460

perim. of the exterior polygon. 6.2831867904

Hence

Hence the diameter of a circle being 1, the circumference is 3.1415925, or 3.1416 nearly; and its area .7854.

8. The diameter of a circle being given, to find its circumference.

Because the circumferences of circles are to each other as their diameters, and the circumference of a circle, whose diameter is 1, is 3.1416; therefore, as 1 is to 3.1416, so is the diameter of any circle to its circumference. And, because the first term of the proportion is 1, therefore, multiply 3.1416 by the given diameter, the product is the circumference.

EXAM. 1. What is the circumference of a circle, whose diameter is  $19\frac{1}{2}$  inches?

Ans. 61.2612 inches.

2. The diameter of the earth is 7964 miles; what is the circumference of one of its great circles?

Ans. 25019.7 miles.

9. The diameter of a circle being given, to find its area.

A circle is equal to a plain triangle, whose base is equal to the circumference, and its perpendicular height equal to the radius; but similar triangles are to each other as the squares of their correspondent sides; therefore, circles are to each other as the squares of their radii; or, (because the radius is half of the diameter), as the squares of their diameters. And, since the area of a circle, whose diameter is 1, is .7854; as the square of 1 is to the square of any diameter so is .7854 to the area; and, because the first term of the proportion is 1,

Multiply the square of any diameter by .7854, the product is the area of the circle.

EXAM.

**EXAM. 1.** What is the area of a circle whose diameter is 12 inches?

*Ans.* 113.0976 square inches.

2. What is the area of a circle whose diameter is 7964 miles?

*Ans.* 49814227.4784 square miles.

There are other numbers expressing the proportion of the diameter of a circle to its circumference. *Archimedes* found, that the diameter being 7, the circumference would be 22; and therefore, as 7 is to 22, so is the diameter of any circle to the circumference; and, if the area of the circumscribed square be 14, the area of the circle would be 11; and therefore, as 14 is to 11, so is the square of any diameter to the area of the circle. But these numbers are not very accurate.

*Metius* found, that the diameter being 113, the circumference would be 355; therefore, as 113 is to 355, so is any diameter to the circumference; which is nearer the truth than 7 to 22.

10. To find the length of any arc of a circle, the number of degrees in that arc, and the radius of the circle being given.

Multiply the number of degrees in the given arc, the radius of the circle, and the number .01745329 continually; the product is the length of the arc.

**EXAM.** Required the length of an arc of a circle of  $121^{\circ} 26'$ , the radius of the circle being 25.455?

*Ans.* 53.9496, &c.

*Note.* .01745329 is the length of an arc of 1 degree, the radius being 1.

11. The area of a circle being given, to find the diameter and circumference

Divide

Divide the given area by .7854; the square root of the quotient is the diameter; which, being multiplied by 3.1416, the product is the circumference.

EXAM. 1. What is the diameter and circumference of a circle whose area is 10000 square feet?

Ans. Diameter 112.8 feet. Circumference 354.372 feet.

2. What length of a cord tied to a cow's tail, the other end being fixed in the ground, will allow her to eat a Scots acre of grafs; the length of the cow and tail being  $3\frac{1}{2}$  yards?

Ans. 39.3 yards.

Note. A Scots acre of land is 5760 square yards.

12. The circumference of a circle being given, to find its area.

Multiply the square of the circumference by .0795775, the product is the area.

EXAM. 1. What is the area of a circle, whose circumference is 54 inches?

Ans. 232.04799 square inches.

2. What is the area of a circle whose circumference is  $42\frac{1}{2}$  inches?

Ans. 143.7368, &c.

To bring all the problems about the Circle into one view.

Put  $d$  = the diameter,  $p$  = the circumference,  $A$  = the area: and let  $a = 3.1416$ .  $b = \frac{a}{4} = .7854$ .

1.  $1 : a :: d : p$  or  $ad = p$ .

2.  $1 : b :: d^2 : A$  or  $bd^2 = A$ .

3.  $d = \frac{p}{a}$  or  $d = .3183 p$  because  $\frac{1}{3.1416} = .3183$ .

$$4. \quad a^2 d^2 = p^2, \text{ or } d^2 = \frac{p^2}{a^2}, \text{ that is } d^2 = .10132 p^2.$$

$$\text{for } \frac{1}{9.86965} = .10132.$$

$$5. \quad d^2 = \frac{A}{b} = 1.2732 A \text{ for } \frac{1}{.7854} = 1.2732.$$

$$6. \quad ad^2 = \frac{aA}{b} = \frac{p^2}{a} \text{ then } \frac{a^2 A}{b} = p^2, \text{ and by}$$

substituting  $\frac{a}{4}$  for  $b$ , we have

$$7. \quad \frac{4a^2 A}{a} = p^2 \text{ that is } 4aA = p^2; \text{ therefore}$$

$$8. \quad A = \frac{p^2}{4a} \text{ that is } A = \frac{p^2}{12.5664} \text{ or } .0795775 p^2 = A.$$

$$9. \quad d = \sqrt{\frac{A}{b}} = \sqrt{1.2732 A}.$$

$$10. \quad p = \sqrt{4aA} \text{ or } p = \sqrt{\frac{A}{.0795775}}.$$

13. To find the area of any sector of a circle, the length of the arc and the radius of the circle being given:

Multiply the radius by half the length of the arc, the product is the area of the sector.

EXAM. *Fig. 61.* It is required to find the area of the sector BCR, the length of the arc BC being 52.994555, and the radius RC=25.455?

*Ans.* 674.488, &c.

14. To find the area of the segment of a circle less than a semicircle, its base and height being given. *Fig. 61.*

Let CPO be the segment, whose area is to be found.

1. Divide the square of CI by OI, the quotient is IB, which, added to OI, gives BO, the diameter.

2. Having found the diameter, find the circumference of the circle by problem 9. p. 127.

3. In



3. In the right angled triangle CRI, all the sides are known, from which the angle CRI may be found, whose double is the angle CRP, or the arc COP.

4. As  $360^\circ$  is to the arch COP in degrees, so is the circumference of the circle to the length of the arch COP.

5. Multiply the radius RO by half of the arch COP, the product is the area of the sector CRPO.

6. Find the area of the triangle CPR, and subtract it from the area of the sector CRPO; the remainder is the area of the segment COP; but when the area of a segment greater than a semicircle is required, such as, CBP. To the area of the sector RCBP, add the area of the triangle CPR, the sum is the area of the segment CBP.

EXAM. Let the chord or base of the segment CP=44.4, and the versed sine or height OI=13; what is the area of the segment CPO? *Ans.* 410, &c.

Otherwise put  $r$ =the radius RO.  $c$ =half the chord =CI and  $d$ =RO—OI: and the area of the segment COP will be  $\frac{2\frac{1}{2}rr - 1\frac{1}{2}rd - dd}{1\frac{1}{2}r + d} \times C$

And the last example wrought by this rule comes out 409.49 square inches.

2. It is required to find the area of a segment of a circle, its base being 25.455, and its height 5.28?

*Ans.* 92.469.

15. To find the area of any zone of a circle included between two parallel chords; the diameter of the circle, the including chords, and the breadth of the zone being given. *Fig.* 61.

CASE 1. When the including chords are equally distant from the center, as GP and EF.

R 2

Since

Since  $IG$ , the breadth of the zone, is given,  $RI$  will be known; which being subtracted from the semidiameter of the circle, the height of the segment cut off,  $OI$  will remain: Therefore, find the area of the whole circle, and also the area of the segment  $COP$ : Subtract twice the area of the segment  $COP$  from the area of the circle, and there will remain the area of the zone  $CEFP$ .

**CASE 2.** When the zone is included by a diameter and any chord, as  $AD$ ,  $CP$ .

Find the area of the segment  $COP$ , and subtract it from the area of the semicircle, the remainder is the area of the zone  $ACPD$ .

**CASE 3.** When the including chords are unequally distant from the center, and both on the same side of it.

In this case, the distance of each of the including chords from the center of the circle must be known; and then having found the area of each of the segments cut off by the chords, the difference of these areas is the area of the zone.

**CASE 4.** When the including chords are unequal, and on different sides of the center.

The distance of each chord from the center must be known; and in this case compute the area of each of the segments cut off, and subtract their sum from the area of the whole circle, the remainder is the area of the zone.

The same rules will serve for computing the area of any part of a circle included between chord lines which are not parallel; the distance of each chord from the center being known.

**EXAM. 1.** Let the diameter of a circle be 16 inches; it is required to find the area of its middle zone, included

ed between two parallel chords equally distant from the center, each chord being 12.49 inches, and the breadth of the zone 10 inches?

*Ans.* 148.867 square inches.

2. Let the diameter of a circle be 25, it is required to find the area of a zone of it contained between two parallel, but unequal chords, lying on different sides of the center; the greatest chord being 20, and its distance from the center  $7\frac{1}{2}$ ; and the least chord 15, and its distance from the center 10?

*Ans.* 395.4369.

16. The diameter of a circle being given, to find the diameter of another circle, such, that the area of the given circle shall be to the area of the other in a given ratio, suppose of 2 to 3.

Because the areas of circles are to each other as the squares of their diameters; therefore, as 2 is to 3, so is the square of the given diameter to the square of the diameter of the circle required.

EXAM. Let the diameter of a circle be  $19\frac{1}{2}$  inches; it is required to find the diameter of another circle, whose area shall be to the area of the first circle in the ratio of 3 to 2?

*Ans.* 23.882 inches.

2. The area of a circle is 50 square feet; it is required to find the diameter of another circle, whose area shall be to the area of the given circle, as  $3\frac{1}{2}$  to 5?

*Ans.* The diameter is 6.675 feet.

## OF THE ELLIPSE.

*Definition 1.* An ellipse is a plain figure bounded by a curve line, like a circle; but has different properties.

*Fig. 24.*

2. There is a point in an ellipse called its center, through which its diameters pass; but every part of its circumference is not equally distant from this point, as in the circle.

3. All the diameters of a circle are equal to each other, but the diameters of an ellipse are unequal.

4. The ellipse hath two principal diameters, called axes, which cut each other at right angles in its center.

5. The longest diameter AB is called the transverse axis, and the shortest diameter GH is called the conjugate axis.

17. The two axes of an ellipse being given, to find its circumference.

1. Take half the sum of the two axes, and multiply it by 3.1416, the product is the circumference nearly. Generally too little.

2. Square the two axes, and multiply the square root of half their sum by 3.1416, and the product is the circumference nearly. Generally too much.

3. Take the half sum of these two products, it is the circumference, very near the truth.

*Exam.* Let the transverse axis  $AB = 31\frac{1}{4}$  inches, and the conjugate axis  $GH = 23\frac{3}{4}$  inches; what is the circumference of the ellipse?

*Ans.* 87.2114 inches.

AB

$$\begin{array}{r} AB = 31.5 \\ GH = 23.75 \\ \hline 2) 55.25 \\ \hline \end{array}$$

Half sum of the two axes  $\doteq 27.625$   
then  $27.625 \times 3.1416 = 86.7867$  the 1st product.

The square of AB  $= 992.25$   
the square of GH  $= 564.0625$

Sum of the squares of AB and GH  $= 1556.3125$

half ditto  $778.15625$

and the square root of  $778.15625$  is  $27.8954$   
now  $27.8954 \times 3.1416 = 87.63618864$  the 2d product  
to which add the 1st product  $86.7867$

$$\begin{array}{r} 2) 174.4228 \\ \hline \end{array}$$

*Ans.* The circumference is  $87.2114$  inches.

6. The area of any ellipse, is equal to the area of a circle, whose diameter is a mean proportional between the two axes of the ellipse. Or,

The rectangle, or product of the two axes of any ellipse is equal to the square of the diameter of a circle, whose area is equal to the area of the ellipse.

18. To find the area of an ellipse, its two axes being given. *Fig. 24.*

Multiply the longer axis by the shorter, and the product by  $.7854$ ; the last product is the area of the ellipse?

EXAM. Let the longest axis  $AB = 31\frac{1}{2}$  inches, and the shortest  $GH = 23\frac{3}{4}$  inches; what is the area of the ellipse?

*Ans.*  $587.577$  square inches.

## SURVEYING OF LAND.

Small parts of the earth's surface are estimated in acres, and parts of an acre.

A right angled parallelogram, whose length is 10 chains, and breadth 1 chain, or any space containing 10 square chains, or 100,000 square links, is an acre.

## TABLE of Scots Land Measure.

Sq. Links.    Sq. Feet.    Sq. Ells.

$$17\frac{5}{8} = 9\frac{7}{4} = 1$$

$$625 = 342\frac{1}{4} = 36 = 1 \text{ Sq. Fall.}$$

$$25000 = 13690 = 1440 = 40 = 1 \text{ Sq. Rood.}$$

$$100000 = 54760 = 5760 = 160 = 4 = 1 \text{ Sq. Acre.}$$

N. B. In this table the Scots ell is 37 inches, and the feet are English.

The square ell expressed in square links and decimal parts is 17.36111, or 9.50694 English square feet.

## TABLE of English Land Measure.

Sq. Links.    Sq. Feet.    Sq. Yards.

$$20\frac{8}{17} = 9 = 1$$

$$625 = 272\frac{1}{4} = 30\frac{1}{4} = 1 \text{ Sq. Pole.}$$

$$25000 = 10890 = 1210 = 40 = 1 \text{ Sq. Rood.}$$

$$100000 = 43560 = 4840 = 160 = 4 = 1 \text{ Sq. Acre.}$$

The English square yard expressed in square links, and decimals of a link, is 20.661157 square links of the English chain.

When the dimensions of a field are taken with the surveying chain, they are commonly expressed in links; and the area computed from these dimensions is square links.

To reduce square links to acres, roods, &c. If it be Scots measure, divide the square links by 100000, which  
may

may be done by pointing off five figures to the right hand. The figures on the left of the point are acres, and those on the right are decimals of an acre. Multiply these decimals by 4, 40, and 36; and, from each product, point off five figures to the right hand, and you have the roods, falls, and ells.

If it be English measure, divide the square links by 100000 as before, and multiply the remaining decimals by 4, 40, and  $30\frac{1}{4}$ ; and from each product point off five decimals, and you have the area in English acres, roods, poles or perches, and yards.

EXAMPLES.

1. To find the area of a rectangular field, bounded by four straight lines. *Fig. 56*

Measure its length and breadth, and multiply the one by the other.

What is the area of a rectangular field whose length is 2496 links, and breadth 1689 links of the Scots chain?

$$2496 \times 1689 = 4215744 \text{ square links.}$$

		<i>Ac.</i>	<i>R.</i>	<i>F</i>	<i>E.</i>
<i>Acres</i>	42.15744	<i>Ans.</i> 42	0	25	6
	4				
<i>Roods</i>	0.62976				
	40				
<i>Falls</i>	25 190.0				
	36				
	114240				
	57120				
<i>Ells</i>	6.85440				

**EXAM. 2.** What is the area of a rectangular field, whose length is 2798 links, and breadth 1894 links of the English chain?

$$2798 \times 1894 = 5299412 \text{ square links.}$$

	<i>Ac.</i>	<i>R.</i>	<i>P.</i>	<i>℥.</i>
<i>Acres</i> 52.99412	<i>Ans.</i> 52	3	39	℥
4				
<i>Roods</i> 3.97648				
40				
<i>Poles</i> 39.05920				
30 $\frac{3}{4}$				
177600				
1480				
<i>Yards</i> 1.79080				

3. How many acres are there in a square field whose side is 1976 links of the Scots surveying chain?

*Ans.* 39 acres 7 falls.

4. How many acres are there in a square field whose side is 2675 links of the English chain?

*Ans.* 71 acres, 2 roods, 9 poles.

5. What is the area of a rectangular field, the length being 1868 links, and the breadth 1496 links of the Scots chain?

*Ans.* 27 acres, 3 roods, 31 falls.

6. What is the area of a rectangular field, its length 2867 links, and its breadth 1984 links of the English chain?

*Ans.* 56 acres, 3 roods, 21 poles.

7. How



7. How many English acres are there in a rectangular field, its length being 2468 links, and breadth 1426 links of the Scots chain?

*Ans.* 35 acres, 30 poles, 29 yards.

8. How many acres are there in a square mile?

*Ans.* 640 acres.

For 80 chains, or 8000 links is a mile, and the square of 8000, is 64000000 square links, which is 640 acres.

The English acre is to the Scots acre as 1089 to 1369; or 1369 English acres are equal to 1089 Scots acres.

1. To reduce Scots land measure to English, and the contrary.

Because the English chain is 66 feet, and the Scots 74; as 66 is to 74, so is any number of Scots links to the number of English links, in the same line or length; and, as 74 is to 66, so is any number of English links to the number of Scots links, in the same line.

EXAM. 1. How many links of the English chain are equal to 2496 links of the Scots chain?

$66:74::2496:2798$  the answer.

2. How many links of the Scots chain are equal to 1894 links of the English chain?

$74:66::1894:1689$  the answer.

3. To reduce Scots acres to English acres; as 1089 is to 1369, so is any number of Scots acres to their equivalent English acres; and, as 1369 is to 1089, so is any number of English acres to their equivalent Scots acres.

EXAM. 1. Reduce 42.15744 Scots acres to English acres.

$1089:1369::42.15744:52.99681$  the answer;

or, 52 acres, 3 roods, 39 poles, &c.

2. Reduce 500 English acres to Scots acres.

$1369:1089::500:397.73559$  the answer;

or, 397 acres, 2 roods, 37 falls, 25 ells, nearly.

3. To reduce lineal links of the Scots chain to feet. As 100 is to 74, so is any number of links to the feet equal to them.

And to reduce links of the English chain to feet; as 100 is to 66, so is any number of links to the feet.

Lineal feet may be reduced to links of either kind, by inverting the proportions.

4. To reduce square feet to square links, Scots measure.

2500 square links are equal to 1369 square feet; therefore, as 1369 is to 2500, so is any number of square feet to square links.

In English measure, 2500 square links are equal to 1089 square feet; therefore, as 1089 is to 2500, so is any number of square feet to the square links.

5. When the dimensions of a field are taken in feet, and the area computed in square feet; to reduce them to Scots acres. Divide the square feet by 9.50694, or by 9.5, the quotient is square ells; divide the ells by 36, the quotient is falls; divide the falls by 40, the quotient is roods; and divide the roods by 4, the quotient is acres.

6. To reduce square feet to English acres. Divide them by 9,  $30\frac{1}{4}$ , 40, and 4, the quotients are square yards, poles, roods, and acres.

EXAM. What is the area of a rectangular space of ground, whose length is 596 feet, and breadth 156 feet, in Scots and English acres?

596 x

$596 \times 156 = 92976$  square feet.  
 $9.50694)92976.00000(9779$  square ells.  
 $36)9779(271$  falls.  
 remainder 23.  
 $4)6 \quad 31$   
*Ac. R. F. E.*  
*Ans. 1 2 31 23 Scots.*  
 $9)92976$   
 $30.25)10330 \quad 6$   
 $40)341 \quad 14$   
 $4)8 \quad 21$   
*Ac. R. P. Y.*  
*Ans. 2 0 21 14 Eng.*  
 $2 \quad 0 \quad 21 \quad 14$

2. How many Scots acres are there in a square field, each of its sides being 716 feet?

*Ans.* 9 acres, 1 rood, 18 falls, 35 ells.

3. How many English acres are there in a square field, its side being 596 feet?

*Ans.* 8 acres, 24 poles.

4. What is the area of a rectangular field, its length being 598 feet, and breadth 386 feet, in Scots and English acres?

*Ans.* 4 acres, 34 falls, 33 ells, Scots; and  
5 acres, 1 rood, 7 poles, 25 yards, English.

5. It is required to find the area of a rectangular field in Scots acres; its length being 374 ells, and breadth 275 ells?

*Ans.* 17 acres, 3 roods, 16 falls, 34 ells.

6. *Fig. 57.* What is the area of a field in form of an oblique angled parallelogram, whose longest side is 2368 links, and the perpendicular falling from the opposite angle

angle upon that side 1648 links of the Scots chain; the acute angle of the parallelogram being  $63^{\circ} 30'$ ?

*Ans.* 39 acres, 0 roods, 3 falls, 33 ells.

7. *Fig 31.* What is the area of a triangular field, whose base is 2896 links, and the perpendicular falling upon it from the opposite angle 1896 links of the English chain? See the rule, p. 117.

*Ans.* 27 acres, 1 rood, 32 poles.

8. What is the area of a triangular field whose base is 1768 links, and the perpendicular distant 795 links from the nearest extremity of the base, is 1674 links of the English chain?

*Ans.* 14 acres, 3 roods, 7 poles.

9. What is the area of a triangular field, whose three sides are 2464, 1968, and 1764 links of the Scots chain?

*Ans.* 17 acres, 33 falls. See page 118.

10. A farm of a triangular form, has its three sides  $2\frac{1}{2}$  miles, 3 miles, and  $1\frac{1}{2}$  miles; how many acres does it contain? *Ans.*  $1197\frac{1}{3}$  acres, nearly.

$$\begin{array}{r}
 2.5 \quad 3.5 \quad 3.5 \quad 3.5 \\
 3. \quad 2.5 \quad 3. \quad 1.5 \\
 \hline
 1.5 \quad \hline
 1. \quad .5 \quad 2. \\
 \hline
 2)7.0 \\
 \hline
 3.5
 \end{array}$$

$3.5 \times 2 \times .5 = 3.5$ , Extract the square root,  
 then  $\sqrt{3.5} = 1.870826$  square miles  
 multiply by 640

$$\begin{array}{r}
 74833040 \\
 11224956 \\
 \hline
 1197.328640 \text{ Acres.}
 \end{array}$$

11. The

11. The kingdom of England, and the principality of Wales together, are of a triangular form, the south coast being the base, and the north point of Northumberland the vertex of the triangle: Now, supposing the length from North to South is 360 miles, and the breadth from East to West is 300 miles: How many acres are in the whole?

$$\begin{array}{r}
 \text{Length } 360 \text{ miles} \\
 \text{Multiply by } \frac{1}{2} \text{ the base } 150 \\
 \hline
 \text{area } 54000 \text{ square miles} \\
 640 \\
 \hline
 2160000 \\
 324000 \\
 \hline
 \text{Ans. } 34,560,000 \text{ acres.}
 \end{array}$$

12. *Fig. 58.* What is the area of the quadrilateral field ABCD, whose diagonal BD=3096 links, the side AB=1976 links, BC=1760 links, CD=2016 links, and DA=2340 links?

*Ans.* 39 acres, 3 roods, 5 falls. See p. 120.

13. What is the area of a field of five sides, (*Fig. 59.*) the side AB being 1740, BC=1500, CD=1520, DE=1480, and EA=1600 links; also the diagonal AC=2460, and AD=2200 links?

*Ans.* 41.23654 square links, or 41 acres 37 poles.

14 To measure a polygonous field, such as ABCDEF, by the chain. *Fig. 62.*

Having walked over and taken a view of the field, draw a random, or eye-draught of it, in which draw proper diagonals.

Measure all the sides and diagonals with the chain, and the whole field is divided into triangles; in each of which

which all the sides are known from, whence their areas may be found, and consequently the area of the whole field.

**EXAM.** Let  $AB=2175$ ,  $BC=1925$ ,  $CD=1275$ ,  $DE=1675$ ,  $EF=1575$ ,  $FA=1725$ ,  $AC=3715$ ,  $FC=4085$ , and  $FD=3025$  links of the Scots chain; what is the area of the field? *Ans.* 69 acres, 1 rood, 37 falls.

8. To survey a field, by measuring every angle and side. *Fig. 63.*

Begin at some convenient angle, which call your first station, and mark it in your field-book thus,  $o^1$ .

Having chosen your first station, place poles in proper places, and measure the angle at this station; which write in your field-book below the former character.

When you begin to measure the sides with the chain, proceed with your left hand to the dyke, or boundary of the field; and, if the line which you measure does not coincide with the side of the field, you must measure the distance between them at proper places; these distances are called *offsets*, and must be entered in your field-book; and over against them, in the proper column, write down the distances from the beginning of the line you are then measuring.

In the same manner, measure every angle and side, until you have gone round the whole field, and are arrived at your first station.

To know if the angles have been measured truly. From twice the number of sides of the field, subtract 4, and multiply  $90^\circ$  by the remainder. Add together the angles of the field, as they stand in your field-book; and, if the sum be equal to the product, they have been truly measured.

IF

If there be any considerable difference, such as a degree or more, it must be corrected by measuring the angles again.

## Form of the FIELD-BOOK.

Offsets.		N. E. corner.
	95	
	0	
32	84° 50'	
0	400	
	1090	
	0	
	85° 15'	
	1339	
	0	
	68° 55'	
90	90	
113	130	
65	400	
25	888	Goes on 145, as in next offset.
145	0	
	121°	
0	328	
200	450	
232	558	
0	620	
pond.	700	
	956	
	0	

*To draw a Plan of the Field on Paper. Fig. 63.*

1. Draw the right line EAB. Set 95 links from E to A, and make the angle  $BAC=84^{\circ} 50'$ .

Having raised a perpendicular at the point A, on it set the offset 32 links from A to *b*; then set 400 links from E to *t*; and because you find 0 offset in the field-book, the line EB coincides with the boundary of the field from that place; set 1090 links from E to B.

2. At B make the angle  $ABD=85^{\circ} 15'$ , and set 1339 links from B to D.

3. At D make the angle  $BDC=68^{\circ} 55'$ . Set the several numbers of links found in your field-book, between  $o^3$  and  $o^4$ , from the point D on the line DC; and from the points *l*, *n*, *m*, raise perpendiculars, on which set the offsets as they stand in your field-book; and set 145 links from C to *f*.

4. Set the several numbers of links in the field-book between  $o^4$  and  $o^5$  from C towards A; and at the points *r* and *s*, raise perpendiculars, on which set the offsets 200 and 232 links. The extremities of the offsets being every where joined, will give the outlines, or boundaries of the field.

*To find the area of the Field.*

Join AD; and in the triangle ABD, there is given the side AB=995 links, the side BC=1339 links, with the angle  $ABD=85^{\circ} 15'$ ; and, by the third case of oblique triangles, the side AD will be 1600 links. Then, in each of the triangles ABD and ADC, all the sides are known; and so their areas may be found.

For



For the areas of the parts where the offsets were taken. Multiply  $Df$  by half of  $ma$ , the product is the area of the triangles  $Dfa$ . In like manner, find the areas of the other triangles  $fCg$ ,  $bgr$ ,  $iKs$ ,  $EKA$ , and  $Azh$ ; and in the quadrilateral figure  $hrrs$ ; multiply the half sum of the offsets  $rh$  and  $s$  by  $rs$ . The sum of the areas of all these different parts is the area of the field, which is near 11 acres 3 roods.

*Note 1.* In surveying a quadrilateral field, it is not necessary to measure all the angles. If one angle and the four sides are known, the diagonal can be calculated, and from thence the area.

2. Although a field has more than four sides, the greatest part of it may, by taking proper stations, be brought within four straight lines; and the rest measured by taking offsets. Hence,

The measuring of all the angles of a field, although much practised, and generally proposed by writers on this subject, is not necessary in surveying, except in cases where some of the sides are inaccessible.

When a four-sided field has two of its angles at the extremity of any side, equal to two right angles, but the other sides unequal, as in *Fig. 57*. the area may be found thus:

Having measured the sides, multiply the half sum of the two parallel sides by the breadth of the field, the product is the area; that is,  $\frac{AF + BC}{2} \times AB = \text{the area.}$

9. To survey any small field by measuring diagonals, and the perpendicular falling from the opposite angles on them. *Fig. 59.*

T 2

Having

Having made choice of the most proper diagonals, such as AC and AD, measure the diagonal AC, beginning at the point A ; and, when you come to the point *b*, measure the perpendicular *bB* ; also, when you come to the point *g*, measure the perpendicular *gD*, marking in your field-book, or on an eye-draught of the field, the lengths of *Ab*, *bB*, *Ag*, *gD*. In like manner, measure AD ; and, when you come to the point *f*, measure *fE*, marking down their lengths as before.

To find the area of the field, multiply AC by the half sum of *bB* and *gD* ; also multiply AD by half of *Ef*, and the sum of these products is the area.

In the same manner, a field of four, or any number of sides, wherein the ground is not incumbered with great ascents or descents, may be accurately measured.

The points *b*, *g*, and *f*, where the perpendiculars meet the diagonals, are to be found by trials ; or, more accurately, by a cross, which is a square board, whereon sights are placed at the extremities of two right lines cutting each other at right angles. This being fixed on a staff stuck in the ground, at the place where the perpendicular is supposed to meet the diagonal, by looking through one pair of sights in the direction of the diagonal, the other pair will be in a line perpendicular to it. When this line falls on the angle, the station is right ; but, if not, by moving the cross backward or forward, the point where the perpendicular falling from any angle on the diagonal may be accurately found.

This method of measuring fields is both accurate and easy, and therefore, preferable to any other, when it can be applied.

10. When

10. When two sides of a triangular field, AB and AC can be measured; but the third side BC is inaccessible; to find the length of that side, and the area of the field. *Fig. 70.*

Measure AB and AC, and take AD and AE in the same right lines produced; so that AD be to AE, as AB to AC, and measure ED; then, because the two triangles ADE and ABC are similar, as  $AD : DE :: AB : BC$ . And the three sides being known, the area of the triangle may be found.

By this method, the diagonal of any four-sided field may be found, without measuring it.

*Note.* In order to have AD and AE in just proportion to AB and AC, take AD equal to some part of AB, such as  $\frac{1}{2}$ ,  $\frac{1}{3}$ , or  $\frac{1}{5}$ ; and take AE the same part of AC.

11. To find the perpendicular and area of a triangular field, by measuring only that side of it on which the perpendicular falls. *Fig. 64.*

Let ABC represent a triangular field, and only its side AC accessible; it is required to find the perpendicular BD, and the area of the field.

Measure CA, and mark the point D, where the perpendicular meets it; then take AF in the right line CA produced, of any length, and make the right angle AFE. Measure along the line FE, until you find the point E in the same right line with A and B. Then the triangles AFE and ABD being similar, as  $AF : FE :: AD : DB$  the perpendicular; which being multiplied by half of AC, the product is the area of the triangle ABC.

By this method, the perpendicular BD, whether it be that of a triangular field, or a distance of any moderate magnitude,

magnitude; not exceeding 2 or 3 miles, may be accurately investigated by the surveying chain.

12. To survey a triangular field, one of whose sides is inaccessible.

Let  $ABC$ , *Fig. 64.* represent a triangular field, whose side  $BC$  is inaccessible; measure the two sides  $AB=769$  links, and  $AC=1178$  links, and the angle  $BAC=37^{\circ} 45'$ . Then, supposing a perpendicular to fall from  $B$  upon  $AC$ , as  $R$  : Sine  $A :: AB$  : the perpendicular. Multiply  $AC$  by half of the perpendicular, the product is the area of the field : which, in this example, is 2.77243, or 2 acres, 3 roods, 3 falls.

Otherwise. Having measured the two sides  $AB$  and  $AC$ , with the contained angle  $BAC$ , work thus :

As radius		10.0000000●
Is to the sine of $BAC$	$37^{\circ} 45'$	9.7869056
So is the product of $AB$ and $AC$	$\left\{ \begin{array}{l} 769 \\ 1178 \end{array} \right.$	$\left\{ \begin{array}{l} 2.8859263 \\ 3.0711453 \end{array} \right.$

To twice the area of the triangle 554596 5.7439772  
the half of which is 277298 square links, the area of the triangle.

The following problem may be wrought by this rule.

13. To survey a field, by taking a station within it, from whence all its angles can be seen, and the lines drawn from the station to the angles measured.

Let  $ABCDE$ , *Fig. 65.* represent the field to be surveyed. Chuse some convenient station, as at  $O$ , from whence you can see all the angles of the field; place poles in  $A, B, C, D, E$ , and measure all the angles  $AOB, BOC$ , &c. at the point  $O$ ; and measure all the lines  $AO, OB, OC, OD, OE$ ; then the field is divided  
into

into as many triangles as it hath sides; in each of which, two sides with the contained angle are given, to find the area, which may be done by the last problem; and the sum of all these is the area of the field.

EXAM. Let the angle  $AOB=78^\circ$ ,  $BOC=62^\circ 50'$ ,  $COD=79^\circ 20'$ ,  $DOE=77^\circ 30'$ , and  $EOA=62^\circ 20'$ ; also, let  $AO=580$  links,  $BO=580$ ,  $CO=420$ ,  $DO=395$ , and  $EO=512$  links; what is the area of the field ABCDE?

Ans. 5 acres, 3 roods, 15 falls.

Note 1. The station may be taken any where, as at one of the angles, if all the other angles of the field can be seen from thence.

2. A field may also be surveyed from two stations; but this method is seldom practised.

14. When the boundary of a field is an irregular curve line, *Fig. 66.* the space between it and the station-line AB must be measured, by taking the offsets *bc*, *de*, and *fg*. To find the area of the space ABgeC, multiply *Ab* by half of *bC*, and *Bf* by half of *fg*, the products are the areas of the triangles *AbC*, *Bfg*; then, if *bd=df*, add *bC*, *de*, and *fg*; take a third part of the sum, and multiply it by *bf*, the product is the area of *bdfgeC*; to which add the areas of the two triangles *AbC*, *Bfg*, the sum is the area of the space ABgeC. If *bd* be not equal to *df*, compute the areas of *Cd* and *dg* separately.

15. To reduce Scots or English acres to square links; multiply them by 100000.

EXAM. Reduce 28 acres to square links.

Ans. 2800000.

When

When acres, and parts of an acre are to be reduced to square links; reduce the parts to decimals of an acre, Write down the acres on the left hand of the decimals, and multiply the whole by 100000, which is done by removing the decimal point five places toward the right hand; the product is square links.

EXAM. 1. Reduce 32 acres, 3 roods, 19 fells, and 30 ells, to square links.

$$\begin{array}{r} 36)30.00000 \\ \hline \end{array}$$

*Ans.* 3287395.

$$\begin{array}{r} 40)19.83333 \\ \hline \end{array}$$

$$\begin{array}{r} 4)3.49583 \\ \hline \end{array}$$

$$3287395.$$

2. Reduce 40 acres, 2 roods, 23 poles, English measure, to square links?

*Ans.* 4064375.

## OF LAYING OUT GROUND.

1. To find the side of a square field, which shall contain any given quantity of land; reduce the given quantity to square links; the square root of these is the side of the square required.

EXAM. 1. What is the length of the side of a square field, which shall contain 30 acres? *Ans.* 1732.

30 acres = 3000000 square links.

And the square root of 3000000 is 1732.

2. What

2. What is the length of the side of a square field that shall contain 27 acres, 1 rood, 19 falls?

*Ans.* 1654 links.

2. One side of a rectangular field, and the quantity of land it must contain being given; to find the other side.

Reduce the given quantity of land to square links, and divide them by the given side, the quotient is the other side.

EXAM. It is required to lay out 29 acres, 3 roods of land, in form of a right angled parallelogram, one of whose sides is 1128 links; what is the length of the other side? *Ans.* 2637.

2. It is required to lay out 31 acres, 2 roods, 15 falls of ground, in form of an oblique angled parallelogram, whose acute angle is  $57^\circ$ , one of its sides being 1672 links; required the perpendicular falling from the opposite angle upon that side?

*Ans.* 1889 links; found as in the last example.

3. To lay out any quantity of land in form of a triangle; its base being given.

Divide the square links in the given area by one half of the base, the quotient is the perpendicular.

EXAM. It is required to lay out 31 acres, 2 roods of land, in form of a triangle, whose base is 1726 links, and the acute angle at the base  $51^\circ 20'$ ; required the perpendicular? *Ans.* 3650.

2. To lay out one acre of land in an equilateral triangle; it is required to find the length of its side?

*Ans.* 480.56 links.

Put  $a = 100,000$  square links, the area of the triangle,  $x =$  the length of the side; then the triangle being equilateral, the sum of its sides is  $3x$ , and the half sum  $= \frac{3x}{2}$ , from which subtracting each side, the three remainders are  $\frac{x}{2}, \frac{x}{2}, \frac{x}{2}$ , therefore  $\frac{3x}{2} \times \frac{x}{2} \times \frac{x}{2} \times \frac{x}{2} = \frac{3x^4}{16}$  the square of the given area:

$\frac{3x^4}{16} = a^2$  or  $3x^4 = 16a^2$ , and by extracting the square root of both sides  $x^2 \times \sqrt{3} = 4a$ .

$$x^2 = \frac{4a}{\sqrt{3}} = \frac{400000}{1.73205} = 230940.21$$

$$x = \sqrt{230940.21} = 480.56 \text{ links the side.}$$

Hence, Having the area of an equilateral triangle given, to find the length of its side. Multiply the given area by 4; divide the product by the square root of 3.  $= 1.7320508$ , and the square root of the quotient is the length of the side of the triangle.

*Exam.* It is required to find the side of an equilateral triangle, whose area shall be one square mile, or 640 acres of land? *Ans.* 12157.37 links.

Given area  $= 64000000$  square links

4

$$1.73205) 256000000 (147801737.82$$

and the square root of 147801737.82 is 12157.37 links.

*Note.* For the angles of the polygons mentioned in the following problems, See page 13.

4. The area of a regular pentagon being given, to find its side. *Plate 1. fig. 21.*

*Exam.*



*Exam.* Let the area of the pentagon = 3601 square feet, what is the length of its side AB? *Ans.* 45.74 feet.

The angle of the pentagon FAB is  $108^\circ$ , its half CAG =  $54^\circ$ .

Put  $a = 3601$ ,  $b =$  the tang. of  $54^\circ = 1.3763819$ .  $x = AB$ .

In the triangle CAG : As rad. : tan. CAG :: AG : GC ;

that is, as  $1 : 1.3763819 = b :: \frac{x}{2} : \frac{bx}{2} = CG =$  the perpendicular.

$5x =$  the sum of the sides, and  $\frac{5x}{2} =$  the half sum

$$\text{then } \frac{5x}{2} \times \frac{bx}{2} = a \text{ or } \frac{5bx^2}{4} = a$$

$$5bx^2 = 4a$$

$$x^2 = \frac{4a}{5b} = \frac{14404}{6.88190} = 2092.8813$$

And the square root of 2092.8813 is 45.74 feet, the side of the pentagon.

5. The area of a regular hexagon being given, to find its side ?

*EXAM.* Let the area of the hexagon  $a = 100000$  square feet ; required the length of its side.?

*Ans.* 196.188 feet.

Let fall a perpendicular from the centre upon one of its sides ; the angle of a hexagon is  $120^\circ$  its half is  $60^\circ$ , the tangent of  $60^\circ$  is  $1.73205 = b$ . Put  $x =$  the side.

as  $1 : b :: \frac{x}{2} : \frac{bx}{2} =$  the perpendicular.

The half sum of the sides is  $3x$ , then  $3x \times \frac{bx}{2} = a$

$$\frac{3bx^2}{2} = a \text{ and } x^2 = \frac{2a}{3b} = \frac{200000}{5.19615} = 38490.0358$$

then  $x = \sqrt{38490.0358} = 196.188$  feet.

6. The area of a regular heptagon being given, to find the length of its side.

EXAM. Let the area of a heptagon be 100,000 square yards: What is the length of its side?

*Ans.* 165.894 yards.

Half the angle of the heptagon is  $64^{\circ} 17' \frac{1}{2}$ , of which the tangent is  $2.0763215 = b$ ,  $a = 100000$  and  $x =$  the side.

$$\text{as } 1 : b :: \frac{x}{2} : \frac{bx}{2} = \text{the perpendicular}$$

$$\frac{7x}{2} \times \frac{bx}{2} = \frac{7bx^2}{4} = a$$

$$x^2 = \frac{4a}{7b}$$

$$\text{and } x = \sqrt{\frac{4a}{7b}} = \sqrt{\frac{400000}{14.53425}} = 165.894 \text{ yards.}$$

7. The area of a regular octagon being given, to find the length of its side.

Half the angle of an octagon is  $67^{\circ} 30'$ , whose tangent is  $2.4142136 = b$ , the given area  $= a$ , and  $x =$  the side.

Half the sum of the sides of an octagon is  $4x$ .

$$\text{Then } 4x \times \frac{bx}{2} = 2bx^2 = a, \text{ and } x^2 = \frac{a}{2b}$$

$$\text{or } x = \sqrt{\frac{a}{2b}}$$

EXAM. It is required to lay out 4 acres of land in an octagon: What must be the length of its side?

*Ans.* 287.824 links.

In this example  $a = 400000$  square links.  $2b = 4.8284272$

$$x^2 = \frac{400000}{4.8284272} = 82842.71 \text{ and } x = \sqrt{82842.71} =$$

287.824 links, the side.

8. The

8. The area of a regular nonagon being given, to find the length of its side.

$x$  = the side of the nonagon.

Half the angle of a nonagon is  $70^\circ$ , of which the tangent is  $2.7474774 = b$ . The half sum of the sides is  $\frac{9x}{2}$ , and the perpendicular falling from the centre is  $\frac{bx}{2}$

$$\frac{9x}{2} \times \frac{bx}{2} = \frac{9bx^2}{4} = a. \quad x^2 = \frac{4a}{9b}. \quad \text{and } x = \sqrt{\frac{4a}{9b}}.$$

EXAM. The area of a nonagon is 10,000 square feet, what is the length of its side?

Ans. 40.214 feet.

$$a = 10,000 \quad 9b = 24.7272966$$

$$\text{Then } \sqrt{\frac{4a}{9b}} = \sqrt{\frac{40,000}{24.7272966}} = 40.214 \text{ feet.}$$

9. The area of a regular decagon being given, to find the length of its side.

The angle of a decagon is  $144^\circ$ , its half  $72^\circ$ , whose tangent is  $3.0776835 = b$ .  $\frac{bx}{2}$  = the perpendicular as before. The half sum of the sides is  $5x$ .

$$\text{Then } \frac{5bx^2}{2} = a \text{ and } x^2 = \frac{2a}{5b} \text{ also } x = \sqrt{\frac{2a}{5b}}$$

EXAM. The area of a decagon is 50,000 square feet, what is the length of its side? Ans. 80.208 feet.

$$a = 50000. \quad 5b = 15.3884175.$$

$$x^2 = \frac{100000}{15.3884175} = 6433.4106$$

$$\text{and } x = \sqrt{6433.4106} = 80.208.$$

10. The area of an undecagon being given, to find the length of its side.

The

The angle of an undecagon is  $147^{\circ} 16' \frac{4}{11}$ ; its half  $= 73^{\circ} 38' \frac{2}{11}$  whose tangent is  $3.4051423 = b$ . The perpendicular  $= \frac{bx}{2}$  and the half sum of the sides is  $\frac{11x}{2}$

$$\text{Then } \frac{11x}{2} \times \frac{bx}{2} = \frac{11bx^2}{4} = a$$

$$x^2 = \frac{4a}{11b} \quad x = \sqrt{\frac{4a}{11b}}$$

EXAM. The area of an undecagon is 5280 square yards, what is the length of its side?

Ans. 23.747 yards.

$$a = 5280. \quad 11b = 37.4565653$$

$$\text{then } x^2 = \frac{21120.}{37.4565653} = 563.8530 \text{ and } x = \sqrt{563.8530} = 23.747 \text{ yards.}$$

11. The area of a dodecagon being given, to find the length of its side.

Let  $a$  = the area,  $x$  = the side as before. Half the angle of a dodecagon is  $75^{\circ}$  whose tangent is  $3.7320508 = b$ .

The perpendicular  $= \frac{bx}{2}$ , and  $6x$  is the half sum of the sides.

$$6x \times \frac{bx}{2} = 3bx^2 = a \quad x^2 = \frac{a}{3b} \text{ and } x = \sqrt{\frac{a}{3b}}$$

EXAM. Let the area of a dodecagon be 360 square yards, what is the length of its side?

Ans. 5.67043 yards.

$$x = \sqrt{\frac{360}{11.1961524}} = 5.67043$$

12. To reduce any quadrilateral figure ABCD to a triangle. Fig. 67.

Join AC; through B draw BE parallel to AC; produce DC to E, and join AE; then the triangle AED is equal to the given figure ABCD.

13. To

13. To reduce any five sided figure  $ABCDE$  to a triangle. *Fig. 68.*

Join  $AC$  and  $AD$ ; and through  $B$  and  $E$  draw  $BF$  and  $EG$  parallels to  $AC$  and  $AD$ ; join  $AF$  and  $AG$ ; then the triangle  $AFG$  is equal to the pentagon  $ABCDE$ .

14. To divide a triangular field  $ABC$  into any number of equal parts, by lines drawn from one of its angles, suppose  $A$ . *Fig. 69.*

Divide the opposite side  $BC$  into the number of equal parts required, in the points  $e$  and  $f$ ; and lines drawn from  $A$  to  $e$  and  $f$  will divide the field.

If it were required to divide the field into two parts, having a given ratio, such as 3 to 2; divide  $BC$  in the point  $g$ , so that  $Bg$  is to  $gC$  as 3 to 2; and a line drawn from  $A$  to  $g$  will divide the field into the parts required.

15. To divide a triangular field  $ABC$  into two parts, having a given ratio to each other, suppose 6 to 5, by a line drawn parallel to one of its sides  $AC$ . *Fig. 70.*

Divide the area of the whole field into parts in the given proportion, by the rule of Fellowship, in common arithmetic, and determine which of the parts must lie next the angle  $B$ ; then, because similar triangles are to each other as the squares of their like sides, as the area of the whole field is to the area of the triangle  $BFD$ , so is the square of  $BC$  to the square of  $BD$ ; and, by extracting the square root,  $BD$  will be known. Through  $D$  draw  $DF$  parallel to  $AC$ , and  $DF$  divides the field into the parts required.

16. To divide a triangular field  $ABC$  into two parts, having a given proportion to each other, such as 3 to 4, by a line drawn from a given point  $E$  in one of its sides  $BC$ . *Fig. 71.*

Draw

Draw  $AE$ , and divide  $BC$  in  $D$ , so that  $BD$  be to  $DC$  as 3 to 4. Through  $D$  draw  $DF$  parallel to  $EA$ ; and join  $EF$ ; and  $FE$  will divide the field into the parts required.

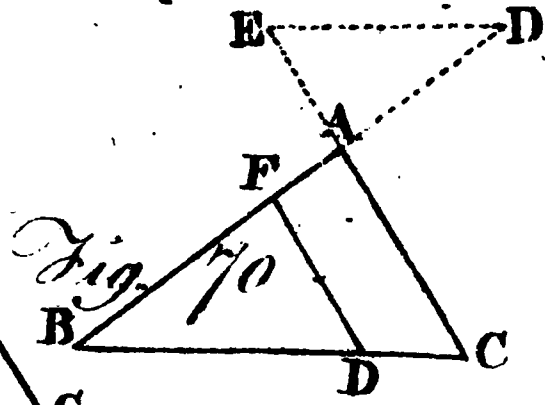
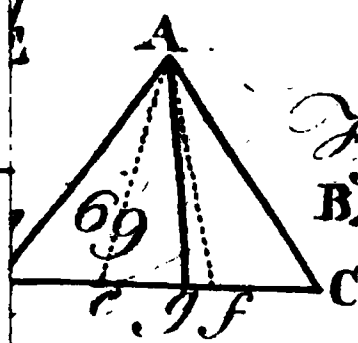
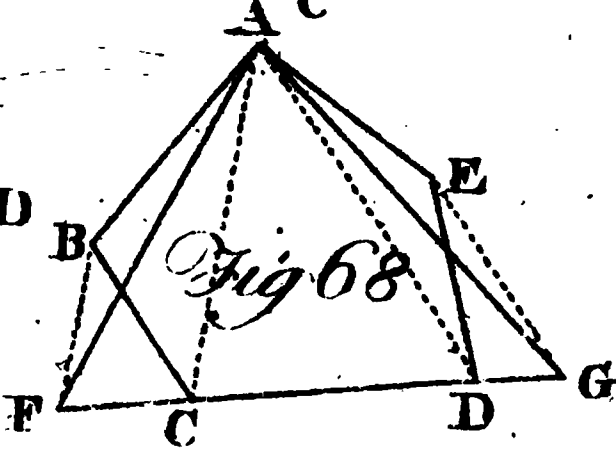
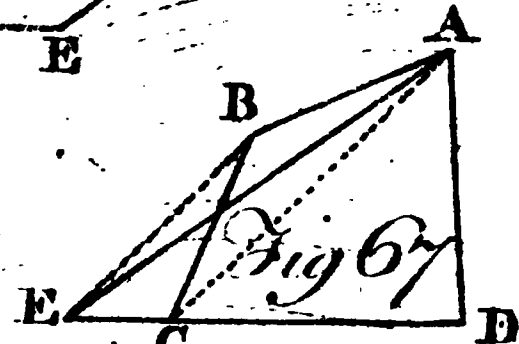
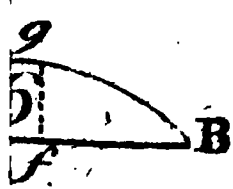
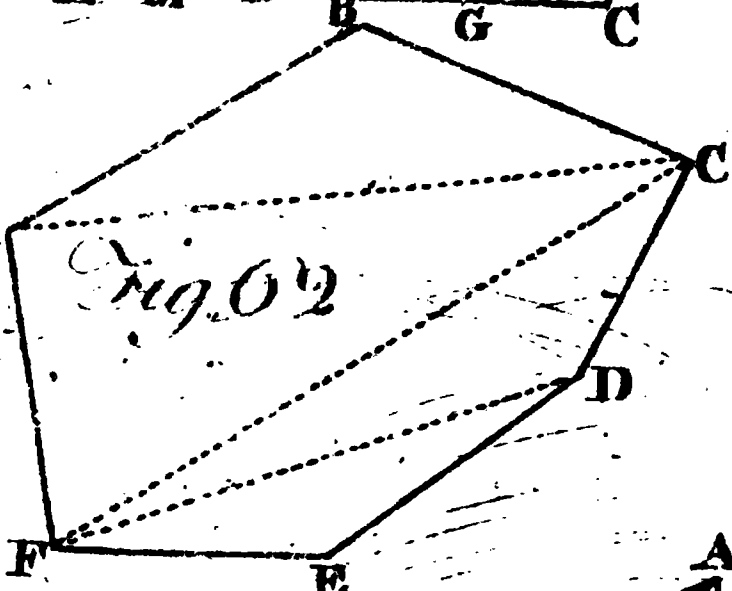
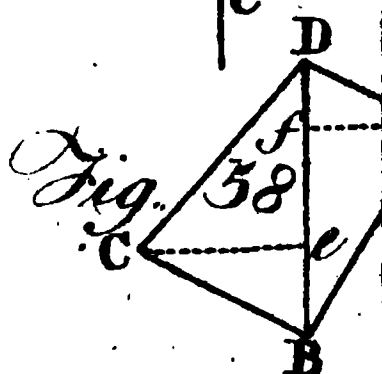
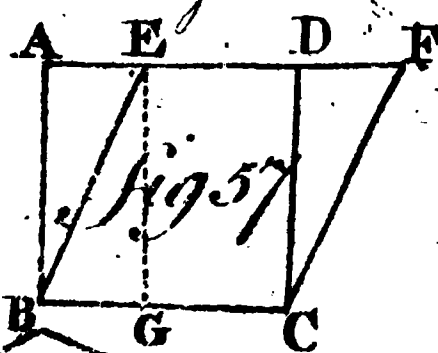
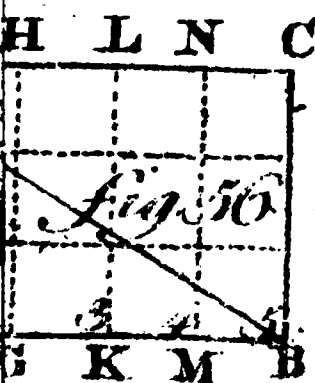
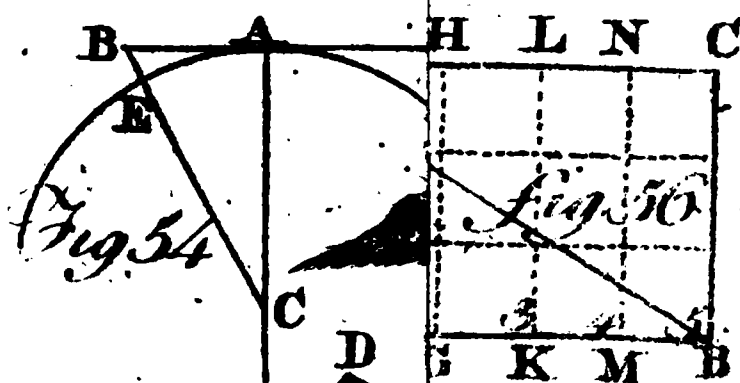
17. To divide a four or five sided field into parts, having any given proportion to each other. The general rule is, Reduce the given field to a triangle; divide the triangle into the parts required, and the same lines will divide the field. For the particular cases, see *Wilson's Trigonometry*.

In the rectangular field  $ABCD$ , *Fig. 72*. whose length is 3619 links, and breadth 2649 links, there is a well at  $E$ , distant from  $A$  2230 links, and from  $D$  1950 links; it is required to divide the field into three equal parts, by right lines drawn from  $E$ .

To solve this problem; describe the figure accurately; find the point  $E$ ; join  $EA$ ,  $ED$ , and  $EB$ ; likewise through  $E$  draw the perpendiculars  $hl$  and  $ik$ . Measure the perpendiculars  $Eh$  and  $Ei$ ; which being subtracted from the length and breadth of the field, will leave  $El$  and  $Ek$ .

2d, Multiply  $AD$  by half of the perpendicular  $Eh$ , the product is the area of the triangle  $AED$ . If this be equal to the third part of the field, the problem will be solved, by dividing the remaining part of the field into two equal parts. But, if the triangle  $AED$  be less than the third part of the field, subtract it from the third part, the remainder is an area which must be added to the triangle  $AED$ .

3d, Divide the remainder by half of the perpendicular  $Ei$ ; the quotient being taken from the scale, and set from  $D$  to







D to F, join EF, and AEFD is one third part of the field.

4th, Multiply AB by half of EF, the product is the area of the triangle AEB; if this be less than the third part of the field, subtract it from the third part, and divide the remainder by half of EF; the quotient being taken from the scale, and set from B to G; join EG, and the field is divided into three equal parts by the right lines EA, EF, and EG.

When either of the triangles AED or AEB is greater than the third part of the field, the difference between the area of the triangle and the third part of the field must be divided by half of the perpendicular of that triangle; and the quotient being taken from the scale, must be set the contrary way from what it is in this example.

In this manner, any field of whatever number of sides, may be divided into any number of equal parts; or parts having a given ratio to one another, by right lines drawn from a given point, either within the field, or in any of its sides or angles.

## S E C T. V.

## MENSURATION OF SOLIDS.

**A** CUBE is a solid body, bounded by six equal squares.

As every superficies is measured by a square, whose side is unity, (as 1 inch, 1 foot, 1 yard, &c.) so every solid is measured by a cube, whose side is also a unit, as a solid or cubic inch, foot, yard, &c.

## TABLE of Solid or Cubic Measure.

1728 Cubic lines = 1 Cubic inch.

1728 inches = 1 foot.

27 feet = 1 yard.

216 yards = 1 rood.

If the base of a rectangular solid be one square inch, and its altitude 5 inches, the whole will contain 5 cubic inches; but, if the base consist of 15 square inches, and the altitude of 5, the whole solid will contain 15 times 5, or 75 cubic inches. Hence,

1. The rule for computing the solid content of a cube, parallelopipedon, and prism, is, multiply the area of the base by the altitude; the product is the solid content.

For the cube and parallelopipedon, the rule may be expressed thus.

Multiply the length by the breadth, and the product by the thickness or height; the last product is the solidity.

E X A M.

EXAMPLES.

1. What is the solid content of a cube, whose side is 3 feet  $9\frac{1}{2}$  inches? *Fig. 73:*

*Ans.* 54 cubic feet,  $884\frac{3}{4}$  inches.

2. What is the solidity of a log of wood, whose length is 17 feet 6 inches, breadth 2 feet 3 inches, and thickness 1 foot 9 inches? *Ans.* 68 cubic feet, 1566 inches.

3. A cellar was dug, whose length was 49 feet 6 inches, breadth 23 feet 9 inches, and depth 9 feet 5 inches; how many solid yards were taken out of it; and what was the expence, at 3d. *per* cubic yard?

*Ans.* 410 yards, which comes to L. 5 : 2 : 6.

2. Every solid body, whose opposite ends are equal and similar rectilineal figures, parallel to each other, and its other sides parallelograms, is called a *prism*. *Fig. 74.*

EXAM. What is the solidity of a triangular prism; one side of its triangular base being 19.5 inches, and the perpendicular falling from the opposite angle upon that side 16.89 inches; also the length of the prism 39.25 inches?

*Ans.* 3 cubic feet 1279.59 inches.

Perpendicular 16.89

Side 19.5

—————

8445

15201

1689

—————

2)329.355

—————

Area of the base = 164.6775

Multiply by the length 39.25

—————

Ft. inches.

1728)6463.591875(3 1279.59

X 2

2. What

2. What is the solidity of a triangular prism, its base being an equilateral triangle, having each of its sides 12 inches, and the length of the prism  $48\frac{1}{2}$  inches?

*Ans.* 1 foot 1296 inches.

Area of the base = 62.3538

Then  $62.3538 \times 48.5 = 3024.1593$  cubic inches.

Ft. inches.

and  $1728)3024.1593(1.1296$

3. What is the solidity of a prism whose bases are regular pentagons, one side being 2 feet  $9\frac{1}{4}$  inches, and the perpendicular falling from the center on that side 23.05 inches; also the length of the prism 6 feet 8 inches?

*Ans.* 89 cubic feet 643 inches.

2. To find the solidity of a pyramid. *Fig. 75.*

Because every pyramid is the third part of a prism of the same base and altitude, multiply the area of its base by one third part of its height, the product is the solid content.

1. What is the solid content of a triangular pyramid, one side of its triangular base being 24.5 inches, and the perpendicular falling upon that side 21.217 inches; also the height of the pyramid 35.5 inches?

*Ans.* 3075.58 cubic inches.

Perpendicular 21.217

Side 24.5

106085

84868

42434

2)519.8165

Area of the base = 259.90325

$$\begin{array}{r}
 259.90825 \\
 35.5 \\
 \hline
 129954125 \\
 129954125 \\
 77972475 \\
 \hline
 3)9226.742875
 \end{array}$$

*Ans.* 3075.580958 cubic inches.

2. What is the solidity of a pyramid, its base being a square whose side is 12 feet 4 inches, and its height 29 feet 7 inches?

*Ans.* 1499 cubic feet 1701 $\frac{1}{2}$  inches.

3. To find the solidity of a cylinder, the diameter of its base and height being given. *Fig. 76.*

Find the area of its base, and multiply it by the height; the product is the solidity.

EXAM. 1. What is the solid content of a cylinder, whose length is 5 feet 5 inches, and the diameter of its base 2 feet 3 inches?

*Ans.* 21 cubic feet, 928 inches.

2. What is the solidity of a cylinder, its length being 21 feet 10 inches, and the diameter of its base 5 feet 6 inches? *Ans.* 548 cubic feet, 1251 inches.

4. To find the solid content of a cone, the diameter of its base and height being given *Fig. 77.*

Because every cone is the third part of a cylinder of the same base and altitude.

Multiply the area of its base by one third part of its height; the product is the solidity.

EXAM. 1. What is the solid content of a right cone, whose axis is 40 inches, and the diameter of its base 32 inches? *Ans.* 6 solid feet, 355 inches.

2. What

2. What is the solidity of a cone, whose height is 3 feet 9 inches, and the diameter of its base 3 feet 11 inches? *Ans.* 15 cubic feet 104 inches.

5. To find the superficies of any solid body bounded by plains; find the area of each bounded plain; the sum of these is the superficies of the solid.

EXAM. A squared piece of timber is 21 feet 9 inches long; the greater end is 25 inches by 19.75 inches, and the less end 19 inches by 15: It is required to calculate the superficies of the whole? *Ans.* 148 feet 20½ inches.

In this example, the perimeters of the two ends may be considered as the parallel, but unequal sides of a trapezium, and the length of the piece of timber, as the perpendicular breadth of the trapezium; therefore, multiply the half sum of the perimeters by the length, the product is the superficies of the sides, to which add the superficies of the two ends, and the sum is the superficies of the whole.

			inches.
Perimeter of the greater end	89.5		
of the less	68.		
	<u>Sum</u>	<u>157.5</u>	
F. I. half	78.75		
Length 21.9 =	261 inches		
	<u>7875</u>	19.75	19
	47250	25	15
	<u>15750</u>	<u>9875</u>	<u>95</u>
Superficies of the sides	20553.75	3950	89
— of the greater end	493.75	493.75	285
— of the less	285.		
Superficies of the whole	21332.50 square inches		
	or 148 square feet 20½ square inches.		

6. To find the superficies of a cylinder, the diameter of the base, and height of the cylinder being given. *Fig. 76.*

Find the circumference of the base, and multiply it by the length of the cylinder, the product is the curve superficies: Then find the areas of the two bases; the sum of these three is the superficies of the cylinder.

EXAM. 1. What is the superficies of a cylinder, its length being 7 feet 9 inches, and the diameter of its base 3 feet 5 inches?

*Ans.* 14619.43 square inches, or 101 square feet, 75.43 square inches.

2. Required the superficies of a cylinder, its length being 6 feet  $9\frac{1}{2}$  inches, and the diameter of its base 5 feet  $1\frac{1}{2}$  inches?

*Ans.* 21687.6429 square inches, or 150 square feet, 87 inches.

7. To find the superficies of a right cone, its axis and the diameter of its base being given. *Fig. 77.*

Find the area and circumference of the base, and multiply the circumference of the base by half the length of the conic superficies; the product is the curve superficies, to which add the area of the base, the sum is the superficies of the cone.

*Note.* The length of the conic superficies is equal to the square root of the sum of the squares of the axis of the cone and the semidiameter of the base.

EXAM. 1. What is the superficies of a right cone, the diameter of its base being 36 inches, and its axis 48 inches.

*Ans.* 3916.796 square inches, or 27 square feet, 28.796 inches.

2. Required

2. Required the superficies of a cone, the diameter of its base being 12 feet 4 inches, and its axis, or perpendicular height, 24 feet 9 inches?

*Ans.* 28360.39 square inches, or 613 square feet, 88.39 inches.

8. To find the superficies of a globe, its diameter being given. *Fig. 61.*

1. The superficies of a globe is equal to four times the area of one of its great circles: Wherefore, find the area of a circle of the same diameter with the globe, and multiply it by 4, the product is the superficies of the globe. Or,

2. Because the area of a circle is equal to the product, made by multiplying the circumference by half of the radius, or one fourth part of the semidiameter; if the circumference of a circle be multiplied by its diameter, the product will be four times the area of that circle: Therefore, multiply the circumference of any globe by its diameter, and the product shall be the superficies of the globe.

EXAM. 1. What is the superficies of a globe whose diameter is 12 inches?

*Ans.* 452.39 square inches.

2. Required the superficies of a globe, its diameter being 7 feet 6 inches?

*Ans.* 176 square feet, 102.96 square inches.

3. The diameter of the terraqueous globe is 7964 miles; what is its superficies in square miles?

*Ans.* 199256909.9136 square miles.

9. To find the curve superficies of any frustum, or zone of a globe or sphere, the height of the frustum, or zone, and the circumference of the sphere being given.

Multiply



Multiply the circumference of the sphere by the height of the frustum, or zone, the product is the curve superficies. *Vide MacLaurin's Fluxions*, p. 13. and 14.

EXAM. 1. The diameter of the terraqueous globe being 7964 miles, and consequently its circumference 25019.7024 miles; it is required to find the superficies of either frigid zone? *Plate 5. Fig. 16.*

Let PÆSQ represent the solstitial colure, ÆQ the equator, PS the earth's axis, P the north pole, and *ar* the diameter of the arctic circle. Join *Ca*; then, in the right angled triangle *aCn*, we have *Ca*=3982 miles, and the angle *aCn*=23° 28'; to find *Cn*, which being subtracted from CP, the semidiameter, there will remain *nP*, the height of the frigid zone.

As radius	10.0000000
Is to cosine <i>aCn</i> =23° 28'	9.9625076
So is <i>Ca</i> =3982	3.6001013
To <i>Cn</i> =3652.65	<hr/>
	3.5626089

*nP*=329.35 miles, the height of the zone.

Then 25019.7024 × 329.35=8240238.98544 square miles, the surfate of either frigid zone.

EXAM. 2. The same things being given, it is required to find the superficies of either temperate zone?

In the right angled triangle *Cto*, there is given *Ct*=3982 miles, and the angle *tCo*=66° 32', to find *Co*, which being subtracted from *Cn*, the remainder is, *no*, the height of the zone.

Y

As

As radius.

10:

Is to cosine  $\angle C = 66^\circ 32'$ 

9.6001188

So is  $\angle C = 3982$ 

3.6001013

To  $C_0 = 1585.69$ 

3.2002194

From  $C_n = 3652.65$ Subtr.  $C_0 = 1585.69$ 

Remain  $m = 2066.96$  miles, the height of the zone, and  $25019.7024 \times 2066.96 = 51714724.072704$  square miles, the surface of each temperate zone.

EXAM. 3. It is required to find the superficies of the torrid zone?

Since by the last operation  $C_0 = 1585.69$ , the double of this is  $3171.38$  miles, the height of the zone; and  $25019.7024 \times 3171.38 = 79346983.797312$  square miles, the surface of the torrid zone.

Square Miles.

The surface of the torrid zone is

79346983.797312

of the temperate zones

{ 51714724.072704

{ 51714724.072704

of the frigid zones

{ 8240238.985440

{ 8240238.985440

The surface of the whole globe

199256909.913600

10. If the whole superficies of the polar segment, or frustum of a globe, be required; having found the curve superficies, find the area of its circular base, and add it to the curve superficies, the sum is the answer. Or work by the following rule.

11. To find the superficies of the polar segment, or frustum of a globe COPI, its height OI, and the diameter of its base CP being given. Fig. 61.

The

The curve superficies is equal to the area of a circle whose radius is  $CO$ ; and  $CO$  is equal to the square root of the sum of the squares of  $CI$  and  $OI$ . Wherefore,

Find the area of a circle whose diameter is double of  $CO$ . Find also the area of the circular base, whose diameter is  $CP$ ; the sum of these is the superficies of the segment.

EXAM. It is required to find the superficies of the segment of a globe, its diameter  $CP$  being 44.4 inches, and its height  $OI = 13$  inches?

*Ans.* 3627.5344 square inches.

12. To find the solidity of a globe, its diameter being given.

The solidity of a globe, whose diameter is 1, has been found to be .5236 nearly; and globes are in proportion to one another as the cubes of their diameter; therefore, as 1 is to the cube of the given diameter, so is .5236 to the solidity; or, because the first term of the proportion is 1,

Multiply the cube of the given diameter by .5236, the product is the solid content of the globe.

EXAM. 1. What is the solidity of a globe, its diameter being 12 inches?

*Ans.* 904.78 cubic inches.

2. Required the solidity of a globe, its diameter being  $18\frac{1}{2}$  inches.

*Ans.* 3315.23 cubic inches.

3. What is the solidity of the terraqueous globe, its diameter being 7964 miles?

*Ans.* 264480338425.3184 cubic miles.

A globe may be considered as a body consisting of an infinite number of small cones, whose bases are in the superficies of a globe, and their vertices at its center; and, because the solidity of a cone is the product of its base into the third part of its height, the solidity of the whole globe will be the product of its superficies into the third part of its radius, or into the sixth part of its diameter. Therefore,

To find the solidity of a globe, its diameter being given.

Find the superficies of the globe, by Prob. 8. and multiply the superficies by the 6th part of the diameter, the product is the solidity of the globe.

In the last example, the diameter is 7964 miles, its sixth part is  $1327\frac{1}{3}$ , and the superficies of the globe is 199256909.9136; and

$199256909.9136 \times 1327\frac{1}{3} = 264480338425.3184$ , the same as before.

13. To find the solidity of the polar segment, or frustum of a globe  $\text{COPI}$ , its height  $\text{OI}$ , and the diameter of its base  $\text{CP}$  being given. *Fig. 6r.*

Divide the square of  $\text{CI}$  by  $\text{OI}$ , the quotient is  $\text{IB}$ , which being added to  $\text{OI}$ , the sum is the diameter of the globe. And then,

From the triple product of the diameter of the globe ( $\text{OB}$ ) into the square of the frustum's height ( $\text{OI}$ ), subtract twice the cube of its height, and multiply the remainder by .5236, the product is the solidity of the segment.

EXAM.

**EXAM.** It is required to find the solidity of the segment of a globe, the diameter of its base CP being 44.4 inches, and its height OI = 13 inches?

*Ans.* 11214.1349 solid inches.

2. It is required to find the solidity of either frigid zone, the axis, or height of it being 329.35 miles, and the diameter of the earth (taken as a perfect sphere) being 7964 miles.

$$(1) \ 329.35 \times 329.35 = 108471.4225,$$

$$\text{and } 108471.4225 \times 329.35 \times 2 = 71450126.$$

$$(2.) \ 108471.4225 \times 7964 \times 3 = 2591599226.37$$

$$\text{Difference} = 2520149100.37$$

$$(3.) \ 2520149100.37 \times .5236 = 1319550068.9837 \text{ cubic miles the answer.}$$

14. To find the solidity of the middle zone of a globe CPFE, the diameter of the globe, the diameter of the zone's base, and the height of the zone being given. *Fig. 61.*

Multiply the square of the globe's diameter by 2, and to the product add the square of the diameter of the zone's base; divide the sum by 3.8197, and multiply the quotient by the height of the zone; the product is the solidity of the zone.

**EXAM. 1.** The diameter of a globe AD is 16 inches, and two polar segments being cut off, the diameter of the base of the middle zone CP or EF is 13.8564 inches, and the height 8 inches: What is the solid content of the zone?

*Ans.* 1474.46 cubic inches.

2. It is required to find the solidity of the torrid zone, the diameter of the earth being 7964 miles, the diameter

diameter of either tropic = 7305.3 miles, and the axis, or height of the zone = 3171.38 miles.

$$\begin{array}{rcl}
 (1.) & 7964 \times 7964 \times 2 & = & 126850592 \\
 & 7305.3 \times 7305.3 & = & 53367408.09 \\
 & & & \hline
 & & & 180218000.09
 \end{array}$$

$$(2.) \frac{180218000.09}{3.8197} \times 3171.38 = 149629472782.1242$$

cubic miles, the answer.

15. To find the solidity of any zone of a sphere contained between two parallel and unequal plains, (such as *artd*, Plate 5. Fig. 16.) the diameters of the bases and height of the zone being given.

To the sum of the squares of the semidiameters of the two bases, add one third part of the square of the zone's height; multiply the sum by the height, or axis of the zone, and multiply the product by 1.5708; the last product is the solidity of the zone.

EXAM. It is required to find the solidity of either temperate zone, the semidiameter of the polar circle being 1585.69 miles; the semidiameter of the tropic = 3652.65 miles; and the height, or axis of the zone = 2066.96 miles?

$$\begin{array}{rcl}
 (1.) & \text{The square of } 1585.69 & \text{is } 2514412.7761 \\
 & \text{Ditto of } 3652.65 & \text{is } 13341852.0225
 \end{array}$$

$$\begin{array}{rcl}
 & \text{Sum} & 15856264.7986 \\
 \text{The square of } 2066.96 & & \\
 \text{is } 4272323.6416 & & \\
 \text{The third part whereof is} & & 1424107.8805 \\
 & & \hline
 & & 17280372.6791
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{add}$$

$$(2.) 17280372.6791 \times 2066.96 \times 1.5708 = 56105581678.3745, \text{ the solidity of the temperate zone in cubic miles.}$$

From

From the three last problems, we have

The solid content of the torrid zone	Cubic Miles.
=	149629472782.1242
Of the two temperate zones	{ 56105581678.3745
	{ 56105581678.3745
Of the two frigid zones	{ 1319550068.9537
	{ 1319550068.9537

Solidity of the terraqueous globe = 264479736276.7806

*Note.* The reason why this sum falls short of the solid content of the terraqueous globe (in Exam. 3. of Art. 12) is, in this calculation, the diameters of the polar circles and tropics, and the heights or axes of the several zones are computed only to two places of decimals. If these lines had been taken nearer to the truth, the sum of the solidities of the zones would have come nearer to the solidity of the whole globe.

If a semi-ellipse be revolved round its longer axis, the solid generated by that revolution is called an *oblong spheroid*; but, if it be revolved round its shorter axis, the solid is called an *oblate spheroid*.

16. To find the solidity of any spheroid, its two axes being given.

Multiply the axis round which the generating ellipse was revolved by the square of the other axis, and multiply the product by .5236; the last product is the solidity of the spheroid, whether it be oblong or oblate.

EXAM. It is required to find the solidity of an oblong spheroid, whose axes are 42 and  $19\frac{1}{2}$  inches?

*Ans.* 8362.15. cubic inches.

2. What is the solid content of an oblate spheroid, whose axes are  $19\frac{1}{2}$  and 42 inches?

*Ans.* 18010.79 cubic inches.

17. The

17. The solidity of the middle zone of a spheroid is found by the same rule as that of a globe or sphere.  
*Art. 14 of this Section.*

EXAM. It is required to find the solidity of the middle zone of an oblong spheroid, (*Fig. 24*). the conjugate diameter GH being 43.4 inches, the diameter of either base NP or MO=32.6 inches, and the length of the zone  $kl=38$  inches?

*Ans.* 48049.67 cubic inches.

For twice the square of GH is 3767.12, to which add the square of NP=1062.76, and divide the sum 4829.88 by 3.8197; the quotient 1264.465 being multiplied by 38 gives 48049.67 for the answer.

18. To find the solidity of any frustum, or segment of a spheroid, cut off by a plain perpendicular to one axis, and parallel to the other, the diameter of its base and its height being given.

Find its solidity as if it were the frustum of a globe; and then, as the solidity of a globe whose diameter is equal to the axis of the spheroid perpendicular to the base of the frustum, is to the solidity of the spheroid; so is the solidity of the frustum of the globe to the solidity of the like frustum of the spheroid. Or,

The frustums are to one another as the cube of the fixed diameter of the spheroid is to the product of the same diameter into the square of its other diameter.

EXAM. It is required to find the solidity of the frustum of an oblong spheroid, whole axes are 50 and 30 inches; the diameter of its base (parallel to the lesser axis) being 18 inches, and its height 5 inches?

*Ans.* 659.736 cubic inches.

For



For  $50 \times 5 \times 5 \times 3 = 3750$ , from which  
 Subtract  $5 \times 5 \times 5 \times 2 = 250$

Remains  $3500$

And  $3500 \times .5236 = 1832.6 =$  the solidity, if it was the segment of a sphere whose diameter is 50 inches.

The cube of 50 is 125000, and  $50 \times 30 \times 30 = 45000$ .

Lastly, as  $125000 : 45000 :: 1832.6 : 659.736$ , the answer.

If a semi parabola be revolved round its axis, the solid generated by that revolution is called a *parabolic conoid*. Fig. 78.

19. To find the solidity of a parabolic conoid.

Multiply the area of its circular base by one half of its altitude, the product is the solid content; because this solid is one half of a cylinder, having the same base and altitude.

EXAM. Required the solidity of a parabolic conoid, the diameter of its base being 31 inches, and its height 43 inches? Ans. 16227.54 cubic inches.

20. To find the solidity of the lower frustum of a parabolic conoid, cut off by a plain parallel to its base, its height and the diameters of its bases being given.

Find the areas of the two bases, and divide the sum by 2; then multiply the quotient by the height or length of the frustum; the product is the solidity.

EXAM. It is required to find the solidity of the lower frustum of a parabolic conoid, the diameter of its greater base being 24 inches, and of its lesser 16 inches, and the length of the frustum 35 inches?

Ans. 11435.42 cubic inches.

If a parabola be revolved round one of its ordinates, the solid generated is called a *parabolic spindle*.

21. To find the solidity of a parabolic spindle, its greatest diameter and its length, or axis, being given.

*Fig. 79.*

Find the solidity of a cylinder, the diameter of whose base is equal to the greatest diameter of the spindle, and its height equal to the length of the spindle, and  $\frac{8}{15}$  of this cylinder is the solid content of the spindle.

EXAM. Required the solidity of a parabolic spindle, its length being 60 inches, and its greatest diameter 24 inches? *Ans.* 14476.4928 cubic inches.

22. To find the solidity of the middle zone of a parabolic spindle, its greatest diameter, the diameter of its base, and the length of the zone being given.

To twice the square of its greatest diameter, add the square of the diameter of its base; and, from the sum subtract  $\frac{4}{15}$  of the square of the difference between these two diameters; divide the remainder by 3.8197, and multiply the quotient by the length of the zone; the product is the solidity.

EXAM. It is required to find the solidity of the middle zone of a parabolic spindle, its greatest diameter being 32 inches, the diameter of either base = 24 inches, and the length of the zone = 40 inches?

*Ans.* 27210.48 cubic inches.

When a pyramid or cone is cut by a plain parallel to its base, the part between that plain and the base is called a *frustum*.

23. To find the solidity of a frustum of a pyramid or cone, its length, and the dimensions of its bases, being given.

Find the areas of its bases, and multiply the one by the other. Extract the square root of the product, and add

add the root to the areas of the bases ; then multiply the sum by one third part of the length of the frustum ; the product is the solidity.

EXAM. 1. It is required to find the solidity of the frustum of a pyramid, the greater base being a rectangle, whose sides are 12 and 9 inches ; and the lesser base, the same figure whose sides are 10 and  $7\frac{1}{2}$  inches ; and the length of the frustum 19 inches ? *Ans.* 1729 solid inches.

2. How many cubic feet of timber are there in a log of wood 18 feet long, the base at the greater end being 32 inches by 20, and at the lesser end 16 inches by 10 ?

*Ans.*  $46\frac{2}{3}$  cubic feet.

*Note.* This rule is general, and serves to find the solid content of the frustum of any pyramid or cone ; but in some cases the problems may be solved with less labour. For example,

1. When the bases of the frustum are squares. Multiply the side of the greater base by the side of the less, and add the product to the squares of these sides, then multiply the sum by one-third part of the length of the frustum, the product is the solid content.

EXAM. 1. Suppose the frustum of a pyramid hath square bases ; the side of the greater base being 12 inches, and the side of the less, 9 inches, and its length 17 inches : What is the solid content ?

*Ans.* 1887 cubic inches.

Product of the sides	108	333
Square of the greater base	144	17
of the less	81	—
	—	331
Sum	333	333
		—
		3)5661

*Ans.* 1887

2. What is the solid content of the frustum of a pyramid having square bases; the side of the greater base being 30 feet 9 inches, and of the less 20 feet 10 inches, and the height, or length of the frustum 42 feet 3 inches?

*Ans.* 28451 cubic feet, 631 inches.

2. The solid content of the frustum of a cone may be found thus; multiply the diameter of the greater base by the diameter of the less, and to the product add the squares of the two diameters; multiply the sum by .2618 (the 3d part of .7854), and multiply the product by the length of the frustum, the last product is the solid content.

EXAM. What is the solid content of the frustum of a cone, its length being 19 inches, the diameter of the greater base = 46 inches, and of the lesser base 9 inches?

*Ans.* 2392.59 cubic inches.

Product of the diameters = 144	.2618
Squares of the diameters { 256	481
81	<hr style="width: 50%; margin: 0 auto;"/>
Sum 484	2618
	20944
	10472
	<hr style="width: 50%; margin: 0 auto;"/>
	125.9258

Then  $125.9258 \times 19 = 2392.5902$  cubic inches.

2. Let the length of the frustum of a cone be 6 feet 9 inches; the diameter of the greater base 3 feet 8 inches, and the diameter of the lesser base 2 feet 7 inches: What is the solid content of the frustum?

*Ans.* 52 cubic feet 501.9 inches.

A solid resembling the frustum of a pyramid, having parallel bases, but not similar to one another, is called a *prismoid*.

24. To find the solidity of a prismoid, its length and the sides of its bases being given.

1. To

1. To the longest side of the greater base add half the longest side of the lesser base, and multiply the sum by the breadth of the greater base.

2. To the longest side of the lesser base add half the longest side of the greater base, and multiply the sum by the breadth of the lesser base. Add this product to the former, and multiply the sum by one third part of the given length, the product is the solidity of the prismoid.

*Note.* This rule holds only when the bases are quadrilateral figures. If the bases consist of more than four sides, use the following rule:

To the sum of the areas of the two bases, add four times the area of a section parallel to the bases, taken in the middle of the solid; multiply the sum by the given length, and one sixth part of the product is the solidity of the prismoid.

A solid resembling the frustum of a cone, and having parallel bases, and these bases both elliptical, (or the one base an ellipse and the other a circle), but disproportional; the diameters of the one base not having the same proportion to one another as the correspondent diameters of the other base, is called a *cylindroid*.

25. To find the solidity of a cylindroid, its length, and the dimensions of its bases, being given.

1. To the longest diameter of the greater base, add half the longest diameter of the lesser base, and multiply the sum by the shortest diameter of the greater base.

2. To the longest diameter of the lesser base, add half the longest diameter of the greater base, and multiply the sum by the shortest diameter of the lesser base. Add this product to the former, and multiply the sum by

.7854,

.7854, then multiply the product by one third part of the given length ; the last product is the solidity of the cylindroid.

*Note.* However useful the two last rules may be on some occasions, they are seldom applied in practice ; for trees have generally their bases, either perfectly, or nearly similar ; and, therefore, are frustums of pyramids or cones : And, when they are not similar, their contents, when considered as frustums of pyramids or cones, come so near to the same, when considered as prismoids and cylindroids, that the difference is quite inconsiderable ; not amounting to half a cubic foot in sixty.

## OF MEASURING TIMBER.

When large trees are of any regular form, as a parallelopiped, prism, cylinder, pyramid, or cone, their solid contents are found by the foregoing rules. When their forms are different, they must be considered as frustums of some regular solid.

### I. *Of Equal SQUARED TIMBER.*

By equal squared timber, is understood all trees cut into the form of a parallelopiped, whose bases are equal squares or rectangles ; and the solid content is found by *Art. 1. of this section.*

**EXAM. 1.** Required the solid content of a squared piece of wood, its length being 19 feet, and the side of its square base 16 inches ?

*Ans.* 33 cubic feet, 1344 inches.

2. In.

2. In a piece of squared timber, 32 inches broad, 18 inches thick, and 14 feet 6 inches long; how many solid feet?

*Ans.* 58 cubic feet.

*Note.* In measuring squared timber, unskilful measurers take a fourth part of the circumference for the side of a mean square base; this quarter girt being multiplied by itself, and the product by the length, they account the solid content; but this method gives an answer always greater than the truth, except when the breadth and the thickness are nearly equal; and the greater difference there is between the breadth and thickness, the greater will the error be. In the last example  $32 + 18 = 50$ , and half of 50 is 25, the side of the mean square base; and  $25 \times 25 = 625$  for the area of the base. Lastly,  $\frac{625 \times 14.5}{144} = 62$  solid feet, 16 1/4 inches, which is near 5 feet above the true solidity.

## 2. Of Unequal SQUARED TIMBER.

By unequal squared timber is meant, any piece of squared timber whose bases are unequal, whether these bases be squares or rectangles; and such are most trees when hewn into what is called a Square Form. Any solid of this kind is truly, or very nearly, the frustum of a pyramid; and its solid content may be found by *Art. 23. of this section.*

*EXAM.* It is required to find the solid content of a tree 18 feet 6 inches long, the base at the greater end being (a rectangle) 32 inches by 19 1/2, and at the lesser end 17 inches by 9? *Ans.* 46.506 cubic feet.

The

The usual customary way of measuring such timber is, to take the square or rectangle at the middle of the tree for a mean base, and multiply this by the length.

The last Example wrought by this method.

$$(1.) \frac{3^2 + 17}{2} = 24.5, \text{ and } \frac{2 + 19\frac{1}{2}}{2} = 14.25$$

$$(2.) \frac{24.5 \times 14.25 \times 18.5}{144} = 44.852 \text{ cubic feet, too little by } 1.654 \text{ feet.}$$

Hence the error of this customary way of measuring is abundantly evident.

EXAM. 2. How many solid feet are there in a tree 24 feet long, its bases being squares; the side of the greater = 15 inches, and the side of the less = 6 inches?

Ans. 19 cubic feet, 864 inches.

3. What is the solid content of a tree 27.36 feet long; the breadth at the greater end being 1.78 feet, and the thickness 1.23 feet; and the breadth at the lesser end 1.04 feet, and thickness .91 feet.

Ans. 41.723 solid feet.

### 3. Of Round TIMBER.

1. When round trees are equally thick from one end to the other, they are cylinders; and their lengths being measured in feet, and their circumferences in inches, their solidities may be found thus:

Multiply the square of the circumference, or girt, by .0795775; multiply the product by the length, and divide the last product by 144, the quotient is the solid content of the tree in cubic feet.

EXAM. 1.



**Exam. 1.** What is the solidity of a round tree, its length being 17 feet 6 inches, and its circumference 41 inches?

*Ans.* 16.25 cubic feet.

2. Required the solidity of a round tree, its circumference being 52 inches, and its length 30 feet?

*Ans.* 44.828 solid feet.

3. The length of a round tree is 25 feet 5 inches, and its girt 4 feet  $5\frac{1}{2}$  inches; required its solidity?

*Ans.* 40.202 cubic feet.

2. When round trees are smaller at one end than the other, (commonly called Tapering Timber), they are frustums of cones; and their solid contents may be found by *Art. 23. of this section*; but, because it has been always thought best to take the circumference and length of a round tree, to find its solidity,

Measure the circumference at both ends, and in as many places between the two as may be thought necessary; divide the sum of the circumferences by their number, the quotient is the mean girt; then measure the length, with which, and the mean girt, find the solidity by the last rule.

**Exam. 1.** It is required to find the solidity of a round tree, its length being 17 feet 3 inches, and the circumference being taken in five places, the several girts are 113.16, 95.04, 73.8, 56.88, and 37.92 inches, and consequently the mean girt = 75.36 inches?

*Ans.* 54.1375 cubic feet.

2. What is the solidity of a round tree, its length being 19 feet 9 inches, and mean girt = 47.124 inches?

*Ans.* 24.237 cubic feet.

*Ans.*

3. The

3. The customary way of measuring round timber is this: Take a fourth part of the mean girt in inches, and square it; multiply this square by the length in feet, and divide the product by 144, the quotient is the solidity of the tree.

But this rule, although universally used, gives the answer always too little by about one fourth part of the whole.

### E X A M P L E S.

Length.		Mean Girt.	Solidity in
Feet.	Inches.	Inches.	Cubic Feet.
17	9	$30\frac{5}{8}$	7.1666
21	7	$41\frac{1}{4}$	15.94
19	6	50	21.1588
22	10	49	23.7946
23	11	$67\frac{1}{2}$	47.6471
25	0	39	16.5039
27	3	59	41.1706

These examples are all wrought by the common rule, and the answers are therefore less than the truth. It may be a very good exercise for a learner to work them by the former, or following rule, namely,

4. Multiply the square of one fifth part of the circumference by twice the length; the product is the solidity, very near the truth.

EXAM. The length of a round tree is 24 feet 9 inches, and its mean girt  $57\frac{1}{2}$  inches; what is the solidity?

Ans. 45.46 cubic feet.

One 5th of 57.5 is 11.5, and twice the length is 49 feet 6 inches; then  $\frac{11.5 \times 11.5 \times 49.5}{144} = 45.46$ , true to the fourth part of a foot.

*More*

*More EXAMPLES wrought by this Rule.*

Length.		Mean Girt.	Solid content in
Feet.	Inches.	Inches.	Cubic Feet.
25	5	$48\frac{1}{2}$	33.214
27	6	51	39.7375
26	9	$59\frac{3}{4}$	53.055
23	3	65	54.5729
21	4	70	58.074
19	10	75	61.9791
18	8	$81\frac{1}{2}$	68.2675

*Of the Five REGULAR BODIES, viz.*

1. The *Tetrahedron*, or regular triangular pyramid, contained under four equal equilateral triangles.
2. The *Hexahedron*, or Cube, contained under six equal squares.
3. The *Octahedron*, contained under eight equal equilateral triangles.
4. The *Dodecahedron*, contained under twelve equal equilateral pentagons.
5. The *Icosahedron*, contained under twenty equal equilateral triangles.

#### 1. Of the TETRAHEDRON.

To find the superficies of the Tetrahedron. Let the

A a 2

side

side of one of the containing triangles be  $a$ , then the sum of the three sides will be  $3a$ , and the half sum  $\frac{3a}{2}$ ; from which each side being subtracted, the three remainders are  $\frac{a}{2}$ ,  $\frac{a}{2}$ ,  $\frac{a}{2}$ . The continual product of the half sum and three remainders is  $\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2} = \frac{3a^4}{16}$ , and the square root of  $\frac{3a^4}{16}$  is  $\frac{a^2 \times \sqrt{3}}{4}$  = the area of one of the containing triangles; and this root being multiplied by 4, gives  $a^2 \times \sqrt{3}$  for the superficies of the tetrahedron. Therefore, multiply the square of the side of one of the containing triangles by the square root of 3; the product is the superficies of the tetrahedron.

EXAM. What is the superficies of a tetrahedron, each side of its triangular faces being 12 feet?

Ans. 249.4152 feet

To find the solidity of a Tetrahedron.

1. Measure the perpendicular height of the solid, and multiply the area of the base by one third part of the perpendicular, the product is the solidity. Or,

2. Multiply the cube of a side of one of the containing triangles by the square root of 2, and divide the product by 12, the quotient is the solidity.

EXAM. Required the solidity of a tetrahedron, each side of its triangular faces being 12 feet, and the perpendicular height of the solid being 9.8 feet?

Ans. 203.68 cubic feet, by rule 1st; and by rule 2d, the answer is 203.64 cubic feet.

2. Of

2. *Of the HEXAHEDRON, or CUBE.*

Six times the square of its lineal side is the superficies of the cube; and the cube of the same side is the solid content.

3. *Of the OCTAHEDRON.*

To find the superficies of the Octahedron.

Since this solid is contained under eight equal equilateral triangles, find the area of one of these triangles, (by the rule for the tetrahedron), and multiply it by 8, the product is the superficies of the Octahedron. Or,

Multiply twice the square of the lineal side by the square root of 3; the product is the superficies of the Octahedron.

**EXAM.** Required the superficies of an octahedron, each side of its triangular faces being 12 feet?

*Ans.* 498.83 square feet.

To find the solid content of an Octahedron.

The octahedron consists of two equal pyramids, joined together on the same square base; namely, the square of a side of one of the triangular faces. Therefore, measure the length of the octahedron, and multiply two thirds of it by the square of a side of one triangular face; the product is the solid content.

**EXAM.** What is the solidity of an octahedron, each side of its triangular faces being 12 feet, and the length of the solid 8.485 feet?

*Ans.* 814.579 cubic feet.

Otherwise:

Otherwise : Multiply one third part of the cube of the side of one triangular face by the square root of 2 ; the product is the solid content of the octahedron.

#### 4. *Of the DODECAHEDRON.*

To find the superficies of a Dodecahedron.

Because this solid is contained under 12 equal regular pentagons, multiply the area of one of these pentagons by 12 ; the product is the superficies of the dodecahedron.

Find the area of one of the containing pentagons thus : The angle of a pentagon is  $108^\circ$ , its half is  $54^\circ$  ; therefore, as the radius is to the tan.  $54^\circ$ , so is half of the side of the pentagon to the perpendicular falling from the center on that side. Multiply half the sum of the sides by the perpendicular, the product is the area of the pentagon.

EXAM. Required the superficies of a dodecahedron, each side of its pentagonal faces being 12 feet.?

*Ans.* 2972.984 square feet.

To find the solid content of a Dodecahedron.

The dodecahedron consists of 12 equal pyramids, their bases being pentagons, and their vertices in the center of the inscribed sphere. When the solid is set on one of its sides, its height will be the diameter of the inscribed sphere, and the half of this diameter will be the common height of the pyramids. Therefore,

Measure the diameter, and multiply the area of one of the containing pentagons by one sixth part of the diameter,

meter, and multiply the product by 12; the last product is the solidity of the dodecahedron. Or,

Multiply the area of one base by twice the diameter, the product is the solidity of the dodecahedron.

EXAM. Required the solid content of a dodecahedron, the diameter being 26.72424, and each side of the containing pentagons 12?

*Ans.* 13241.79.

### 5. Of the ICOSAHEDRON.

To find the superficies of an Icosahedron.

This body is contained by 20 equal equilateral triangles; therefore, find the area of one of these triangles, and multiply it by 20; the product is the surface of the Icosahedron.

EXAM. What is the superficies of an Icosahedron, each side of its triangular faces being 12 inches?

*Ans.* 1247.076.

To find the solidity of an Icosahedron.

The Icosahedron consists of twenty equal triangular pyramids, whose vertices are in the center of the inscribed sphere; therefore, measure the diameter of the solid, and multiply the area of one face by one sixth part of the diameter, and multiply the product by 20; the last product is the solid content of the icosahedron.

EXAM. Required the solidity of an icosahedron, its diameter being 18.138264, and the side of each triangular face 12 inches?

*Ans.* 3769.96 cubic inches.

The

The superficies and solidity of any of the above bodies may be easily had from the following table, wherein the side of each of the containing plains is 1.

Names.	Superficies.	Solid content.
Tetrahedron	1.732050	0.1178511
Hexahedron	6.000000	1.0000000
Octahedron	3.464101	0.4714045
Dodecahedron	20.645728	7.6631189
Icosahedron	8.660254	2.1816950

To use this Table, observe the following rules :

1. When the superficies is wanted, multiply the tabular number by the square of the lineal side ; the product is the superficies.

2. When the solidity is required, multiply the tabular number by the cube of the lineal side ; the product is the solidity.

**EXAM.** Required the superficies and solidity of a dodecahedron, the lineal side being 3?

*Ans.* The superficies is 185.81, and the solidity is 206.904.



## OF GAUGING.

1. To gauge any vessel, or to find the quantity of liquor it can contain;

Find how many solid inches will fill the cavity of the vessel, and divide these by the number of solid inches which make a pint or gallon; the quotient is the content of the vessel in pints or gallons.

### TABLE of Cubic Inches in several Measures.

104	Cubic inches = 1 Scots pint
2210	Ditto = 1 Scots wheat firlo
282	Ditto = 1 English ale gallon
231	Ditto = 1 English wine gallon
268.8	Ditto = 1 English corn gallon
2150.4	Ditto = 1 English corn or malt bushel.

From these divisors, multipliers may be found thus: Divide 1 by any number in this table, the quotient is a constant multiplier, by which the content of any vessel in cubic inches being multiplied, the product is its content in that measure for which the multiplier was found.

### TABLE of Divisors and Multipliers.

104	.009615 Scots pints
282	.003546 ale gallons
231	.004329 wine gallons
268.8	.0037202 corn gallons
2150.42	.00046502 malt bushels.

N. B. In gauging vessels, the dimensions are always taken in inches and decimals of an inch.

**EXAM. 1.** Suppose a vessel in form of a parallelopiped, the length of its base 27 inches, breadth  $16\frac{1}{2}$  inches, and the depth of the vessel  $32\frac{1}{2}$  inches; required the content in Scots pints, and English ale gallons?

$$27 \times 16.5 \times 32.25 = 14367.375 \text{ cubic inches.}$$

Now, if you divide 14367.375 by 104, or multiply them by .009615, the result will be 138.14 Scots pints, the content of the vessel.

And, if you divide 14367.375 by 282, or multiply them by .003546, the result will be the content of the vessel in English ale gallons, viz. 50.946.

In the same manner, the content of any vessel may be found; but those who practise gauging proceed thus:

In any vessel equally wide from top to bottom, they compute the area of its base in square inches; and, by dividing or multiplying these by the numbers in the tables, get what they call the area in gallons, (that is, the number of gallons which the vessel contains when the liquid is only one inch deep), which multiplied by the depth of the liquid, gives the quantity contained in the vessel,

**EXAM. 2.** Suppose a trough, or cistern, in form of a right angled parallelopiped, its base 27 inches long, and  $16\frac{1}{2}$  inches wide, and the height of the vessel  $32\frac{1}{2}$  inches, but the depth of liquor only 20 inches; required its content in English ale gallons?

*Ans.* 31.594.

$27 \times 16.5 \times .003546 = 1.579743$  the area of the base in gallons, which multiplied by 20, produces 31.594 ale gallons.

3. Suppose

3. Suppose a cylindric vessel hath the diameter of its base 20 inches, and its height 30 inches; required the content in wine gallons? *Ans.* 40.8.

$$20 \times 20 \times .7854 \times 30 \times .004329 = 40.8.$$

Suppose a tub having circular bases, the diameter of the mouth is 60 inches, and the diameter of the bottom is 36 inches, and the perpendicular depth from top to bottom is 30 inches; required its content in Scots pints and English ale gallons?

This vessel is to be considered as the frustum of a cone, and, on this supposition, its content will be 55417.8 cubic inches; which, by reduction, is 532.8 Scots pints, or 196.5 English ale gallons.

The calculation is tedious by common arithmetic, but may be easily performed by logarithms; thus,

To twice the logarithm of the diameter of one base add the logarithm of .7854, the sum is the logarithm of the area of that base. Do the same for the area of the other base, and find the numbers answering to each. Then add the logarithms of the areas of the two bases, and take half of the sum, and find the number answering thereto.

Add the areas of the two bases, and the last found number, and multiply the sum by one third part of the depth, the product is the content in cubic inches.

Operation for the last Example.

Log.	Log.
Diameter 60 1.781513	Diameter 36 1.5563025
1.781513	1.5563025
.7854 9.8950909	.7854 9.8950909
2827.44 3.4513935	1017.8784 = 3.0076959
1917.8784	2827.44 3.4513935
1696.464	2)6.4590894
5541.7824	
Mult. by 10	1696.464 = 3.2293447
Ans. 55417.824	

The content of any vessel of this form may be found with less trouble by this rule:

To the product of the diameters of the two bases, add one third part of the square of their difference; the sum is the square of a mean diameter; which being multiplied by .7854, and the product by the depth of the vessel, gives the content in cubic inches.

EXAM. Let the diameter of the greater base be 60, and of the lesser base 57.6. and the perpendicular height of the tub 29.976 inches; required its content?

Ans. 81410.3 cubic inches.

The content of a vessel of this form may be found without measuring the diameter of the bottom, or least base; thus, Measure the diameter of its mouth, or upper base, AB, the diagonal BC, and the length of the stave AC (Fig. B. plate 4) Then, in the triangle ABC, the three

three sides are known; and, having drawn the perpendicular CE, we have,

As  $AB:BC+CA::BC-CA:BE-EA$ , which is equal to CD; and hence AE and EC may be found. And, when AB, CD, and CE are known, the content of the vessel may be found by the last rule.

EXAM. Let the diameter  $AB=40$ , the diagonal  $BC=42$ , and the length of the stave  $AC=20$  inches, required the content of the vessel?

Ans. 21371.58 cubic inches.

By the above rule,  $BE-EA$ , or  $CD=34.1$  inches, and  $AE=2.95$  inches;  $\overline{AC}^2 - \overline{AE}^2 = \overline{EC}^2 = 391.2975$ ; hence  $EC=19.7812$  inches; and the rest of the operation is the same as in the last example.

When vessels are not equally wide from top to bottom, gaugers consider them as frustums of some regular solid, and compute their contents at every inch of their depth, which contents they enter in a table, and, when they come to survey, they have only to take the depth; and, by comparing the wet inches with the table, have the content by inspection.

## 2. To Gauge a CASK.

Casks are distinguished into the following four varieties:

1. Such as resemble the middle frustum of a spheroid.
2. Such as resemble the middle frustum of a parabolic spindle.
3. Such as being cut through in the middle, the two parts are parabolic conoids.
4. Such as being cut through in the middle, the parts are the lower frustums of two equal cones.

Measure

Measure the head and bung diameters, and the length of the cask in inches; and then,

1. If the staves are very much curved, the cask is supposed to be the middle zone, or frustum of a spheroid; and its content may be found by this rule.

To twice the square of the bung diameter, add the square of the head diameter; multiply the sum by the length of the cask, and divide the product by 3.8197, the quotient is the content in cubic inches.

EXAM. Suppose there is a spheroidal cask, its bung diameter = 31.5 inches, head diameter = 24.5 inches, and the length of cask = 42 inches; required its content in English ale gallons?

*Ans.* 100.78.

2. If the staves of a cask are less curved than was supposed in the last article, the cask is taken for the middle frustum, or zone, of a parabolic spindle; and its content is computed by this rule.

To twice the square of the bung diameter, add the square of the head diameter, and from the sum subtract four tenths of the square of the difference of the diameters; divide the remainder by 3.8197, and multiply the quotient by the length of the cask; the product is its content in cubic inches.

EXAM. Let the bung diameter = 34 inches, the head diameter = 30, and the length of the cask = 40 inches; what is its content in English ale gallons?

*Ans.* 119.039.

3. When the staves of a cask are very little curved, the cask is supposed to consist of the two lower frustums of two equal parabolic conoids, their greatest bases joined together

together in the middle of the cask; the content of such a vessel may be found by this rule.

To the square of the bung diameter, add the square of the head diameter; multiply the sum by .3927, and the product by the length of the cask; the last product is the content in cubic inches.

EXAM. Let the bung diameter of such a cask = 32 inches, the head diameter = 24, and the length of the cask = 42 inches; required its content in cubic inches and ale gallons?

*Ans.* 36760.191 cubic inches, or 109.07 ale gallons.

4. If the staves of a cask are straight between the bung and the ends of the cask, the vessel is supposed to consist of the two lower frustums of equal cones; and its content is found by this rule.

To the sum of the squares of the head and bung diameters add their product; multiply the sum by the length of the cask, and divide the product by 3.8197; the quotient is the content in cubic inches.

EXAM. Required the content of a cask in ale gallons, its bung diameter being 32 inches, its head diameter 24 inches, and the length 40 inches?

*Ans.* 87.93.

By these rules, the content of any cask may be found, it being known to which of the four varieties the cask belongs; but, in common practice,

A mean diameter, whereby the cask is reduced to a cylinder in either variety, is found thus:

5. Multiply the difference between the bung and head diameters by .7 for the spheroid, by .65 for the spindle, by .6 for the conoids, and by .55 for the cones; add the product

product to the head diameter; the sum is a mean diameter; or the diameter of the base of a cylinder equal to the cask, their lengths being the same. The mean diameter being squared and multiplied by .7854, and the product by the length of the cask, gives the content in cubic inches, which may be reduced to gallons by the table.

EXAM. Suppose the bung diameter is 30 inches, the head diameter 20 inches, and the length of the cask 40 inches; required its content in ale gallons according to each variety?

*Difference of BUNG and HEAD DIAMETERS is 10 INCHES.*

*M. D.*

$10 \times .7 = 7.$  and  $20 + 7 = 27$  for the spheroid

$10 \times .65 = 6.5$  and  $20 + 6.5 = 26.5$  for the spindle

$10 \times .6 = 6.$  and  $20 + 6 = 26$  for the conoids

$10 \times .55 = 5.5$  and  $20 + 5.5 = 25.5$  for the cones.

*For the CONTENT in ALE GALLONS.*

$27 \times 27 \times .7854 \times 40 \times .003546 = 81.21$  spheroid

$26.5 \times 26.5 \times .7854 \times 40 \times .003546 = 78.23$  spindle

$26 \times 26 \times .7854 \times 40 \times .003546 = 75.3$  conoids

$25.5 \times 25.5 \times .7854 \times 40 \times .003546 = 72.43$  cones.

Mr. WARR, who had much practice in gauging, says, He never gauged a cask that contained so much as the first variety makes it; and therefore, recommends the 2d and 3d varieties as the best general rules for gauging casks.



*To Gauge MALT.*

1. If the malt lie on the floor in a rectangular form, multiply the length by the breadth, and the product by the depth, all taken in inches; the product is the number of cubic inches in the quantity; which being divided by 2150.42, the quotient is the number of bushels.

The same rule serves for finding the quantity of malt contained in any vessel in form of a parallelopipedon.

EXAM. Suppose a quantity of malt on the floor, 288 inches long, 144 inches broad, and  $9\frac{1}{2}$  inches deep; required the number of bushels?

*Ans.* 183.21.

2. When malt is in a cistern, or any vessel, the content of the vessel is to be found in cubic inches, by some of the former rules, and then divided by 2150.42, the quotient is the number of bushels.

3. To find the solidity of any irregular solid, such as a thorn or whin-bush.

Put the irregular body into any vessel, and fill it with water; take out the body, and the water will fall lower, and leave a part of the vessel empty, equal to the solidity of the body to be measured; then measure so much water by a vessel of a known capacity as shall fill up the empty space, and the number of cubic inches in that space, and consequently in the irregular body, will be known.

4. To find the tonnage or burden of a ship, multiply the length of the keel, taken within board, by the breadth of the ship at the mid ship-beam, taken also within board, or from plank to plank, and multiply the product by the  
C c
depth

depth of the ship taken from the plank below the keelson to the under part of the upper deck plank, and divide the product by 94, the quotient is the tonnage required.

5. To find the dimensions of a vessel that shall contain a given quantity.

**General RULE.** In vessels equally wide from top to bottom: If the base be given, divide the content in cubic inches, by the area of the base in square inches; the quotient is the depth or length of the vessel in inches.

If the length or depth of the vessel be given, divide the content by the depth, the quotient is the area of the base; and the area of the base being known, its side, if it must be a square; its length and breadth, if it must be a rectangle; or its diameter, if it must be a circle, may be found.

### EXAMPLES.

1. It is required to make a vessel in form of a parallelopipedon, its base being 19 inches by 14, which shall contain 40 ale gallons; required its depth or length?

*Ans.* 42.4 inches.

For the area of the base is  $19 \times 14 = 266$  square inches, and 40 ale gallons  $= 282 \times 40 = 11280$  cubic inches; divide 11280 by 266, the quotient 42.4 is the length.

2. A vessel, equally wide from top to bottom, and that shall contain 40 ale gallons, is to be made, its depth 42.4 inches; required the area of its base?

*Ans.*  $\frac{11280}{42.4} = 266$  square inches.

If the base be a square, its side will be the square root of 266, which is 16.3 inches.

If the base be a rectangle, and one of its sides 19 inches, the other will be  $\frac{266}{19} = 14$  inches.

If the vessel be a cylinder, its base is a circle; and, to find its diameter, divide 266 by .7854; the square root of the quotient, viz. 18.4 inches is the diameter.

6. To find the dimensions of a round vessel, having unequal bases, that shall contain a given quantity.

CASE 1. When the diameters of the two bases are given, to find its depth.

To the product of the diameters of the bases, add one third part of the square of their difference; multiply the sum by .7854, and divide the given content in cubic inches by the product; the quotient is the depth of the vessel.

EXAM. A tub is to be made that shall contain 196.5 English ale gallons, the diameter of its greater base = 60 inches, and of its lesser base = 36 inches; required its perpendicular depth?

Ans. 30 inches.

CASE 2. When the depth, and the diameter of one base are given, to find the diameter of the other base.

Multiply .7854 by the perpendicular depth, and divide the given content of the vessel in cubic inches by the product, the quotient is the square of a mean diameter, which consists of the product of the two diameters, and one third part of the square of their difference; and, when the diameter of one base is given, the other is found by the solution of a quadratic equation; thus,

C. c. 2.

Put

Put  $a$  = the given diameter,  $x$  = the diameter sought, and  $S$  = the quotient of the given content, divided by the product of the depth into .7854.

Then  $ax + \frac{a^2 - x^2}{3} = S$ , that is,  $ax + \frac{aa - 2ax + xx}{3} = S$

$$3ax + aa - 2ax + xx = 3S$$

$$xx + ax = 3S - aa$$

$$xx + ax + \frac{aa}{4} = 3S - aa + \frac{aa}{4} = 3S - \frac{3aa}{4}$$

$$x + \frac{a}{2} = \sqrt{3S - \frac{3aa}{4}}$$

$$x = \sqrt{3S - \frac{3aa}{4}} - \frac{a}{2} \text{ which gives this rule:}$$

Multiply the given depth by .7854, and, divide the content of the vessel by the product; multiply the quotient by 3, and subtract  $\frac{3}{4}$  of the square of the given diameter from the product; extract the square root of the remainder, and subtract one half of the given diameter from the root; the remainder is the diameter sought.

EXAM. It is required to find the diameter of the lesser base of a tub, which shall contain 60 ale gallons; the diameter of the greater base being 40 inches, and the depth 24 inches?

Ans. The diameter of the lesser base is 18.637 inches.

1st, The content of the vessel is  $282 \times 60 = 16920$  cubic inches.

$$2d, .7854 \times 24 = 18.8496 \text{ and } \frac{16920}{18.8496} = 897.634$$

$$3d, 897.634 \times 3 = 2692.902, \text{ and } \frac{3}{4} \text{ of } 40^2 \text{ is } 1200.$$

$$4th, 2692.902 - 1200 = 1492.902.$$

$$5th, \sqrt{1492.902} = 38.637, \text{ and } 38.637 - 20 = 18.637.$$

The

The content of a cask, with the bung and head diameters being given, to find its length ; the cask being of the 3d variety.

Multiply the sum of the squares of the bung and head diameters by .3927, and divide the content of the cask in cubic inches by the product ; the quotient is the length of the cask.

EXAM. A cask is to be made which shall contain 109.07 English ale gallons, its bung diameter = 32 inches, and its head diameter = 29 inches ; what must its length be ?

*Ans.* 41.99, or 42 inches.

SECT.

## S E C T. VI.

## NAVIGATION.

DEFINITIONS *and* PRINCIPLES, *Fig. 80.*

1. **N**AVIGATION is the art of conducting a ship in the sea from one port or place to another.

2. The earth and sea constitute one spherical body, which differs very little from a perfect globe; its diameter is about 7964 miles, and its circumference 25020 miles.

3. The earth hath two motions, one round the sun in a year, called its *annual motion*; and another round its own axis from west to east, once in twenty-four hours, called its *diurnal motion*.

4. The annual motion of the earth, is the cause of the apparent annual motion of the sun; and its diurnal motion is the cause of the apparent revolution of the sun, moon, and fixed stars from east to west in the space of twenty-four hours.

5. Several circles are supposed to be described on the terraqueous globe, and also in the visible heavens, which altogether are called circles of the sphere; and these are either great or small.

6. A great circle divides the globe into two equal parts, as the *equator*, *meridian*, *horizon*, *ecliptic*, and *vertical circles*.

7. A.

7. A small circle divides the globe into two unequal parts, as the *polar circles*; *tropics*, and other parallels of latitude and declination.

8. The *poles* of the globe P and S are the extremities of its axis; or, they are the quiescent points, when the globe turns round its axis; and opposite to these are the celestial poles in the visible heavens.

9. The *equator* is a great circle surrounding the globe in the middle between the two poles, as *ÆQ*.

10. *Meridians* are great circles passing through the two poles, and cutting the equator at right angles, as *PmS*, &c.

11. The *ecliptic* *EC* is the path of the earth's annual motion round the sun, or the path of the sun's apparent annual motion round the earth; one half of it lies on the north, and the other half on the south side of the equator. The sun appears always somewhere in this circle; for, this being the earth's orbit, in whatever point the earth is, the sun is seen in the opposite point. The sun's daily progress is near one degree.

12. The intersections, or points where the ecliptic crosses the equator represented by the point *A*, are called *equinoctial points*; because, when the sun appears in either of these, the day and night are equal.

13. The points of the ecliptic, which are farthest from the equator at *E* and *C*, are called *solstitial points*; because, while the sun passes over a small part of the ecliptic on each side of these points, he is for some time nearly at the same distance from the equator, and so seems to stand still.

14. The

14. The meridian which passes through the equinoctial points is called the *equinoctial colure*, as P A S; and that which passes through the solstitial points is called the *solstitial colure*, as P r E S. These are the only two meridians which divide both equator and ecliptic into four equal parts.

15. The ecliptic is divided into twelve equal parts, called *signs*, each containing  $30^{\circ}$ , namely,

♈	♉	♊	♋	♌	♍	♎
Aries,	Taurus,	Gemini,	Cancer,	Leo,	Virgo,	Libra,
♏	♐	♑	♒	♓		
Scorpio,	Sagittarius,	Capricornus,	Aquarius,	Pisces.		

16. The *Zenith* Z, is that point in the visible heavens directly over the spectator's head, and the *Nadir* N, is the opposite point.

17. There are two horizons, the *rational* and *sensible*. The *rational horizon* H O, is a great circle surrounding the globe in the middle between the zenith and nadir, which are its poles.

18. The *sensible, or visible horizon*, is the boundary of the spectator's view, and is properly a plane touching the earth in the place where the spectator stands.

19. *Latitude* on the earth, is the distance of any place north or south from the equator, reckoned on the meridian. Thus Z r E is the latitude of Z.

20. *Longitude* on the earth, is the distance of any place east or west from the first meridian, reckoned on the equator; or the longitude of any place is the arch of the equator between the first meridian and the meridian of that place.

21. The



21. The first meridian is not fixed to any particular place, but chosen according to the fancy of different authors and nations. In English maps and globes, the meridian of London is the first. In French maps, the meridian of Ferro Island, &c.

22. In the visible heavens, circles passing through the poles of the ecliptic, and cutting it at right angles, are called *circles of latitude*, as *na*.

23. *Latitude* in the heavens, is the distance of any star from the ecliptic, reckoned on the circle of latitude passing through the star.

24. *Longitude* of any star in the heavens, is an arch of the ecliptic between the vernal equinox and the circle of latitude passing through the star, reckoning in the order of the signs.

25. *Declination*, is the distance of the sun, or any star, north or south from the equator, reckoned on the meridian.

26. The obliquity of the ecliptic, or the angle it makes with the equator *CAQ*, is  $23^{\circ} 28'$ , and the same is the sun's greatest declination.

27. Circles parallel to the equator, are called *parallels of latitude on the earth*, and *parallels of declination in the heavens*. One of these is supposed to pass through every point in the meridian; but four are distinguished by particular names, viz. the two *tropics*, and the two *polar circles*.

28. The *tropics* *Ed* and *bC* are distant from the equator  $23^{\circ} 28'$ . That on the north is called the *Tropic of Cancer*, because it touches the ecliptic in the beginning

D. d.

of

of the sign Cancer; and that on the south is called the *Tropic of Capricorn*, because it touches the ecliptic in the beginning of the sign Capricorn. These circles are the apparent paths of the sun on the solstitial days, viz. on the 21st of June, and 22d of December.

29. The tropics are the boundaries of the sun's declination north or south from the equator. The space between them is called the *torrid zone*; because the sun is always vertical to some place within this tract of the earth's surface.

30. The arctic, or north polar circle *ar*, is  $23^{\circ} 28'$  distant from the north pole; the space within it is called the *north frigid zone*. The antarctic, or south polar circle, *te*, is  $23^{\circ} 28'$  distant from the south pole, and the space within it is called the *south frigid zone*.

31. The space between the tropic of Cancer and the arctic circle, is called the *north temperate zone*; and the space between the tropic of Capricorn and the antarctic circle, is called the *south temperate zone*.

32. When the sun appears in either equinox, he hath no declination. In all other points of his apparent path, the arch of the ecliptic between the vernal equinox and the sun's place, is called the *sun's longitude*. The arch of the equator between the same equinox and the meridian passing through the sun's place, is called the *sun's right ascension*, and the arch of the meridian between the equator and the sun's center, is called the *sun's declination*. The same is to be understood of the moon, or any star.

33. When the sun is in either equinox, he rises due east, and sets due west; but in all other points of the ecliptic,

ecliptic, he rises and sets at some distance from the east and west points; and this distance is called the *sun's amplitude*. The greatest amplitude  $Am$  or  $An$ , is when the sun is in either tropic.

34. The amplitude is always of the same name with the declination; that is, when the sun hath north declination, he rises between east and north, and sets between west and north; but, when he hath south declination, he rises between east and south, and sets between west and south. The same is true of the moon, or any star.

35. Vertical or azimuth circles  $ZmN$  and  $ZnN$  pass through the zenith and nadir, and cut the horizon at right angles. Of these, the meridian cuts the horizon in the north and south points; and the prime vertical crosses the meridian at right angles in the zenith, and cuts the horizon in the east and west points.

36. An azimuth circle is supposed to pass through the sun, moon, or any star which appears above the horizon. That part of the azimuth circle between the horizon and the sun at any time, is called his *altitude*; and the other part between the sun and the zenith, is called his *zenith distance*; and the arch of the horizon, between the point where the azimuth circle cuts it, and the north or south points, is called the *sun's azimuth*. This is reckoned either from north or south, the one being always the supplement of the other.

37. The arch of the equator between the vernal equinox and that point of it which rises with the sun, is called the *sun's oblique ascension*; and the arch of the equator between the same equinox and that point of it which sets with the sun, is called the *sun's oblique descension*.

38. The difference between the sun's right and oblique ascension, is called the *ascensional difference*. This quantity reduced to time, at the rate of  $15^{\circ}$  to an hour, shews how long the sun riseth before or after 6 o'clock.

39. Oblique ascension and descension, with ascensional difference, apply only to places between the equator and polar circles. For, at the equator, the right and oblique ascension are always the same; and within the frigid zones, the sun shines for several days and nights together without setting.

40. The mariner's compass, *Fig. 81.* is a representation of the sensible horizon of that place, where it is, by means of a circular piece of paper, called a *card*, its circumference being divided into 32 equal parts, called *points*. This card being properly fixed to a piece of steel, called a *needle*, (which, by the touch of a loadstone, is endued with the property of pointing north and south) and placed on a supporter on which it can turn freely round; the lines drawn from the center to north, south, and all other points on the card, will tend towards the same points of the horizon; and therefore, by the help of this card or compass, a ship may be directed in any proposed tract or course.

41. The several parts of the earth's surface are most conveniently represented on the surface of a small globe, because this body is similar to the earth; but this does not answer the purposes of navigators, and therefore other representations on plain surfaces have been contrived.

42. The first and most simple representation of the earth's surface, is the *plain chart*, wherein the meridians  
are

are parallel to each other, and the parallels of latitude are all of the same length with the equator; but this does not agree with the figure of the earth, and therefore, the extent of lands and seas, the situations, distances, and bearings of places, and consequently the solution of questions in navigation on the principles of the plain chart, cannot be true. The error, however, is not equally great in all places; for, within the torrid zone, the plain chart agrees nearly with the globe: And in maps representing a small part of the earth's surface, of about 2 or 3 degrees of latitude, and as many of longitude, the error is not very considerable, if the place represented lies between the equator and  $40^{\circ}$  of latitude. The error increases with the latitude, and is the greatest of all towards the poles.

43. The resolution of questions in navigation, on the principles of the plain chart, is called *plain-sailing*. The other methods, known by the names of Middle Latitude, and Mercator's Sailing, being only attempts to correct the errors of the plain chart, cannot be understood until plain sailing is learned.

44. A ship's course is known by the compass, and the distance she sails in an hour is found by the log-line thus: A small cord, of a sufficient length, is marked with knots at the distance of  $\frac{1}{120}$  part of a nautical mile, or minute, from each other; and, to the end of the cord, a piece of wood is fixed, called the *log*. The line thus prepared is wound up on a reel, which turns easily round. They heave out the log, and, at the same time, set a half-minute sand-glass a-running, and so find how many knots the ship sails in half a minute; and, because half a minute

minute is the same part of an hour, as a knot is of a mile, the number of knots shows the number of miles the ship runs in an hour.

*Note.* A nautical mile is 6120 feet; and consequently the distance between the knots on the log-line should be 51 feet, viz.  $\frac{6120}{120}$ ; but, as it is safer to have the reckoning rather before the ship than after it, therefore 50 feet may be taken as the proper length between knot and knot on the log-line.

45. A rhumb line, is any right line drawn from the center of the compass; such as N.E. S. S. W. &c.

46. The course is the angle which the rhumb line makes with the meridian; and may be reckoned either in degrees, or points of the compass.

47. There are 32 points in the compass, and each of these is divided into halves and quarters.

48. Because the whole circumference of any circle consists of  $360^\circ$ , and the same is divided into 32 points: to find how many degrees one point contains, divide  $360^\circ$  by 32, and the quotient will be  $11^\circ 15' =$  one point.

49. Distance is the number of miles or leagues sailed on any rhumb in a given time,

50. Difference of latitude is the distance a ship has made to the northward or southward of the place she set out from, and it is reckoned on the meridian.

51. Departure is the distance east or west, which a ship has made from the meridian of the place she sailed from, reckoned on the parallel of latitude which she  
has

has arrived in; and in the plain chart, it is the same as difference of longitude.

52. If a ship sails on a meridian either north or south, she makes no departure, and her distance and difference of latitude are the same.

53. If a ship sails due east or west, she makes no difference of latitude, and her departure and distance are the same; but,

54. *Fig. 82.* If a ship sails from any point A to any other point C, neither lying in the same meridian with A, nor in the same parallel of latitude; draw AB, the meridian of the place sailed from, and from C let fall the perpendicular CB; then AC is the distance sailed; AB the difference of latitude of the places A and C, and BC is the departure or distance of the meridians of A and C; also, CAB is the angle of course, and ACB the complement of the course.

55. The difference of latitude and departure are always the legs of a right angled triangle, and the distance sailed is the hypotenuse. The departure is always opposite to the angle of course, and the difference of latitude is opposite to the complement of the course.

56. When the course is  $45^{\circ}$ , or 4 points, the difference of latitude is equal to the departure.

57. When the course is less than  $45^{\circ}$ , or four points, the difference of latitude is greater than the departure; but when the course is greater than  $45^{\circ}$  or four points, the departure is greater than the difference of latitude.

58. When a figure representing a ship's course is to be constructed, it must first be considered whether the  
ship

ship sails northward or southward; and whether she goes to the eastward or westward of the place she departed from; that the lines of the figure may be drawn accordingly.

Let the upper part of the paper, or whatever the figure is drawn on, represent the north, then the lower part will be south, the right hand east, and the left hand west.

Draw a north and south line to represent the meridian of the place sailed from; then if the course is to the southward, mark the upper end of the line for the place sailed from; but if the course is northward, mark the lower end of the line for that place.

The line representing the distance sailed, must be drawn through the point sailed from; and the parts given must be taken from a scale, and the figure completed according to the conditions of the question.

A TABLE



A TABLE

Of the Proportions for Solving the Cases of PLAIN SAILING : in the Nautical Terms.

Cases	Given.	Sought.	Solutions.
1.	Course and Distance	Diff. lat. and Depart.	$R : \cos. \text{course} :: \text{dist.} : \text{diff. lat.}$ $R : \sin. \text{course} :: \text{dist.} : \text{departure}$
2.	Distance and Diff. lat.	Course and Depart.	$\text{Dist.} : \text{diff. lat.} :: R : \cos. \text{course}$ $R : \sin. \text{course} :: \text{dist.} : \text{departure}$
3.	Course and Diff. lat.	Dist. and Depart.	$\cos. \text{Cour.} : R :: \text{diff. lat.} : \text{distance}$ $R : \tan. \text{cour.} :: \text{diff. lat.} : \text{departure}$
4.	Course and Depart.	Dist. and Diff. lat.	$\sin. \text{cour.} : R :: \text{depart.} : \text{distance}$ $\tan. \text{cour.} : R :: \text{depart.} : \text{diff. lat.}$
5.	Distance and Depart.	Course and Diff. lat.	$\text{Dist.} : \text{depart.} :: R : \sin. \text{course}$ $R : \cos. \text{course} : \text{dist.} : \text{diff. lat.}$
6.	Diff. lat. and Depart	Course and distance.	$\text{Diff. lat.} : \text{depart.} :: R : \tan. \text{course}$ $\sin. \text{cour.} : R :: \text{depart.} : \text{distance.}$

Hence, to solve the six cases of plain sailing, is no more than to solve as many right angled plain triangles.

E. e.

PLAIN

PLAIN, SAILING, *Fig. 82.*

**CASE 1.** Given the course and distance sailed, to find the difference of latitude and departure.

**EXAM. 1.** A ship, from latitude  $12^{\circ} 30' \text{ N.}$  has sailed N. W. by N. 147 miles; what latitude hath she come to, and what departure hath she made?

The course, being three points, is  $33^{\circ} 45'$ ;

To find the difference of latitude :

As Rad. : Cos course :: Dist. : diff. Lat. = 122.2 m.

To find the departure,

As Rad. : Sine course :: Dist. : departure = 81.7.

To latitude failed from  $12^{\circ} 30' \text{ N.}$

Add diff. of latitude  $2^{\circ} 2'$

Sum is the latitude come to  $14^{\circ} 32' \text{ N.}$

**EXAM. 2.** A ship from latitude  $20^{\circ} 20' \text{ N.}$  has sailed 265 miles N. E. by E.  $\frac{1}{2}$  E. ; required the latitude come to, and the departure?

*Ans.* Latitude  $22^{\circ} 25' \text{ N.}$  departure 233.7 miles.

**CASE 2.** Given the distance sailed, and difference of latitude, to find the course and departure.

**EXAM. 1.** A ship from latitude  $15^{\circ} 20' \text{ S.}$  fails 320 miles between S. and W. and then is in latitude  $19^{\circ} 25' \text{ S.}$ ; required her course and departure?

*Ans.* Course  $40^{\circ} 3' \text{ S.}$  Westward, departure 206 miles.

2. A ship from latitude  $20^{\circ} 49' \text{ N.}$  fails 268 miles between S. and E. and then is in latitude  $17^{\circ} 1' \text{ N.}$ ; required her course and departure?

*Ans.* Course S.  $31^{\circ} 43' \text{ W.}$  departure 141.

**CASE**

**CASE 3.** Given the difference of latitude and course, to find the distance and departure?

**EXAM. 1.** A ship from latitude  $14^{\circ} 17'$  S. sails S. E.  $\frac{3}{4}$  E. until she is in latitude  $19^{\circ} 10'$  S.; required the distance sailed, and departure?

*Ans.* Distance 491.8 m. departure 395 m.

2. A ship from latitude  $19^{\circ} 39'$  N. sails W. N. W. until she is in latitude  $23^{\circ} 30'$  N.; required her distance and departure?

*Ans.* Distance 603.6 departure 557.6.

**CASE 4.** Given the course and departure, to find the distance sailed, and difference of latitude.

**EXAM. 1.** A ship from latitude  $16^{\circ} 16'$  S. sails S. W. by W.  $\frac{1}{2}$  W. until she has made 499 miles of Westing; required the distance sailed, and the latitude come to?

*Ans.* Dist. 565.8 m. and lat. come to is  $20^{\circ} 43'$  S.

2. A ship from latitude  $27^{\circ} 37'$  N. sails S. E. by S.  $\frac{1}{4}$  E. until she has made 256 miles of Easting; required her distance sailed, and the latitude come to?

*Ans.* Distance 429.7, latitude  $21^{\circ} 52'$  N.

**CASE 5.** Given the distance and departure, to find the course and difference of latitude.

**EXAM. 1.** A ship from latitude  $1^{\circ} 30'$  N. sails 294 miles between S. and E. and then has made 186 miles of Easting; what course did she steer, and what latitude hath she come to?

*Ans.* Course  $39^{\circ} 14'$ , or S. E. by S.  $\frac{1}{2}$  E. and she is in latitude  $2^{\circ} 18'$  South.

2. A ship from latitude  $19^{\circ}$  S. fails 396 miles between W. and N. and thereby made 219 miles of departure; required her course and latitude come to?

*Ans.* Course N.  $34^{\circ} 51'$  W. latitude  $2^{\circ} 22'$  N.

CASE 6. Given the difference of latitude and departure, to find the course and distance failed.

EXAM. 1. A ship from latitude  $23^{\circ} 28'$  N. has failed between N. and W. until she came to latitude  $27^{\circ} 1'$  N. and made 199 miles of Westing; required her course and distance? *Ans.* Course N.  $43^{\circ} 3'$  W. dist. 291.5 m.

2. A ship from latitude  $48^{\circ} 30'$  N. is arrived in latitude  $49^{\circ} 54'$  N. and is got 54 miles to the westward of her departed meridian; what course did she steer, and what distance hath she failed?

*Ans.* Course  $32^{\circ} 44'$ , or N. W. by N. dist. 99.86 m.

*Of the TABLE of difference of Latitude and Departure.*

This table consists of numbers expressing the sides of right angled triangles, the distance being the hypotenuse, the difference of latitude and departure the two legs, and the course and its complement are the acute angles. Any two of these being given, (except the two acute angles) the other parts of the triangle may be found by inspection, if within the extent of the table.

The distances 1, 2, 3, &c. at top and bottom of each page may be accounted 10, 20, 30, or 100, 200, 300, &c. if the numbers under the titles of difference of latitude and departure are increased in the same proportion.

Hence, if the distance consists of two significant figures, the difference of latitude and departure are to be taken out at twice, and, if of three figures, at thrice.

1. Given

1. Given the course and distance sailed, to find the difference of latitude and departure.

Find the course on the left hand side of the page, if it be less than  $45^\circ$ , or 4 points; and on the right hand side, if it be greater than  $45^\circ$ ; and even with the course, below or above the distance, you have the difference of latitude and departure.

EXAM. 1. A ship sails S..S. W.  $\frac{3}{4}$  W. 50 miles, required her difference of latitude and departure?

Course.	Dist.	Diff. Lat.	Depart.
$2\frac{3}{4} p.$	50	42.85	25.75 the answer.

2. A ship sails E. N. E.  $\frac{1}{4}$  E. 68 miles; required her difference of latitude and departure?

Course.	Dist.	Diff. Lat.	Depart.
$6\frac{1}{4}$	60	20.21	56.49
	8	3.06	7.39
	<hr/>	<hr/>	<hr/>
	68	23.27	63.88 the answer.

3. A ship sails 148 miles on a course  $49^\circ$  S. eastward, required her difference of latitude and departure.

Course.	Dist.	Diff. Lat.	Departure.
$49^\circ$	100	65.60	75.47
	40	26.24	30.18
	8	5.24	6.03
	<hr/>	<hr/>	<hr/>
	148	97.08	111.68 the answer.

2. The difference of latitude and departure being given, to find the course and distance.

Find the difference of latitude and departure together in their columns, or the nearest numbers to them; and  
the

the course is on the same line at the side of the page, and the distance at the top or bottom.

When the difference of latitude and departure cannot be found nearly in the table, take some part of each, such as  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{12}$ ; and these parts being found, the course is even with them on the side of the page, and the same part of the distance on the head or foot of the page, as was taken of the difference of latitude and departure.

EXAM. 1. Given the difference of latitude 59 miles S. and the departure 68 miles W.; required the course and distance?

*Ans.* The course is  $49^\circ$  S. Westward, or S. W.  $\frac{1}{2}$  W. and the distance 90 miles.

In this example, the difference of latitude and departure are found, nearly, in the column marked with 9 of distance, and even with  $49^\circ$  on the side of the page.

2. Given the difference of latitude 30 miles N. and the departure 18 miles E.; required the course and distance?

Here the difference of latitude and departure are not to be found together in the table; but, dividing each by 5, the quotients 6 and 3.6 are found under 7 of distance, and even with  $31^\circ$ ; therefore, the distance is  $7 \times 5 = 35$  miles, and the course is  $31^\circ$  N. Eastward.

3. Given the difference of latitude 86.2 miles S. and the departure 42 miles E.; required the course and distance?

In this example, divide both 86.2 and 42. by 12, the quotients 7.18 and 3.5 are found together under the distance 8, and even with the course  $26^\circ$ ; therefore the course is  $26^\circ$  S. Eastward, and the distance is  $8 \times 12 = 96$  miles.

A table

A table of this extent will do pretty well when the course is even degrees, or points and quarter points of the compass; but, when the course is not exactly found in the table, there will be some error in the answer.

To work a traverse, or to reduce a compound course to a single one.

Find the difference of latitude and departure for each single course, and place them properly in columns, marked N. S. E. W.; find the sum of each column, and the difference of the columns N. and S. will be the difference of latitude; also the difference of the columns E. and W. will be the departure which the ship has made in the whole traverse; and, from these, the direct course and distance may be found.

EXAM. 1. A ship from latitude  $39^{\circ} 30'$  N. meeting with contrary winds, sailed S. E. by S. 67 m. S. E. by E. 53 m. W. S. W. 45 m. N. E. by N. 74 m. W. 57 m. and S. by E. 83 m.; required her difference of latitude, departure, course, and distance from the place she sailed from, and the latitude come to? *Fig. 83.*

<i>Courses.</i>	<i>Dist.</i>	<i>N.</i>	<i>S.</i>	<i>E.</i>	<i>W.</i>
S. E. by S.	67		55.7	37.2	
S. E. by E.	53		29.4	44.1	
W. S. W.	45		17.2		41.6
N. E. by N.	74	61.5		41.1	
W.	57				57.
S. by E.	83		81.4	16.2	
	—		—	—	98.6
		61.5	183.7	138.6	
			61.5	98.6	
			—	—	
			122.2	40.0	

By these different courses, the ship hath got 122.2 m.  
to

to the south, and 40 *m.* to the east of the place sailed from. Hence the course is  $18^{\circ} 7'$  south-eastward, and the distance 128.6 *m.* The latitude come to is  $37^{\circ} 28' \text{ N.}$

2. Yesterday noon we were in latitude  $3^{\circ} 18' \text{ S.}$  and since then, we have plied on the following courses, viz. N. N. E. 22 *m.* N. by W. 30 *m.* N. E. by E. 40 *m.* E. S. E. 25 *m.* S. S. W. 18 *m.* N. W. by N.  $\frac{1}{2}$  W. 50 *m.* N. E.  $\frac{1}{4}$  E. 42 *m.* W. by S.  $\frac{1}{4}$  W. 45 *m.* S. W. by S. 20 *m.* E. by N.  $\frac{1}{4}$  E. 62 *m.*; required our present latitude and departure, with the course and distance made good?

*Ans.* The ship is in latitude  $1^{\circ} 39' \text{ S.}$  departure 58.4 *m.* course  $30^{\circ} 32'$  northeastward, or N. N. E.  $\frac{1}{4}$  E. and distance 115 miles.

3. A ship in latitude  $45^{\circ} \text{ N.}$  is bound for a port in latitude  $50^{\circ} 57' \text{ N.}$  bearing N. W. by W. but, by contrary winds is forced to steer the following courses, viz. N. N. W. 35 leagues, N. E. by N. 42 leagues, W. N. W. 38 leagues, W. 20 leagues, N. W. by N. 23 leagues, N. 12 leagues, E. N. E. 39 leagues, S. W. by W. 48 leagues, N. by E. 31 leagues, N. N. W.  $\frac{1}{2}$  W. 38 leagues, and W. by N. 45 leagues; required the course and distance made good, as also the course and distance from the ship to the port she is bound for?

*Ans.* The course made is  $34^{\circ} 10'$  northwestward, distance 210.7 *m.* and the course from the ship to the port is  $47^{\circ} 38'$  southwestward, and distance 82 miles.

### PARALLEL SAILING.

The distance of two meridians on the equator being given, to find their distance in any parallel of latitude, and *vice versa.* Fig. 84.

Let



Let EP represent a quadrant of the meridian, E a point on the equator, P the pole, and C the centre of the earth, and L any point on the meridian between the equator and the pole, whose latitude is LE, and its complement LP. Draw LB parallel to CE; then, if the quadrant CPE revolve round the axis CP, the point E. will describe the equator, and the point L a parallel of latitude. These circles are, in proportion to each other as their radii CE, and BL. But wholes are as their similar parts; therefore, any part of the equator is to a similar part of the parallel of latitude described by the point L, as CE is to BL; that is, as the radius is to the co-sine of the latitude. Hence, as radius is to the co-sine of any latitude, so is the minutes difference of longitude, or the distance of the two meridians on the equator, to their distance on the parallel of latitude.

CASE 1. Given the latitude and difference of longitude, to find the distance on the parallel.

EXAM. 1. A ship in latitude  $32^{\circ}$  N. sails due W. until her difference of longitude is 384 miles; required the distance sailed?

As Rad.  $\therefore$  Cos. lat.  $32^{\circ} :: m.$  diff. long. 384 : dist. 325.6.

2. How many miles E. or W. in the latitude of  $56^{\circ}$  are equal to  $1^{\circ}$  of longitude?

Ans. 33.5 m.

CASE 2. Given the distance sailed in any parallel of latitude, to find the difference of longitude.

Ff

EXAM.

EXAM. A ship from latitude  $53^{\circ} 36'$  N. longitude  $10^{\circ} 18'$  E. sails due W. 236 m.; required her present longitude?

Ans.  $3^{\circ} 40'$  E

CASE 3. Given the difference of longitude, and the distance sailed, to find the latitude of the parallel sailed on.

EXAM. In what latitude does 384 miles of meridional distance answer to 500 miles difference of longitude?

Ans.  $39^{\circ} 49'$ .

CASE 4. Having the distance of two places in a given parallel, to find the distance of two other places lying under the same meridians with the former in another given parallel of latitude.

EXAM. Two ships in latitude  $32^{\circ} 20'$  N. distant from each other 256 miles, sail directly N. until they are in latitude  $44^{\circ} 30'$  N.; how far are they then asunder?

Ans. 216 m.

## MIDDLE LATITUDE SAILING.

Since the distance of any two meridians varies with the latitude, when a ship sails any distance on an oblique rhumb, and thereby gets into a different latitude; what is called her departure in plain sailing, is not the distance between the meridian of the place sailed from, and the meridian of the ship in her present latitude. To correct this error, the distance of the two meridians in the middle parallel between the latitude sailed from, and that come to, is taken for the true departure; and hence, this correction is called *middle latitude sailing*; which, although

though not perfectly accurate, answers pretty well for keeping an account of the ship's difference of longitude in small distances, such as one day's sailing.

The computations depend on the following rules:

1. Take the half sum of the two latitudes for the middle latitude; and then, by parallel sailing, as co-sine of middle latitude is to the radius, so is the departure to the difference of longitude. Or,

2. Because  $\text{Cos. mid. lat.} : \text{Rad.} :: \text{Dep.} : \text{Diff. long.}$ ; therefore,

$$\text{Cos. mid. lat.} \times \text{Diff. long.} = \text{Rad.} \times \text{Depart.}; \text{ but,}$$

$$\text{Diff. lat.} : \text{Dep.} :: \text{Rad.} : \text{Tan. course}; \text{ therefore,}$$

$$\text{Diff. lat.} \times \text{Tan. course} = \text{Cos. mid. lat.} \times \text{diff. long.}$$

Hence, as  $\text{Cos. mid. lat.}$  is to  $\text{Tan. course}$ , so is  $\text{diff. lat.}$  to  $\text{diff. long.}$

CASE I. Given the latitudes and longitudes of two places, to find their bearing and distance.

EXAM. 1. What is the course and distance from Cape Clear, in Ireland, in latitude  $51^{\circ} 18'$  N. longitude  $9^{\circ} 50'$  W. to the island of St. Mary, one of the Azores, in latitude  $37^{\circ}$  N. and longitude  $22^{\circ} 56'$  W.?

$$\text{Diff. lat. } 14^{\circ} 18' = 858 \text{ m. sum of lats.} = 88^{\circ} 18'. \text{ mid. lat.} = 44^{\circ} 9'. \text{ Diff. long.} = 13^{\circ} 6' = 786 \text{ m.}$$

Then, as  $\text{Diff. lat.} : \text{Diff. long.} :: \text{Cos. mid. lat.} : \text{Tan. course} = 33^{\circ} 19'$ , or S. W. by S. nearly.

As  $\text{Cos. course} : \text{Rad.} :: \text{Diff. lat.} : \text{Dist.} = 1026.7$  miles.

2. What is the course and distance from Leith, in latitude  $56^{\circ}$  N. longitude  $3^{\circ}$  W. to Bergen, in latitude  $60^{\circ}$  N. and longitude  $6^{\circ}$  E.

*Ans.* The course is  $50^{\circ}$  northeastward, or N. E.  $\frac{1}{2}$  E. nearly, and distance is 373.3 miles.

CASE 2. One latitude, course, and distance sailed being given, to find the other latitude and difference of longitude.

EXAM. A ship from the Lizard in latitude  $50^{\circ}$  N. and longitude  $5^{\circ} 14'$  W. sails 150 miles on a course  $50^{\circ} 6'$  south-westward; required her latitude and longitude?

*Ans.* Latitude  $48^{\circ} 24'$ , longitude  $8^{\circ} 10'$  W.

As Rad. : Cos. course :: Dist. : Diff. lat. = 96.2 m. =  $1^{\circ} 36'$ ; therefore, the latitude of the ship is  $48^{\circ} 24'$ , and the middle latitude is  $49^{\circ} 12'$ . As Cos. mid. lat. : Tan. course :: Diff. lat. : Diff. long. 176 m. or  $2^{\circ} 56'$ . In the same manner, examples may be wrought in all the cases.

2. A ship from latitude  $55^{\circ} 57'$  N. longitude  $6^{\circ} 30'$  W. sails N. W. by W. 212 miles; required her longitude and latitude?

*Ans.* Longitude  $11^{\circ} 53'$  W. latitude  $57^{\circ} 55'$  N.

CASE 3. Given the distance and difference of latitude, to find the course and difference of longitude.

EXAM. 1. A ship from latitude  $48^{\circ} 30'$  N. sails 194 miles between N. and W. and then is in latitude  $50^{\circ}$  N. required her course and difference of longitude?

*Ans.* Course  $62^{\circ} 22'$ , or N. W. by W.  $\frac{1}{2}$  W. difference of longitude  $4^{\circ} 23'$  W.

2. A ship from latitude  $47^{\circ} 27' N.$  longitude  $15^{\circ} W.$  fails 424 miles between N. and E. and then is in latitude  $51^{\circ} 59' N.$ ; required her course and longitude?

*Ans.* Course  $50^{\circ} 6'$ , or N. E.  $\frac{1}{2}$  E. longitude  $6^{\circ} 37' W.$

CASE 4. Given the course and difference of latitude, to find the distance and difference of longitude.

EXAM. 1. A ship from a port in latitude  $49^{\circ} 19' N.$  and longitude  $12^{\circ} 14' W.$  fails S. S. E.  $\frac{1}{4}$  E. until she is in latitude  $45^{\circ} 49' N.$ ; required the distance failed, and the longitude of the ship?

*Ans.* Distance 238 m. longitude  $9^{\circ} 28' W.$

2. A ship from latitude  $45^{\circ} 30' N.$  and longitude  $17^{\circ} 30' W.$  fails N. W. by W.  $\frac{1}{4}$  W. until she is in latitude  $49^{\circ} 32' N.$ ; required her distance and longitude?

*Ans.* Distance 470.7 m. longitude  $27^{\circ} 28' W.$

CASE 5. The course and departure being given, to find the distance failed, with the difference of latitude, and of longitude.

EXAM. A ship from latitude  $50^{\circ} 20' N.$  and longitude  $4^{\circ} 12' E.$  fails E. N. E. until her departure is 119 miles: Required the distance failed, with the latitude and longitude of the ship?

*Ans.* Distance 128.8 m. latitude  $51^{\circ} 9' N.$  and longitude  $7^{\circ} 20' E.$

CASE 6. The distance failed and departure being given, to find the course, the difference of latitude and difference of longitude.

EXAM.

**EXAM.** A ship from latitude  $58^{\circ} 20'$  N. and longitude  $4^{\circ} 12'$  W. sails 256 miles between S. and E. and then has made 128 miles of easting: Required her course, latitude and longitude?

*Ans.* Course S.  $30^{\circ}$  E. latitude  $54^{\circ} 40'$ , long.  $0^{\circ} 20'$  W.

**CASE 7.** The difference of latitude and departure being given, to find the course, the distance sailed, and difference of longitude.

**EXAM.** A ship from latitude  $42^{\circ} 30'$  S. and longitude  $43^{\circ} 12'$  W. sails between S. and W. until she is in latitude  $49^{\circ} 2'$  S. and has made 212 miles of westing: Required her course, distance and longitude?

*Ans.* Course S.  $28^{\circ} 24'$  W. distance 445.7 and longitude  $48^{\circ} 16'$  W.

Questions in middle latitude may be solved by the Table of difference of latitude and departure.

**CASE 1.** With the complement of the middle latitude found among the courses, and the difference of longitude among the distances, find the departure in its column.

2. With the difference of latitude and departure found together, you have the course and distance as in plain sailing.

**EXAM.** Let the difference of latitude = 858 m. the difference of longitude = 786 m. the middle latitude =  $44^{\circ} 9'$ , and its complement =  $46^{\circ}$  nearly, as in example 1.

Course.	Diff. Long.	Depart.
$46^{\circ}$	700	503.54
	80	57.54
	6	4.31
	<hr/>	<hr/>
	786	565.39

The

The difference of latitude being 858, and the departure 565.39, if these be divided by 128, the quotients 6.7031 and 4.4171 are nearly found even with the course  $33^\circ$  under the distance 8; therefore, the course is  $33^\circ$ , and distance  $128 \times 8 = 1024$  miles.

CASE 2. With the course found among the degrees, and the distance in its column, find the difference of latitude and departure, as in plain sailing.

2. Find the comp. of the middle latitude among the courses, and the departure, or some part of it, in its column; the corresponding distance is the difference of longitude, or a part of it.

EXAM. A ship from latitude  $50^\circ$  N. sails 150 m. on a course  $50^\circ$  South-westward; required her difference of latitude and of longitude?

To the course  $50^\circ$ , and distance 150 m. the difference of latitude and departure are 96.4 and 114.9 m.; therefore, the ship is in  $48^\circ 24'$  N. The middle latitude is  $49^\circ 12'$ , and its comp. near  $41^\circ$ . Divide the departure 114.9 by 22, the quotient is 5.222; which is found even with  $41^\circ$  below 8; therefore, the difference of longitude is  $8 \times 22 = 176$  m. W.

*To work a TRAVERSE by Middle Latitude*

1. Find the difference of latitude and departure for each course, as in plain sailing.

2. Make two columns marked E. and W.; find the difference of longitude made on each course, and place them in their proper columns; the difference of their sums is the difference of longitude made in the whole traverse.

EXAM.

EXAM. A ship from latitude  $50^{\circ}$  N. and longitude  $6^{\circ} 14'$  W. sails, 1st, W. S. W. 26 m. 2d, S. E. by E. 30 m. 3d, S. W. by W.  $\frac{1}{2}$  W. 36 m. 4th, W. 64 m. 5th, S. W.  $\frac{1}{2}$  W. 25 m. 6th, S. E. by S. 37 m. 7th, S. by W.  $\frac{1}{2}$  W. 40 m.; required the latitude and longitude of the ship, with the direct course and distance made good?

Courses.	E.	W.	Lats.	M. Lat.	E.	W.
			$50^{\circ}$	0		
W S W	1.0	24.0	49	$50^{\circ} 49'$	55	37.27
S E by E	7	24.9	49	$33^{\circ} 49'$	41	38.48
S W by W half W	0	31.8	49	$16^{\circ} 49'$	24	48.86
W		64.0	49	$16^{\circ} 49'$	16	98.07
S W half W	6	19.3	49	$0^{\circ} 49'$	8	29.49
S E by S	8	10.6	48	$29^{\circ} 48'$	45	31.24
S by W 1-4th W.	8	9.7	47	$30^{\circ} 48'$	9	14.53
	129.2	45.5	148.8		69.72	228.22
			45.5			69.72
	Depart. 103.3				Diff. Long. 158.50	

or  $2^{\circ} 38'$ ; hence the ship is in longitude  $8^{\circ} 52'$  W.

*For the Course and Distance.*

Departed latitude =  $50^{\circ} 0' N$

Difference of latitude 129 m. =  $2^{\circ} 9' S$

Present latitude  $47^{\circ} 51' N$

Middle latitude  $48^{\circ} 55'$

- |                                       |                                   |
|---------------------------------------|-----------------------------------|
| 1. As diff. lat. = 129.2              | 2. As sine cour. $38^{\circ} 52'$ |
| Is to diff. long. 158.5               | Is to radius,                     |
| So is Cos. mid. lat. $48^{\circ} 55'$ | So is depart. 103.3               |
| To tang. course $38^{\circ} 52'$      | To distance = 164.6               |
| or S. W. by S. $\frac{1}{2}$ W.       |                                   |

This is the true way of working a traverse by middle latitude; but many seamen work the same way as in plain



plain sailing; and, having found the course and distance made, and the latitude come to, they find the middle latitude; and then,

As co-sine of middle lat. is to tangent of course,  
So is difference of latitude to difference of longitude.

Or, by the Traverse Table,

Find the complement of the middle latitude among the courses, and the departure in its column, then the distance answering thereto is the difference of longitude: And this method is correct enough for any run that can be made in one day in latitudes less than  $54^{\circ}$ ; but, in higher latitudes, it is necessary to find the difference of longitude to each course, as in this example.

### MERCATOR'S SAILING.

This method of solving nautical questions is a correction of the errors of the plain chart, proposed by Mr. GERARD MERCATOR, a *Fleming*, who published the first chart of this kind about the year 1550. Its true principles were discovered, and demonstrated by Mr. EDWARD WRIGHT, about the year 1590.

In Mercator's chart, the meridians are parallel right lines; but to correct the errors arising from thence, the degrees of latitude are enlarged in the same proportion as the degrees of longitude are.

Thus, Northing and Southing, is every where proportional to Easting and Westing; and the bearings of places from each other are the same on this chart as on the globe itself. The length of any degree of latitude in this projection, is determined by the following theorem.

E g

The

The natural length of any part of the meridian, such as  $1'$ , is to its length on Mercator's chart, as radius is to the secant of the latitude of that part of the meridian. *Fig. 85.*

Let  $EP$  be a quadrant of the meridian,  $E$  a point on the equator,  $P$  the pole,  $CE$  the radius of the equator,  $LB$  the co-sine of the latitude of the point  $L$ , and  $CS$  the secant of that latitude. Because the triangles  $CEA$ ,  $CSE$  are similar,  $CA : CL :: CE : CS$ ; that is, as  $\text{Cos. lat.} : \text{Rad.} :: \text{Rad.} : \text{Sec. lat.}$ ; but as  $\text{Cos. lat.}$  is to  $\text{Rad.}$  so is any part of that parallel of latitude to a similar part of the equator; therefore, as radius is to  $\text{sec. lat.}$  so is any part of the parallel to a similar part of the equator. But the equator is equal to the meridian, as being both great circles; and, in Mercator's chart, every parallel of latitude is equal to the equator; consequently, the natural length of any part of a parallel of latitude is to its length on Mercator's chart, as radius is to the secant of the latitude; and therefore, to enlarge the several parts of the meridian in the same proportion, the natural length of any part of the meridian must be to its length on Mercator's chart, as radius is to the secant of the latitude of that part of the meridian.

The lengths of the several parts of the enlarged meridian being calculated and ranged in order in a table, are called *meridional parts*.

Mr. *Edward Wright*, who calculated the first table, made the meridional parts for 1 minute equal to the secant of 1 minute; for two minutes, equal to the sum of the secants of 1 and 2 minutes; and, for 3 minutes, equal to the sum of the secants of 1, 2, and 3 minutes; and so on, by a constant addition of secants.

There

There is in most books of Navigation, a table of meridional parts, from which the meridional parts answering to any latitude may be had, and consequently the meridional difference of latitude by subtraction or addition, according as the latitudes are of the same or of contrary names.

Let A and C. *Fig. 86.* represent any two places lying in different latitudes and longitudes, properly laid down on Mercator's chart. Let AD be the meridian of the place A; draw AC, and from C let fall the perpendicular CD; set the proper difference of latitude from A to B, and draw BE parallel to DC, then AE is the true distance of the places A and C. AC the enlarged distance, AD the enlarged difference of latitude, DC the difference of longitude, and BE the departure.

**CASE 1.** Given the latitudes and longitudes of two places, to find their bearing and distance.

**EXAM.** Required the course and distance from Caithness-point, in the north of Scotland, in latitude  $58^{\circ} 46'$  N. longitude  $3^{\circ} 17'$  W. to New York in North America in latitude  $41^{\circ} 5'$  N. longitude  $74^{\circ} 15'$  West?

The proper diff. lat. is  $17^{\circ} 41' = 1061$  m. and the diff. long. is  $70^{\circ} 58' = 4258$  m.

The meridional parts answering  $58^{\circ} 46' = 4382.1$

Ditto  $41^{\circ} 5' = 2708.3$

Meridional difference of latitude  $= 1673.8$

1. As Merid. diff. lat. : Diff. long. :: Rad. : Tan. course  $= 68^{\circ} 32'$ , or W. S. W.

2. As Cos. course : Rad. :: Prop. diff. lat. : Dist.  $= 2899.2$  m.

**EXAM. 2.** What is the course and distance from Leith, in latitude  $56^{\circ}$  N. longitude  $3^{\circ}$  W. to Bergen, in Norway, in latitude  $60^{\circ}$  N. longitude  $6^{\circ}$  E.?

*Ans.* Course N.  $50^{\circ}$  E. Distance 373.3 miles.

**CASE 2.** Given the latitude and longitude of the place sailed from, with the course and distance sailed, to find the latitude and longitude of the ship.

**EXAM.** A ship takes her departure from a place in latitude  $51^{\circ} 18'$  N. longitude  $9^{\circ} 50'$  W. and steers S.  $33^{\circ} 8'$  W. until she has run 1024 miles; required her present latitude and longitude?

*Ans.* Latitude  $37^{\circ} 1'$  N. longitude  $22^{\circ} 55'$  W.

2. A ship from latitude  $56^{\circ}$  N. longitude  $3^{\circ}$  W. sails N. E. by N. 373; required her present latitude and longitude?

*Ans.* Latitude  $61^{\circ} 10'$  N. longitude  $3^{\circ} 38'$  E.

**CASE 3.** The course and difference of latitude being given, to find the distance sailed, and difference of longitude.

**EXAM. 1.** A ship from latitude  $56^{\circ} 4'$  N. and longitude  $3^{\circ}$  W. sails N. N. E.  $\frac{1}{4}$  E. until she is in latitude  $60^{\circ} 29'$  N.; required the distance sailed, and the longitude come to?

*Ans.* The distance is 293.1 miles, and the ship hath come to  $0^{\circ} 59'$  E. longitude.

2. A ship from latitude  $52^{\circ} 20'$  N. and longitude  $6^{\circ} 24'$  W. sails W. S. W.  $\frac{1}{4}$  W. until she is in latitude  $47^{\circ} 49'$  N.; what distance hath she sailed, and what longitude hath she come to?

*Ans.* Distance sailed is 804.5 miles, and the longitude come to is  $26^{\circ} 5'$  W.

**CASE**

**CASE 4.** Given the difference of latitude and distance, to find the course and difference of longitude.

**EXAM. 1.** A ship from latitude  $53^{\circ} 14' N.$  and longitude  $2^{\circ} E.$  sails 256 miles between N. and E. and then is in latitude  $56^{\circ} N.$ ; required her course and present longitude;

*Ans.* Course  $49^{\circ} 35'$  Northeastward, or N. E.  $\frac{1}{2}$  E. longitude  $7^{\circ} 37' E.$

2. A ship from latitude  $40^{\circ} 25' S.$  and longitude  $40^{\circ} 25' W.$  sails 413 miles between N. and E. and is then in latitude  $36^{\circ} 19' S.$ ; what was her course, and what longitude hath she come to?

*Ans.* Course  $53^{\circ} 27'$ , or N. E.  $\frac{1}{4}$  E. and the ship has come to longitude  $33^{\circ} 22' W.$

**CASE 5.** The latitude sailed from, the course and difference of longitude being given; to find the distance sailed and the latitude of the ship.

**EXAM.** A ship from latitude  $45^{\circ} 31' N.$  and longitude  $1^{\circ} 24' W.$  sails W. N. W.  $\frac{1}{4}$  W. until she is in  $10^{\circ} 25' W.$  longitude; required the distance sailed, and the latitude of the ship?

*Ans.* Distance 392 miles, and the ship is in latitude  $47^{\circ} 43' N.$

**CASE 6.** The latitude and longitude of the place sailed from, with the course and departure being given, to find the distance sailed, with the latitude and longitude of the ship.

**EXAM.** A ship from latitude  $46^{\circ} 24' S.$  and longitude  $4^{\circ} 24' E.$  sails E. N. E.  $\frac{3}{4}$  N. until she has made 256 miles of easting; required the distance sailed, with the latitude and longitude of the ship?

*Ans.*

*Ans.* Distance 298.4 *m.* Latitude  $43^{\circ} 51'$  Long.  $100^{\circ} 26'$  E.

**CASE 7.** The difference of latitude and departure being given, to find the course, distance and difference of longitude.

**EXAM.** A ship from latitude  $60^{\circ} 24'$  N. and longitude  $43^{\circ}$  W. sails between S. and E. until she is in latitude  $56^{\circ} 30'$  N. and has made 226 miles of departure; required her course, distance and longitude?

*Ans.* Course S. E. nearly. Distance 325.4 *m.* and the longitude of the ship  $35^{\circ} 47'$  W.

### *A Traverse by MERCATOR'S Sailing.*

Suppose a ship from latitude  $68^{\circ} 38'$  N. and longitude  $8^{\circ} 40'$  E. has sailed the following courses, viz. N. E. by N. 63 *m.* N. E. 38 *m.* N. N. E. 56 *m.* N. 30 *m.* N. W. by N. 25 *m.* N. N. W.  $\frac{1}{4}$  W. 36 *m.* N. by E. 40 *m.* N. E. by E.  $\frac{1}{2}$  E. 72 *m.* S. E. 50 *m.* and E. N. E. 65 *m.*; required the ship's latitude and longitude, with her course and distance from the place where she has come to, unto the North Cape in latitude  $71^{\circ} 27'$  N. and longitude  $26^{\circ} 30'$  E.?

*Courses.*

Courses.	D	N	S	E	W	Lats.	Merid. diff.	Longitudes.	
								E	W
NE by N	63	52.4		35.0		68 38			
NE	38	26.9		26.9		69 30	145	96.88	
NNE	56	51.7		21.4		69 57	78	8.	
North	30	30				70 49	155	64.2	
NW by N	25	20.8			13.9	71 19	93		44.1
NNW ½ W	36	31.8			17.	71 40	66		55.
N by E	40	39.2		7.8		72 12	103		
NE by E ½ E	72	33.9		63.5		72 51	130	25.86	
SE	50		35.4	35.4		73 25	117	218.8	
ENE	65	24.9		60.1		72 49	124	12.	
						73 14	86	207.2	
		311.6	35.4	250.1	30.9			814.94	99.8
		35.4		30.9				99.1	
		276.2		219.2					
							Diff. Long.	715.8	

or 11 56' E  
 Departed Long. 8 40 E  
 Ship's Long. 20 36

For the course and distance from the ship to the north Cape,

Ship's lat. 73° 14'	mer. ptr.
Cape's lat. 71 27	6583
	6230
1 47 = 107'	353 = merid. diff. lat.
Cape's longitude 26° 30'	
Ship's longitude 20 36	
5 54	
60	
354	

The meridional difference latitude being nearly equal to the difference longitude, the course is 4 points or S. E. and the distance from the ship to the Cape is 152 miles nearly.

When

When a table of meridional parts is not at hand, the defect may be supplied by the logarithmic tangents. See *Robertson's Navigation*, 2d edition, page 532.

1. To find the meridional parts answering to any given latitude.

To half of the given latitude add  $45^\circ$ ; find the logarithm tangent of the sum, reject the index, and of the seven remaining figures, point off three on the right hand for decimals; then divide by 1.2633114, the quotient is the meridional parts required.

*Note.* 1.2633114 is the natural tangent of  $51^\circ 38' 9''$ .

EXAM. Required the meridional parts answering to  $60^\circ$  of latitude? *Ans.* 4527.36.

The half of  $60^\circ$  is  $30^\circ$

To which add  $45^\circ$

Sum  $75^\circ$  whose logarithmic tangent is 10.5719475. This without its index, and pointed by the rule is 5719.475, and then divide it by 1.2633114.

1.2633114)5719.4750(4527.36 the answer.

2. Required the meridional parts answering to latitude  $55^\circ 24'$ ? *Ans.* 4010.019.

The half of  $55^\circ 24'$  is  $27^\circ 42'$

To which add  $45^\circ$

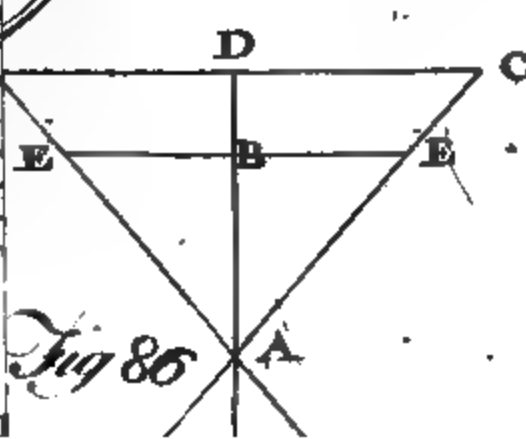
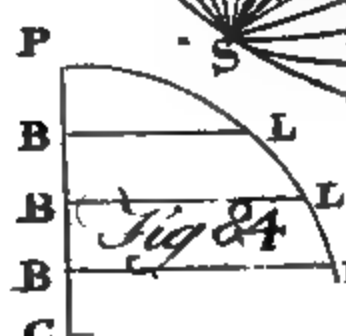
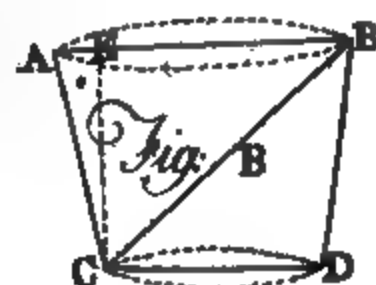
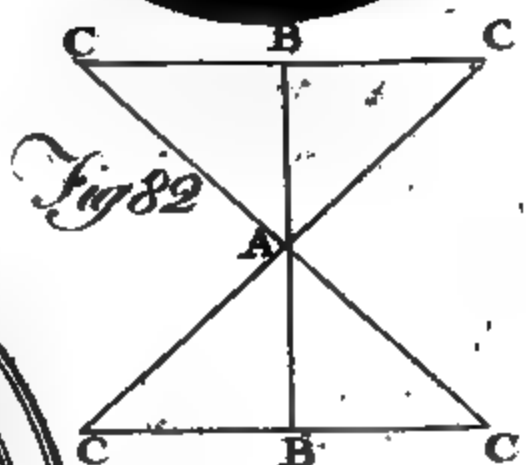
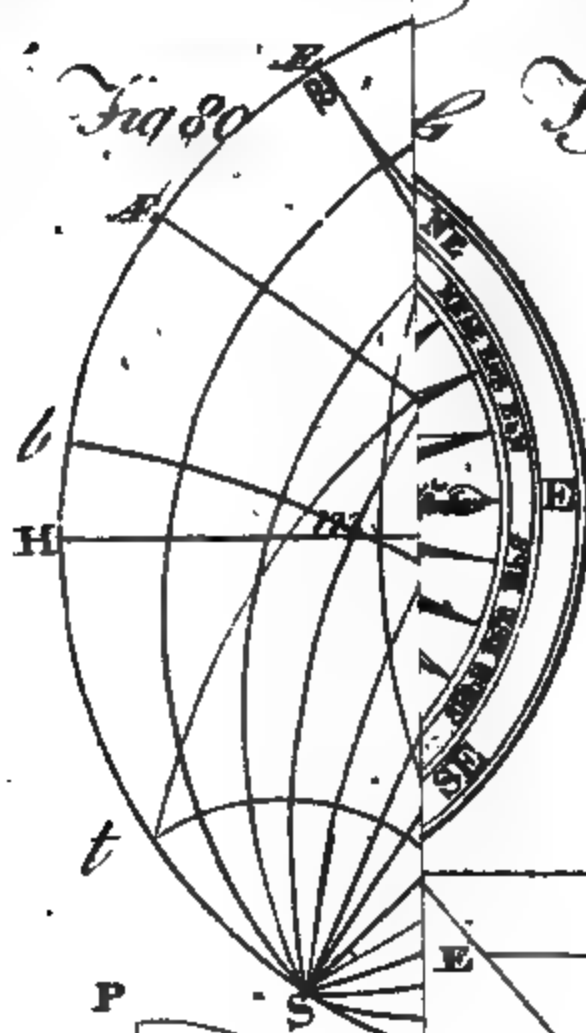
$72^\circ 42'$  log. tan. 10.5065903

1.2633114)5065.9030(4010.019.

2. To find the meridional difference of latitude between any two places, their latitudes being given.

When







When the given places are both in north, or both in south latitude,

Take the difference of the logarithm tangents of the half complements of the latitudes; account four figures of it next the index to be integers, and then divide it by 1.2633114, the quotient is the meridional difference of latitude.

EXAM. 1. Required the meridional difference of latitude between  $13^{\circ} 30'$  and  $50^{\circ}$  North latitudes?

*Ans.* 2656.87.

	Log. Tan.
The half comp. of $13^{\circ} 30'$ is $38^{\circ} 15'$	9.8967116
The half comp. of $50^{\circ} 00'$ is $20^{\circ} 00'$	9.5610659
	3356.457

$$1.2633114)3356.4570(2656.87$$

2. Required the meridional difference of latitude between the Lizard, in latitude  $49^{\circ} 57' N.$  and Cape Verd, in latitude  $14^{\circ} 43' North$ ?

*Ans.* 2576.93.

	Log. Tan.
Half complement of $14^{\circ} 43'$ is $37^{\circ} 38\frac{1}{2}'$	9.8872024
Half complement of $49^{\circ} 57'$ is $20^{\circ} 1\frac{1}{2}'$	9.5616551
	3255.473

$$1.2633114)3255.4730(2576.93$$

When one place is in North latitude and the other in South, add  $90^{\circ}$  to one of the latitudes, and take half of the sum for the half complement of that latitude; take half the complement of the other latitude, and divide the

H h

difference

difference of the logarithm tangents of these two by 1.2633114, the quotient is the meridional difference of latitude. Or,

Take the half complements of the latitudes as before; take the sum of the logarithm co-tangents of these, rejecting the indices, and divide it by 1.2633114

EXAM. 1. Required the meridional difference of latitude between  $50^{\circ} 54'$  N. and  $19^{\circ} 24'$  S. latitude?

*Ans.* 4746.18.

$50^{\circ} 54'$   
add  $90^{\circ} 00'$

140 54 half of this is 70 27 10.4496481  
half comp. of  $19^{\circ} 24'$  is 35 18 9.8500575

Difference 5995.906

1.2633114)5995.9060(4746.18

2. Required the meridional difference of latitude, between the Lizard, in latitude  $49^{\circ} 57'$  N. and the island of St. Helena, in latitude  $15^{\circ} 45'$  South?

*Ans.* 4426.93.

		Co-tang. without index.
Lizard's half comp. lat. is	$20^{\circ} 1\frac{1}{2}'$	4383448
St. Helena's half comp. lat. is	$37^{\circ} 7\frac{1}{2}'$	1209154
		<hr/> 5592.602

1.2633114)5592.6020(4426.93

By these rules, the meridional parts in any latitude, or the meridional difference of latitude between any two places may be found, and all questions in Mercator's sailing answered in the usual way; but the same thing may be done by the logarithmic tangents alone, without the meridional parts. For it is demonstrated in the Philosophical

Philosophical Transactions, N<sup>o</sup>. 219, That the meridional line on *Mercator's Chart* is a scale of the logarithm tangents of the half complements of the latitudes. That these logarithm tangents of Mr. *Briggs'* form, are a scale of the differences of longitude upon the rhumb, which makes an angle of  $51^{\circ} 38' 9''$  with the meridian; and, that the differences of longitude on different rhumbs, are to one another as the tangents of the angles which these rhumbs make with the meridian. Hence,

As the difference of the logarithm tangents of the half complements of the latitudes of any two places,  
Is to the difference of longitude between these places,  
So is the tangent of  $51^{\circ} 38' 9''$  (Log. 10.1015104)  
To the tangent of the course.

By a right application of this rule, all the cases in *Mercator's sailing* may be solved.

E X A M P L E S.

CASE I. What is the course and distance from Caithness-point, in latitude  $58^{\circ} 46'$  N. and longitude  $3^{\circ} 17'$  W. to New York, in latitude  $41^{\circ} 5'$  N. longitude  $74^{\circ} 15'$  W.?

Latitudes.	Log. Tang.
$41^{\circ} 5'$ half complement is $24^{\circ} 27\frac{1}{4}'$	9.6578665
$58^{\circ} 46'$ half complement is $15^{\circ} 37'$	9.4464107
	<hr/>
Difference	2114.558

The difference of longitude is 4258 min.

As diff. of log. tan. of half co-lats. 2114.558 3.3252196

Is to diff. of long. in minutes 4258. 3.6292057

So is the tangent of  $51^{\circ} 38' 9''$  10.1015104

To the tangent of course  $68^{\circ} 32'$  13.7307161

or W. S. W.  $1^{\circ} 2'$  westerly 10.4054965

H h. 2. The

The course being known, the distance may be found as before.

**CASE 2.** A ship from latitude  $50^{\circ}$  N. and longitude  $6^{\circ} 15'$  W. sails 250 miles, on a course  $50^{\circ} 6'$  Southwestward; required the latitude and longitude of the ship?

The difference of latitude, and consequently the present latitude, is found as formerly, in this example.

Departed latitude	$50^{\circ} 00' \text{ N.}$	
Difference of latitude	$2 \quad 40 \text{ S.}$	
Present latitude	$47 \quad 20$	
The half comp. of $47^{\circ} 20'$ is	$21^{\circ} 20'$	Log. Tan. 9.5916812
Ditto of $50 \quad 00$ is	$20 \quad 00$	9.5610659
		<hr/> 306.153
As tangent of	$51^{\circ} 38' 9''$	10.1015104
Is to tangent of course	$50 \quad 6$	10.0777263
So is the diff. of tang.	306.153	2.4859386
		<hr/>
To diff. of longitude	289.83	12.5636649
		<hr/>
or nearly $290' = 4^{\circ} 50' \text{ W.}$		2.4621545
Departed long.	$6 \quad 15 \text{ W.}$	
Present long.	$11 \quad 5 \text{ W.}$	

**CASE 3.** A ship sails from latitude  $49^{\circ} 55' \text{ N.}$  to a place in latitude  $13^{\circ} 10' \text{ N.}$  on a course  $50^{\circ}$  Southwestward; required her difference of longitude?

*Ans.*  $53^{\circ} 1' \frac{1}{2} \text{ W.}$

		Log. Tan.
The half comp. of $13^{\circ} 10'$ is	$38^{\circ} 25'$	9.8993082
Ditto	$49 \quad 55$ is $20 \quad 2 \frac{1}{2}$	9.5620477
		<hr/>
	difference	3372.605
		As

As tang. of	50° 38' 9"	10.1015104
Is to tang. course	50°	10.0761865
So is diff. of log. tang.	3372.605	<u>3.5279648</u>
To the diff. of long.	3181.5	<u>13.6041513</u>
or 53° 1' $\frac{1}{2}$ Westward.		3.5026409

**CASE 4.** A ship from latitude 32° 10' N. and longitude 15° 12' W. sails 3280 miles between South and West, and then is in latitude 20° 19' South; required her course and longitude?

The difference of latitude is 52° 29' = 3149 miles.

As Dist. : Diff. lat. :: Rad. : Cos. course = 16° 15' Southwestward.

Lat. 32° 10' N.  
add 90 00

2)122 10

61° 5'

$\frac{1}{2}$  comp. of 20° 19' is 34° 50'  $\frac{1}{2}$

Log. Tan.

10.2577391

9.8426698

Difference 4150.693

As tang. 51° 38' 9" 10.1015104

Is to tang. course 16° 15' 9.4645990

So is diff. tang. 4150.693 3.6181206

To diff. of longitude 957.65 13.0827196

or 15° 57' W. 2.9812092

Departed long. 15 12 W.

Present long. 31° 9' W.

A TABLE

*A TABLE of the REFRACTION of the HEAVENLY BODIES, to be subtracted from the observed Altitude:*

Height of the eye above the sea in feet.	Dip of the horizon in minutes.		Altit.	Refrac.		Altit.	Refrac.		Altit.	Refr.	Altit.	Refr.
Fect.	M.	S.	D.	M.	S.	D.	M.	S.	D.	M.S.	D.	M.S.
3	1	39	0	33	0	11	4	47	27	1 51	43	1 1
5	2	8	0½	28	22	12	4	23	28	1 47	44	0 59
7	2	31	1	24	29	13	3	57	29	1 42	45	0 57
9	2	52	1½	19	25	14	3	45	30	1 38	46	0 55
12	3	18	2	18	35	15	3	30	31	1 35	47	0 53
15	3	42	2½	16	26	16	3	17	32	1 31	48	0 51
18	4	3	3	14	36	17	3	4	33	1 28	49	0 49
21	4	22	3½	13	6	18	2	54	34	1 24	50	0 48
25	4	46	4	11	51	19	2	44	35	1 21	55	0 40
30	5	14	4½	10	48	20	2	35	36	1 18	60	0 33
35	5	39	5	9	54	21	2	27	37	1 16	65	0 26
40	6	2	6	8	28	22	2	20	38	1 13	70	0 21
50	6	44	7	7	20	23	2	14	39	1 10	75	0 15
60	7	23	8	6	29	24	2	7	40	1 8	80	0 10
70	7	59	9	5	48	25	2	2	41	1 5	85	0 5
80	8	32	10	5	15	26	1	56	42	1 3	90	0 0



*To find the Latitude of a Ship at Sea.*

Observe the meridian altitude of the sun's upper or lower limb by a quadrant.

Correct the observed altitude of the sun's limb, by subtracting the dip of the horizon and the refraction from it; and adding the semidiameter, if the lower limb was observed, or subtracting it, if the upper limb was observed; the result is the true altitude of the sun's center.

Subtract the altitude of the sun's center from  $90^\circ$ , the remainder is the zenith distance, which is north, if the zenith is to the north of the sun; or south, if the zenith is to the south of the sun.

Take the sun's declination out of the Nautical Almanack, or any good Table, for the time and place, noting whether it be north or south; and then,

If the zenith distance and declination are both north, or both south, add them together; but if the one be north and the other south, subtract the less from the greater, and the sum or difference will be the latitude, of the same name with the greater.

EXAMPLES.

1. April 14th, 1790, the meridian altitude of the sun's lower limb was  $40^\circ 20'$ ; the zenith being north of the sun; and the altitude of the observer's eye, 20 feet above the surface of the sea. What was the Latitude?

Altitude

Altitude of sun's lower limb  $40^{\circ} 20' 0''$

dip, subtract  $4 16$

---

$40 15 44$

refraction, subtract  $1 6$

---

$40 14 38$

Sun's semidiameter, add  $15 58$

---

true altitude sun's center  $40 30 36$

$90 0 0$

---

zenith distance  $49 29 24$  N.

sun's declination  $9 34 40$  N.

---

Latitude  $59 4 4$  N.

2. May 26th, 1790, being at sea in longitude  $67^{\circ} 30'$  E. by account; the meridian altitude of the sun's lower limb was  $26^{\circ} 12'$ , the zenith being to the south of the sun, and the observer's eye 18 feet above the surface of the water: required the latitude of the ship?

*h. m.*

Time at the ship  $24 0$  past noon of the 25th

Longitude  $67^{\circ} 30'$  E.  $4 30$

---

time at London  $19 30$

Sun's declination May 25th  $21^{\circ} 1' 57''$  N. daily diff  $16' 57''$

increase in  $19$  h.  $30$  m. add  $8 54$

---

sun's declination at the ship  $21 10 51$  N.

Altitude

Altitude sun's lower limb	26° 12' 0"
dip on 18 feet, subtract	4 3
	<hr/>
	26 7 57
refraction, subtract	1 56
	<hr/>
	26 6 1
Sun's semidiameter, add	15 48
	<hr/>
Altd. of the sun's center	26 21 49
	90 0 0
	<hr/>
zenith distance	63 38 11 S.
declination	21 10 51 N.
	<hr/>
Latitude of the ship	42 27 20 South

When the latitude is to be found from the meridian altitude of a fixed star,

Subtract the dip and refraction from the observed altitude, and then find the latitude from the zenith distance and declination of the star, as before.

3. April 15th, 1790, the meridian altitude of the star Sirius was observed,  $53^{\circ} 35'$ ; the zenith being to the north of the star, and the height of the observer's eye 22 feet above the surface of the sea: What was the latitude?

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Observed

Observed altitude	53° 35' 0"
dip 4' 28"	
refraction 1 28	
	<hr/> 5 56
true altitude of the star	53 29 4
	<hr/> 90
zenith distance	36 30 56 N.
declination	16 26 21 S.
	<hr/>
Latitude	20 4 35 N.

If the meridian altitude of a circum-polar star be observed, when it is below the pole; or the meridian altitude of the sun at midnight, in any place where it does not set: Correct the altitude as above directed, and then, to the altitude of the star, or center of the sun, add the complement of its declination, the sum is the latitude of the place; of the same name with the declination.

4. June 12th, 1786, in a high northern latitude, 23° W. longitude, the eye being 20 feet above the horizon; the meridian altitude of the sun's lower limb, below the pole, was 8° 15'; required the latitude?.

*h. m.*

Ship's time 12. 0 p. m.

Long. 23° W. 1. 32.

London time 13. 32 p. m.

Sun's decl. June 12th 23° 11' 54" N.

increase in 13 h. 32 m. 1 57

Sun's declin. at ship 23 13 51 N.

Altitude:

Altitude sun's lower limb  $8^{\circ} 15' 0''$

dip  $4' 16''$

refraction  $6 19$  —  $10 35$

---

Altitude sun's L. L.  $8 4 25$

Semidiameter  $15 47$

---

Altitude sun's center  $8 20 12$

Co-declination  $66 46 9$

---

Latitude  $75 6 21$  North.

It has been found by observation, that the magnetic needle does not point due north, except in some few places; and that its direction is different at different times in the same place.

The distance between the true north and the direction of the magnetic needle, is called the *variation of the compass*, which, when the needle points to the east of the true north, is called *East variation*; but, when it points west of the true north, it is called *West variation*.

In order to find the variation of the compass, the two following problems are necessary, viz.

1. The latitude of the place, and the declination of any celestial object being given, (that declination being less than the complement of the latitude), to find its amplitude.

**RULE.** As the co-sine of the latitude  
Is to the sine of the declination,  
So is the radius  
To the sine of the amplitude.

I i 2.

EXAM.

EXAM. 1. In latitude  $56^{\circ}$  N. the sun's declination being  $20^{\circ} 12'$  N.; required the sun's amplitude?

As co-sine latitude	$56^{\circ} 0'$	9.7475617
Is to sine declination	$20 12$	9.5381943
So is radius	S. $90 0$	10.0000000

---

To sine amplitude  $38 8$  9.7906326

That is, the sun rises E.  $38^{\circ} 8'$  N. or sets W.  $38^{\circ} 8'$  N. according as the calculation was made for the morning or evening.

2. In latitude  $10^{\circ} 12'$  N. the sun's declination being  $23^{\circ} 12'$  N.; required the sun's amplitude?

Ans. E.  $23^{\circ} 36'$  N.

Note. In the torrid zone, the amplitude and declination are nearly equal.

3. On the 20th of January, 1779, in the latitude of  $34^{\circ} 15'$  S.; what was the sun's amplitude at rising?

In questions of this kind, the sun's declination must be found accurately for the time and meridian. In this example, if the calculation is made for the morning, the declination is  $20^{\circ} 14'$  S. and his amplitude E.  $24^{\circ} 44'$  S.; but, if for the evening, the declination is  $20^{\circ} 7'$  S. and the amplitude E.  $24^{\circ} 35'$  S.

4. In the latitude of  $56^{\circ}$  N.; required the amplitude of the star *Arcturus*, its declination being  $20^{\circ} 22'$  N.?

Ans. E.  $38^{\circ} 29'$  N.

2. The latitude of the place, the sun's declination and altitude being given, to find his azimuth.

Add together the complement of the latitude, the complement of the sun's altitude, and his distance from the

the elevated pole; take half the sum, from which subtract the sun's distance from the elevated pole.

Take the arithmetical complement of the co-sine of the latitude, and of the co-sine of the sun's altitude; take also the sine of the half sum, and the sine of the difference between the half sum and the sun's distance from the elevated pole; the half sum of these four logarithms is the co-sine of half the sun's azimuth from the north in north latitude, and from the south in south latitude.

*Note.* When the latitude and declination are both north or both south, subtract the declination from  $90^\circ$ ; but, when the one is north and the other south, add the declination to  $90^\circ$ ; the difference, or sum, is the sun's distance from the elevated pole.

**EXAM. 1.** In the latitude of  $51^\circ 32' N.$  the sun's altitude was observed to be  $46^\circ 20'$ , when his declination was  $23^\circ 28' N.$ ; what was the sun's azimuth?

*Ans.*  $113^\circ 3'$  from the North.

Comp. latitude  $38^\circ 28'$  Arith. comp. 0.2061683

Comp. altitude  $43^\circ 40'$  Arith. comp. 0.1608604

Sun's dist. from the pole  $66^\circ 32'$

Sum  $148^\circ 40'$

$\frac{1}{2}$  sum  $74^\circ 20'$  Sine 9.9835582

Subtract  $66^\circ 32'$

Difference  $7^\circ 48'$  Sine 9.1326297

2)19.4832166

Co-sine of  $56^\circ 31' 29''$  9.7416083

Which doubled, is  $= 113^\circ 2' 58''$  or  $113^\circ 3'$  nearly.

That

That is, the sun's azimuth is  $113^{\circ} 3'$  from the north, and consequently  $66^{\circ} 57'$  from the south; and, if the observation was made in the forenoon, the sun bears E. S. E. nearly; but W. S. W. if it was made in the afternoon.

*To work an AZIMUTH by Gunter's Scale.*

Find the sum, half sum, and difference as above, and then,

As R. : Cos. lat. :: Cos. alt. : a. 4th line.

As the 4th line : Sine of the  $\frac{1}{2}$  sum :: the sine of the diff. : a. 5th line ;  
against which, on the line of versed sines is the azimuth from the north.

In this example, set one foot of the compasses to line  $90^{\circ}$ , and extend the other to line  $38^{\circ} 28'$ ; with this extent, set one foot to line  $43^{\circ} 40'$ , and the other will fall on line  $25^{\circ} 26'$ ; keep the foot there, and extend the other to line  $74^{\circ} 20'$  (the half sum;) with this extent set one foot to line  $7^{\circ} 48'$ , and the other will fall on line  $17^{\circ} 43'$  against which, on the line of versed sines is  $113^{\circ} 3'$ , the sun's azimuth.

2. In latitude  $39^{\circ} 50'$  N. when the sun's declination was  $13^{\circ} 25'$  N. his altitude was  $50^{\circ}$ ; required the sun's azimuth?

*Ans.*  $121^{\circ} 36'$  from the north; found as in the last example.

3. In the latitude of  $51^{\circ} 32'$  N. the sun's declination being  $19^{\circ} 39'$  S. and his altitude  $15^{\circ} 30'$ ; what is the sun's azimuth?

To



To 90° 0'  
add 19 39

109 39 sun's distance from the pole.

Comp. latitude 38° 28' Arith. comp. 0.2061683

Comp. altitude 74 30 Arith. comp. 0.0160895

Sun's dist. from the pole 109 39

Sum 222 37

$\frac{1}{2}$  sum 111 18 $\frac{1}{2}$  Sine 9.9692473

Difference 1 39 $\frac{1}{4}$  Sine 8.4603922

2)18.6518973

Co-sine of 77° 47' 9.3259486

which being doubled, is 155° 34', the sun's azimuth from the North.

By Gunter's scale, the extent from radius to the sine of 38° 28' will reach from the sine of 74° 30' to the sine of 36° 50', and the extent from 36° 50' to the sine of 111° 18'  $\frac{1}{2}$  (68° 41'  $\frac{1}{2}$ ), will reach from sine 1° 39'  $\frac{1}{4}$  to sine 2° 34', against which, on the line of verfed sines, is 155° 34', the azimuth.

4. In latitude 56° S. the sun's declination was 23° 28' South, and his altitude 44°; required the sun's azimuth?

Ans. 116° 13' from the south, and 63° 47' from the north.

The sun's azimuth being found, the hour of the day when the observation was made may be found thus,

As.

As the sine of the sun's distance from the elevated pole,

Is to the sine of the sun's azimuth ;

So is the sine complement of the sun's altitude,

To the sine of the angle at the pole ;

which, in time, is the hour from noon.

In the last example, to find the time when the sun's altitude was observed.

As sine of  $66^{\circ} 32' 0''$

Is to sine of  $116 13 0$

So is the sine of  $46 0 0$

To the sine of  $44 42 39$

which, reduced to time, at the rate of  $15^{\circ}$  to 1 hour, is 2 hours, 58 minutes, 50 seconds from noon ; which shows, that the observation was made either at 9 hours, 1 minute, 10 seconds in the forenoon, or at 2 hours, 58 minutes, 50 seconds in the afternoon.

Or, the same things being given as for finding the azimuth, the hour of the day may be found by the following Rule :

Add the complement of the latitude, the sun's distance from the elevated pole, and the complement of the sun's altitude, and take half of the sum ; from which subtract the complement of the sun's altitude, and then ;

To the arithmetical complements of the co-sine of the latitude, and of the sine of the sun's distance from the elevated pole, add the logarithm sine of the half sum, and the logarithm sine of the difference between the half sum and the complement of the sun's altitude : the half sum of these four logarithms is the co-sine of half the hour angle, which being doubled and reduced to time, gives,

Fig. 1.

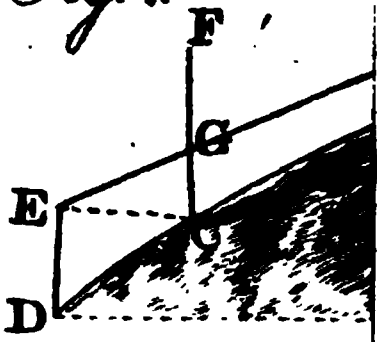


Fig. 5.

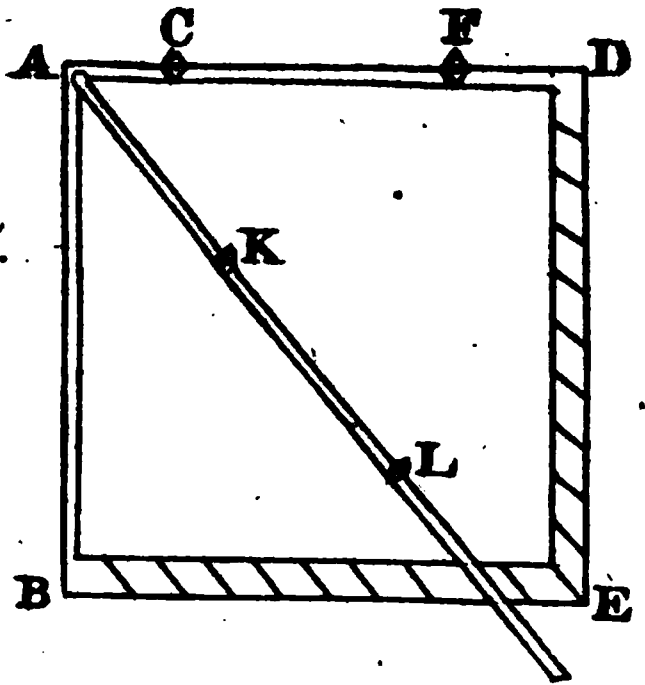


Fig. 2.

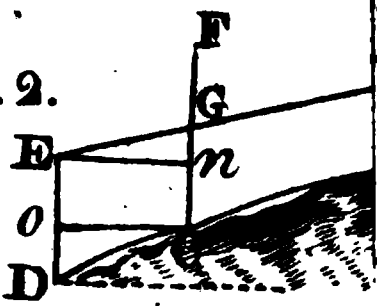


Fig.

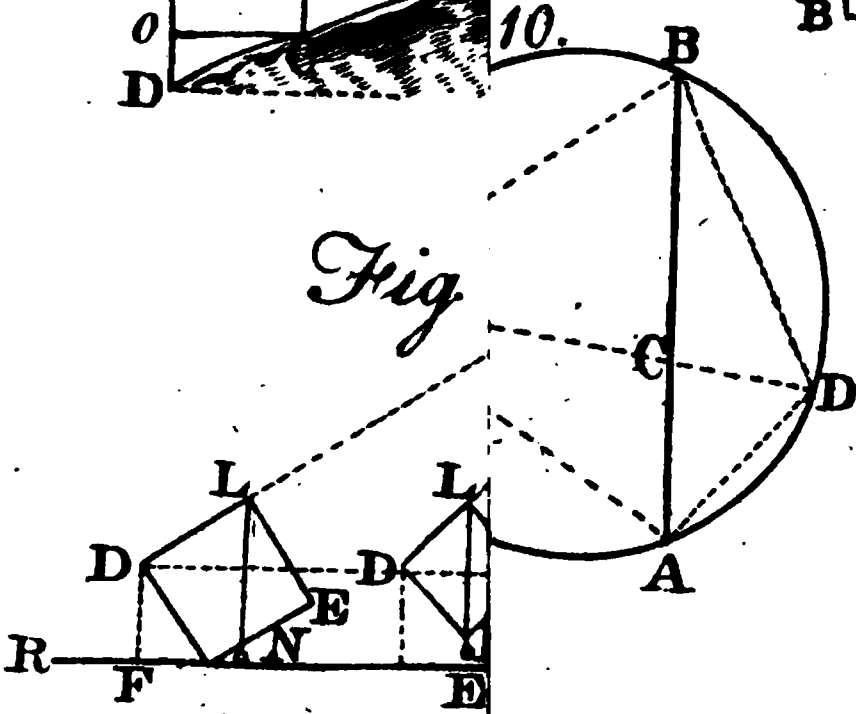


Fig. 11.

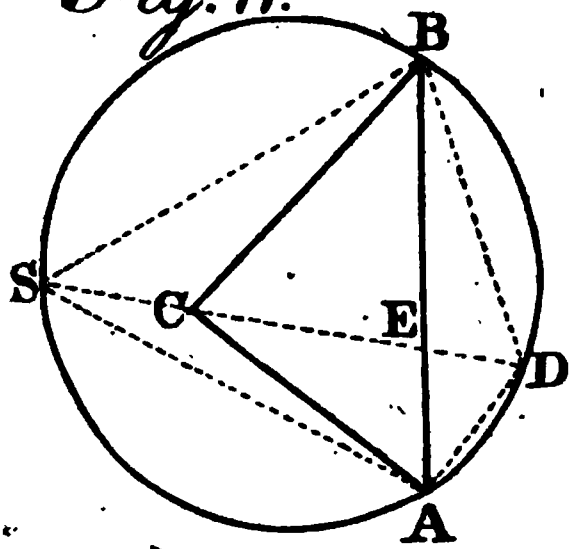


Fig. 12.

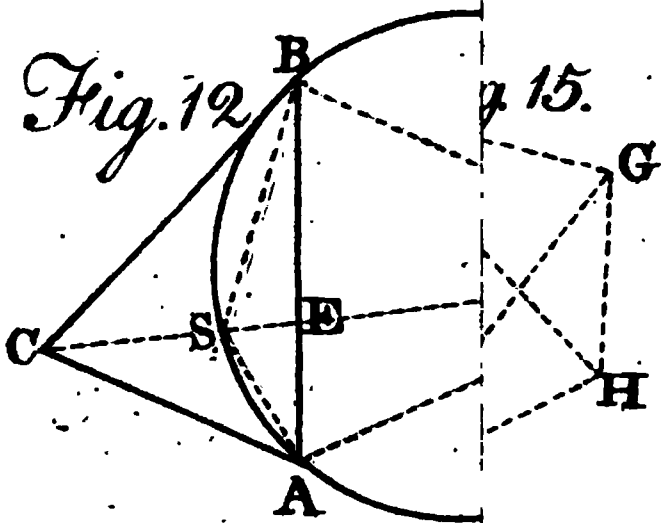
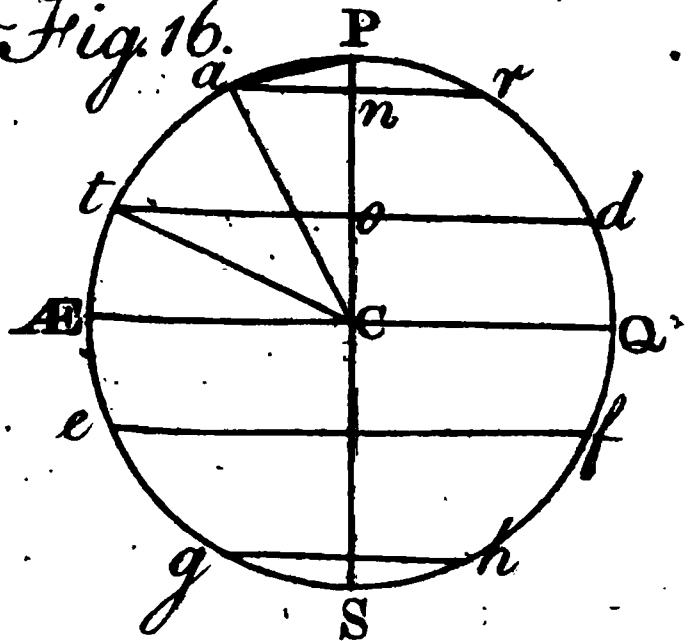
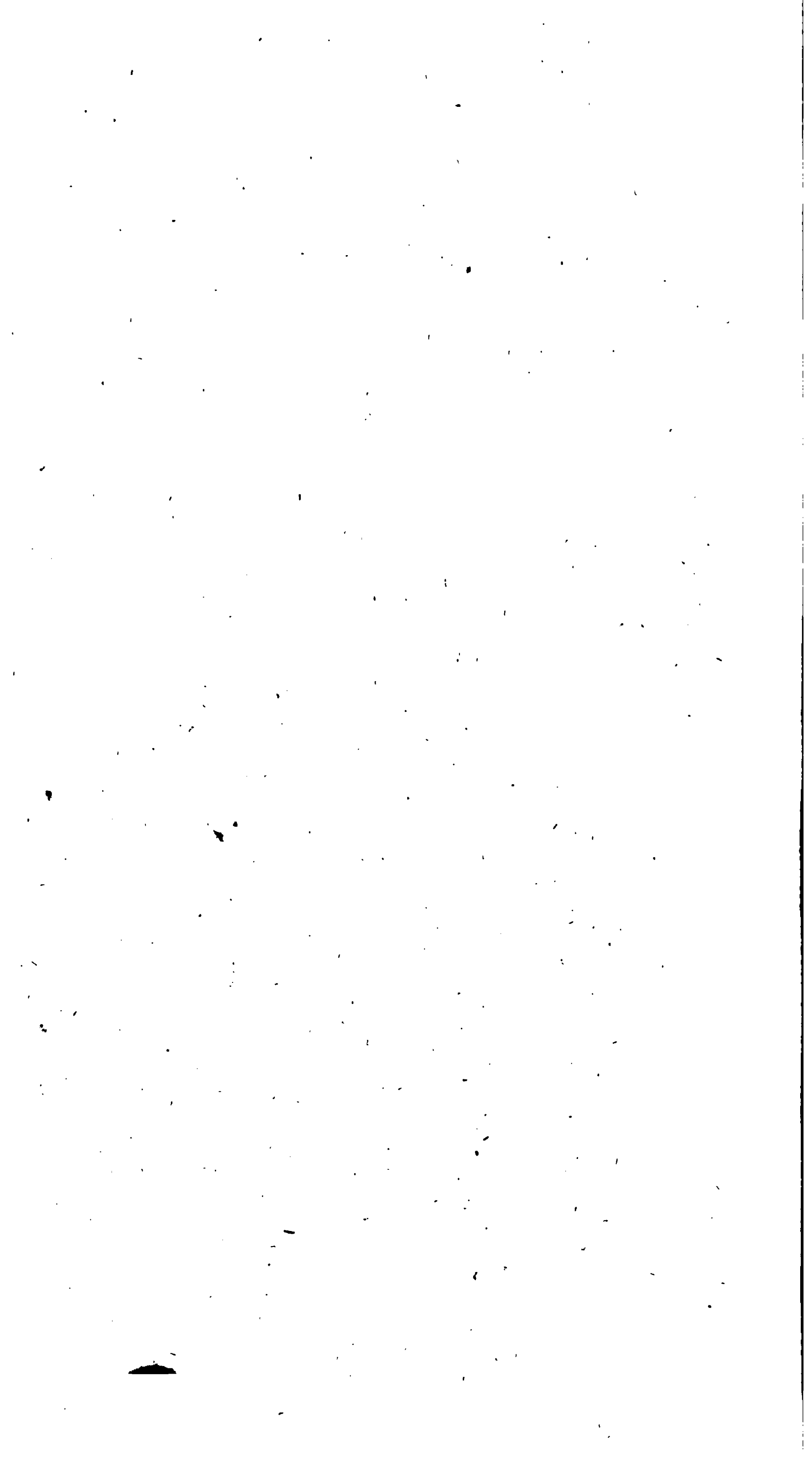


Fig. 16.





gives the apparent time from noon, when the observation was made.

EXAM. In latitude  $39^{\circ} 50' N.$  the sun's declination being  $13^{\circ} 25' N.$  and the true altitude of his center  $50^{\circ}$ : What was the hour from noon?

*Ans.* 2 hours 17 m. from apparent noon.

Comp. lat. $50^{\circ} 10'$	arith. com.	0.1146891
polar dist. $76 \ 35$	ditto	0.0120175
Comp. alt. $40 \ 0$		

Sum  $166 \ 45$

half sum $83 \ 22 \ 30$	sine	9.9970903
-------------------------	------	-----------

difference $43 \ 22 \ 30$	sine	<u>9.8368116</u>
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$2) 19.9606085$

co-sine of  $17^{\circ} \ 7' \ 34'' = 9.9803042$

which doubled is  $34^{\circ} \ 15' \ 8''$  the hour angle, and  $34^{\circ} \ 15' \ 8''$  reduced to time, is 2 hours 17 m. from apparent noon.

The altitude of any fixed star being known by observation, to find the apparent time.

Subtract the dip of the horizon, and the refraction, from the observed altitude of the star; and take its right ascension and declination from any good Table; and then, with the complement of the latitude, the star's polar distance, and the complement of its altitude, find the hour angle, (as in the last problem) which reduce into

K k

time;

time; and to this time apply the star's right ascension in time; that is, add it to the hour angle, if the star be west of the meridian, or subtract it from the hour angle, if the star be east of the meridian; the sum, or difference, is the right ascension of the mid-heaven; from which subtract the sun's right ascension in time, the remainder is the apparent time, nearly; then say, as 24 hours is to the daily difference of the sun's right ascension, so is the time past noon to a fourth, which being subtracted from the former time, the remainder is the true apparent time.

EXAM. April 14th, 1790, in latitude  $48^{\circ} 56'$  N. longitude  $66^{\circ}$  W. the altitude of Aldebaran was observed  $22^{\circ} 24' 30''$ ; the star being west of the meridian, and the height of the observer's eye above the surface of the sea being 21 feet; what was the apparent time, when that observation was made?

Dip on 21 feet	4' 22"
Refraction	2 18

---

Subtract 6 40

Observed altitude  $22^{\circ} 24' 30''$

---

true alt. of the star  $22^{\circ} 17' 50''$ , Declin. N.  $16^{\circ} 4' 24''$

---

polar distance  $73^{\circ} 55' 36''$

Compe

# NAVIGATION.

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Comp. latitude	41° 4'	arith. comp.	0.1824765
polar distance	73 55 36	arith. comp.	0.0173182
Comp. altitude	67 42 10		

Sum 182 41 46

half sum	91 20 53	Sine	9.9998802
----------	----------	------	-----------

difference	23 38 43	Sine	9.6032235
------------	----------	------	-----------

2) 19.8028984

Co-sine of	<u>37° 9' 26"</u>		9.9014492
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doubled is = 74 18 52 = the hour angle.

*b. m. f.*

which in time is	4 57 15	April 14th,	<i>b. m. f.</i>
------------------	---------	-------------	-----------------

Star's R. A.	<u>4 23 53</u>	Sun's R. A.	1 31 28
--------------	----------------	-------------	---------

Long. 66° W.			<u>+41</u>
--------------	--	--	------------

R. A. of mid heav. 9 21 8

Sun's R. A.	<u>1 32 9</u>	Sun's R. A. at ship	1 32 9
-------------	---------------	---------------------	--------

As 24 *b.* 3' 41" :: 7 *b.*

7 48 59      49 *m.* : 1' 12"

Subtract — 1 12

*Ans.* Apparent time 7 47 47 P. M.

To find the time of apparent noon, and thereby regulate a watch or clock, by equal altitudes of the sun.

K k 2

With

With any good instrument observe the sun's altitude any time before noon, and mark the time shewn by the watch; and in the afternoon observe when the sun has the same altitude, and mark the time shown by the watch: the middle point between the two observations is the time of apparent noon, or mid-day.

*Note 1.* The altitude of the sun's lower, or upper limb, or both, may be observed, and from either of these, the altitude of the center may be found by adding or subtracting the sun's semidiameter at the time.

2. If the observations are taken near noon, on any day from the 20th of May to the 20th of July; or from the 20th of November to the 20th of January; the time of apparent noon will be found without any considerable error; but if the observations be made on any day within two months of either equinox, the variation of the sun's declination during the interval between the observations will cause a small error; which in any habitable part of the earth will not be so much as half a minute; and if the observations are made within about an hour of noon, the error will be very small.

For correcting this error, there is a table in ROBERTSON'S Navigation, Vol. I. p. 328.

When equal altitudes are taken at sea in order to find the time of apparent noon; the ship should lie by during the interval, and then, add the times shown by the watch and 12 hours; the half sum of these is the apparent time shown by the watch when the sun was on the meridian of that place; and hence the watch may be rectified to apparent time.

EXAM.



EXAM. 1. May 20th, 1790, at 8 hours 40 *m.* forenoon, and 3 hours 16 *m.* afternoon, by a watch, the sun had equal altitudes; required the time of noon and going of the watch?

	<i>h.</i>	<i>m.</i>	
add together	{	8	40 A. M.
		3	16 P. M.
		12	0

---

2) 23 56

Time of app. noon 11 58

The watch noon 12 00

Watch too fast 2 minutes.

EXAM. 2. July 1, 1790, at 8 hours 10 *m.* 58 *f.* forenoon, and at 3 hours 58 *m.* 34 *f.* afternoon, by a watch, the sun had equal altitudes; required the going of the watch?

<i>h.</i>	<i>m.</i>	<i>f.</i>
8	10	58
3	58	34
12	0	0

---

2) 24 9 32

Time of appar. noon 12 4 46

The watch noon 12 0 0

The watch too slow 4 46

*To observe the SUN'S AMPLITUDE with an AZIMUTH COMPASS.*

Move the box about till the four cardinal points of the card coincide with the four lines in the sides of the compass box ; keep the box steady in this position, and turn the index until the center of the sun, at its rising or setting, is seen through both the sights ; then the degrees intercepted between the north point of the compass and the end of the index pointing to the sun, will be the Magnetic Amplitude sought.

*For an AZIMUTH.*

Place the box and compass as before, and hold it steady ; turn the index until the sights being directed to the center of the sun, the shadow of the thread joining the sights fall directly along the middle line of the index ; and then the degrees intercepted between the north point of the card and that end of the index next the sun, shall give the Magnetic Azimuth of the sun.

The azimuth of any star is found in the same manner, only you are to view the star through the sights.

Having the true and magnetic amplitude, or azimuth, of any celestial object, to find the variation of the compass.

If the true and magnetic amplitude, or azimuth, be the same, then there is no variation, and the needle points due north ; but, if they are not the same, let both be reckoned from the north, eastward when the observation is made in the forenoon, and westward in the afternoon ; and then the difference between the true and magnetic

magnetic amplitude, or azimuth, is the variation of the compass.

*To know whether the VARIATION be EAST or WEST.*

If the amplitude be taken at sun-rising, or the azimuth in the forenoon, and if the magnetic amplitude, or azimuth, be farther from the north than the true, the variation is westward; but, if it be nearer to the north, the variation is eastward.

If the observation be made in the afternoon, or at sun-setting, and if the magnetic amplitude, or azimuth, be farther from the north than the true, the variation is eastward; but, if it be nearer to the north, the variation is westerly.

EXAM. 1. Suppose at sun-setting his amplitude, by the compass, is  $7^{\circ} 10'$  from West towards North, and his true amplitude is  $W. 25^{\circ} 15' N.$ ; required the variation, and which way?

Magnetic amplitude.  $N. 82^{\circ} 50' W.$

True amplitude.  $N. 64 \quad 45 \quad W.$

---

Variation  $18 \quad 5 \quad E.$

2. Suppose at sun-rising the amplitude, by the compass, is  $E. 16^{\circ} 12' N.$  and his true amplitude  $E. 30^{\circ} 10' N.$ ; required the variation, and which way?

Magnetic amplitude  $N. 73^{\circ} 48' E.$

True amplitude  $N. 59 \quad 50 \quad E.$

---

Variation  $13 \quad 58 \quad W.$

3. Supr

3. Suppose the sun's magnetic azimuth is N.  $124^{\circ} 30'$  E. and his true azimuth is N.  $113^{\circ} 3'$  E.; what was the variation of the compass, and which way?

Magnetic azimuth. N.  $124^{\circ} 30'$  E.

True azimuth        N.  $113^{\circ} 3'$  E.

Variation                 $11^{\circ} 27'$  W.

To know whether the variation is E. or W. the common rule is, let the observer's face be turned towards the sun, and if the true amplitude, or azimuth, be to the right hand of the magnetic, the variation is East; but, if the true be to the left hand of the magnetic, the variation is West.

Having the course set, and the variation of the compass, to find the true course made.

If the variation is W. call the N. W. quarter the 1st, the S. W. the 2d, the S. E. the 3d, and the N. E. the 4th; but, if the variation is East, call the N. E. quarter the 1st, the S. E. the 2d, the S. W. the 3d, and the N. W. the 4th; then, if the course be in the 1st or 3d quarter, add the variation to the given course set; but, if it be in the 2d or 4th quarters, subtract; and the sum, or difference, shall be the course, corrected by the variation.

EXAM. 1. Suppose the variation is  $1\frac{1}{4}$  points W. it is required to correct the following courses steered by the compass, viz. N. W.  $\frac{1}{4}$  W. S. S. W. S. E. by E.  $\frac{1}{4}$  E. and N. E. by E.

Courses

Courses	N. W. $\frac{1}{4}$ W. = $4\frac{1}{4}$ Pt.	S. S. W. = 2 Pt.
Variation add	$\frac{1}{4}$	Var. sub. $1\frac{1}{4}$
Cor. course N. W. by W. $\frac{1}{4}$ W. = $5\frac{1}{4}$		S. $\frac{3}{4}$ W. = $0\frac{3}{4}$

Courses S. E. by E. $\frac{1}{4}$ E. = $5\frac{1}{4}$ Pt.	N. E. by E. = 5 Pt.
Variation add $1\frac{1}{4}$	Var. sub. $1\frac{1}{4}$
Cor. course E. S. E. $\frac{3}{4}$ E. = $6\frac{1}{4}$	N. E. by N. $\frac{3}{4}$ E. = $3\frac{3}{4}$

3. Suppose the variation is  $21^{\circ} 30'$  E. it is required to correct the following courses steered by the compass.

Course set N. E. by N. = $33^{\circ} 45'$	S. S. E. = $22^{\circ} 30'$
Variation add $21 \ 30$	Var. sub. $21 \ 30$
Correct course N. $55^{\circ} 15'$ E.	Cor. course S. $1^{\circ} 00'$ E.

Course set W. S. W. = $67^{\circ} 30'$	N. W. by W. = $56^{\circ} 15'$
Variation add $21 \ 30$	Var sub. $21 \ 30$
Correct course S. $89^{\circ} 00'$ W.	Cor. course N. $34^{\circ} 45'$ W.

S E C T. VII  
G U N N E R Y.

D E F I N I T I O N S *and* P R I N C I P L E S.

I. **T**HE path of every projectile depends on two different forces, namely, the impellent force, whereby the motion is first begun, and which, acting alone, would carry the body forward in a straight line; and the force of gravity, by which the projectile during the whole time of its flight is urged downwards toward the center of the earth; and because both forces act continually whilst the body is in motion, its path is a curve line lying between the directions of both. It has been proved by many authors, that the path of a projectile *in vacuo* is a curve line called a *parabola*; but in air, the resistance it meets with changes the form of its path.

Most writers on this subject have thought that the action of the air on such projectiles as cannon-balls, was too small to be taken notice of; and therefore have supposed that the path of a ball in the air was truly, or very nearly, the same, as *in vacuo*.

But, although it is now known, that the resistance of the air to the motions of large and heavy bodies, such as bombs and cannon-balls, is much greater than has been commonly represented, and therefore must affect both the magnitude and figure of their path: Yet, if the  
amplitude

amplitude of the projection answering to any given elevation be first found by experiment, (which is always supposed in gunnery), the amplitude, in all other cases, where the elevations and velocities do not differ very much from the first, may be determined to a sufficient degree of exactness, by the rules arising from the parabolic hypothesis; because, in all such cases, the effects of the air's resistance will be nearly as the amplitudes; and, were they accurately so, the proportions of the amplitudes at different elevations would then be the same as *in vacuo*. Hence, the common rules given by the writers on gunnery, although not mathematically true, may be safely admitted in practice.

2. The point where the projectile begins to move, is called the *point of projection*.

3. The impetus of a piece is the height to which, with its proper charge of powder, it could make the ball ascend, when fired off in a direction perpendicular to the horizon; or the perpendicular height from which the ball must fall, to acquire the velocity it hath at the point of projection.

4. The elevation is the angle contained between the axis of the gun and a right line drawn from the point of projection in the horizontal level.

5. The amplitude, random, or range of a projectile, is the distance between the point of projection and the place where it impinges on the ground; Or, it is the horizontal distance which the projectile passeth over in its flight.

6. Different elevations of the same piece give different amplitudes; for if, when a piece is fired off, its axis be

either in the plane of the horizon, or perpendicular to it, there will be no amplitude; because, in the first case, the ball, by the action of gravity, will meet the ground immediately on leaving the mouth of the gun; and, in the last, it will fall down on the point of projection. At equal distances from either of these directions, such as, at an elevation of 25 or 65 degrees, the amplitudes will be equal. And,

When the elevation is 45 degrees, the amplitude will be the greatest possible.

7. The greatest amplitude is double to the impetus of the piece; and, when the elevation is 45 degrees, the greatest altitude of the ball is one-fourth part of the amplitude.

8. If a body be projected oblique to the horizon, it will fall there again in the same oblique direction, and with the same velocity with which it was projected.

9. The time which a heavy body, projected at an elevation of  $45^\circ$ , will continue in the air, before it arrives at the horizon, will be equal to the time that the same body would take to descend by the force of gravity, through a space equal to its amplitude, or horizontal range.

10. In most problems in gunnery, it is supposed that the gunner makes an experiment on every gun he has the care of, with its ordinary charge of powder, at an elevation of  $45^\circ$ , in order to find the greatest amplitude of the piece, and then half of this is the impetus.

The rules made use of in solving the following problems, are demonstrated by the writers on projectiles.

Of



*OF PROJECTIONS made on the Plane of the Horizon.*

**PROB. 1.** The impetus of the piece and the distance of the object aimed at being known, to find the elevation, so as to strike the object.

**RULE.** The horizontal ranges of equal bodies projected with the same velocity at different elevations, are to one another as the sines of twice the angles of elevation.

**EXAM.** Required the elevation necessary to strike an object on the horizon at the distance of 5170 yards, the impetus of the piece being 3375 yards.

As twice the impetus, or greatest amplitude 6750,

Is to the given distance of the object 5170,

So is the radius or sine of twice  $45^\circ$ ,

To the sine of twice the elevation  $= 50^\circ$ .

Hence, the lower elevation is  $25^\circ$ , and the higher  $65^\circ$ ;

For these are equally distant from  $45^\circ$ .

**PROB. 2.** The angle of elevation, and the amplitude being given, to find the greatest altitude of the ball.

**RULE.** As radius is to the tangent of the angle of elevation, so is one fourth part of the correspondent amplitude to the greatest altitude of the ball.

**EXAM.** If a piece fired off at an elevation of  $25^\circ$  throws the ball to the distance of 5170 yards; what is the greatest altitude of the ball?

*Ans.* 602.8 yards. One fourth part of 5170 is 1292.5.

As Rad. : Tan. elev.  $= 25^\circ :: 1292.5 : 602.8$ .

**EXAM.**

EXAM. 2. If the impetus of a piece be 4000 feet ; what is the proper elevation to hit an object at the distance of 4200 feet ? and what is the greatest altitude of the ball ?

*Ans.* The lower elevation is  $15^{\circ} 50'$ , and the greatest altitude of the ball is 298 feet.

PROB. 3. The impetus of a piece, and its elevation being given, to find the amplitude.

EXAM. 1. Suppose the impetus of the piece to be 3375 yards ; how far will the same piece, with an equal charge of powder, range the shot, at an elevation of  $25^{\circ}$  ?

*Ans.* 5170 yards.

As radius or sine of twice  $45^{\circ}$ ,  
Is to the sine of twice the given elevation  $50^{\circ}$  ;  
So is twice the impetus, 6750 yards,  
To the amplitude required = 5170 yards.

EXAM. 2. Let the impetus be 4000 feet, and the given elevation  $30^{\circ} 16'$  ; required the amplitude ?

*Ans.* 6965 feet.

PROB. 4. The angle of elevation, and the distance of an object aimed at being given, to find the impetus, so as to strike the object.

EXAM. 1. Let the angle of elevation be  $32^{\circ} 12'$ , and the distance of the object aimed at be 6500 feet ; required the impetus of the piece ? *Ans.* 3604 feet.

As the sine of twice the angle of elevation,  $64^{\circ} 24'$ ,  
Is to radius or sine of twice  $45^{\circ}$  ;

So is the given amplitude 6500,

To twice the impetus required 7208

Whence 3604 feet is the impetus.

EXAM.

**EXAM. 2.** Let the angle of elevation be  $30^{\circ} 16'$ , and the distance of the object or amplitude = 6965 feet; what is the impetus of the piece?

*Ans.* 4000 feet.

Hence, the impetus of a piece may be found by firing it off at any elevation, and measuring the horizontal range of the ball.

When the elevation is  $15^{\circ}$ , the amplitude is equal to the impetus; because the sine of twice 15 or of  $30^{\circ}$  is equal to half of the radius.

**PROB. 5.** The amplitude of a piece at any elevation being given, to find the amplitude at any other elevation; and the contrary.

**EXAM. 1.** If a piece, fired off at an elevation of  $25^{\circ} 12'$ , throws the ball to the distance of 5250 feet; how far will the same piece, with an equal charge of powder, throw the ball when fired off at an elevation of  $36^{\circ} 15'$ ?

*Ans.* 6498 feet.

As the sine of twice the first elevation  $50^{\circ} 24'$

Is to the sine of twice the second  $72^{\circ} 30'$

So is the first amplitude 5250,

To the amplitude required 6498

**EXAM. 2.** If a piece fired off at an elevation of  $37^{\circ} 28'$  throws the ball to the distance of 2956; at what elevation must the same piece be fired off to throw the ball to the distance of 2550 feet?

*Ans.*  $28^{\circ} 12'$ .

**PROB. 6.** The elevation of the piece, and the greatest altitude of the ball being given, to find its greatest altitude at any other elevation.

**RULE.**

**RULE.** The heights to which equal bodies projected with equal velocities rise in the air, are, in proportion to one another, as the versed sines of twice the angles of elevation.

**EXAM.** If the greatest altitude of a ball when fired off at an elevation of  $24^{\circ} 5'$  be 180 yards; at what elevation will the greatest altitude be 400 yards?

As the first altitude 180,

Is to the second 400;

So is the versed sine of twice the first elevation  $48^{\circ} 10'$ ;

To the versed sine of twice the elevation required  $74^{\circ} 56'$ .

Half of which  $37^{\circ} 28'$  is the answer.

This proportion may be wrought on Gunter's scale, thus: The extent of the compasses from 180 to 400, on the line of numbers, will reach from the versed sine of  $48^{\circ} 10'$  to  $105^{\circ} 4'$ , the supplement of which  $74^{\circ} 56'$  is the double of the elevation.

**PROB. 7.** The elevation and amplitude being given, to find the time of the flight, or of the ball's continuance in the air.

**RULE.** As the radius is to the tangent of the elevation, so is the given amplitude in feet to the square of four times the number of seconds required.

**EXAM.** Let the elevation be  $32^{\circ}$ , and the amplitude 5280; required the time of the flight?

*Ans.*  $14\frac{1}{3}$  seconds.

As Rad. : Tan.  $32^{\circ}$  :: 5280 : 3300, the square root of which is 57.44; and one fourth of 57.44 is 14.36, or  $14\frac{1}{3}$  nearly.

But

But when the elevation is  $45^\circ$ , (which is commonly the case in throwing bombs), then one fourth part of the square root of the distance in feet, is the number of seconds which the projectile continues in the air. The knowledge of this is necessary in adjusting the fusee.

*Of PROJECTIONS when the Object aimed at is above or below the level of the Piece.*

**PROB. 8.** The horizontal distance, and the angle of elevation or depression of an object being given, together with the impetus, to find the elevation of the piece, so as to hit the object.

**RULE.** As the radius is to the tangent of the elevation or depression of the object, so is twice the impetus to a fourth number; which add to, or subtract from, the given horizontal distance, according as the object is elevated or depressed. Then say,

As twice the impetus is to the sum, or remainder, so is the co-sine of the given elevation or depression, to the co-sine of an angle, which added to and subtracted from, the angle included between the object and the zenith, gives the doubles of the complements of two different elevations whereon the ball will hit the object.

**EXAM.** Let the horizontal distance of the object be 5600 feet, and its elevation  $80^\circ 15'$ , and let the impetus of the piece be 4000 feet; required the elevation?

1. As Rad. : Tan.  $80^\circ 15'$  :: 8000 : 1160; which added to 5600, gives 6760.

2. As 8000 : 6760 :: Cos.  $80^\circ 15'$  : Cos.  $33^\circ 15'$ .

M m

Add

Add and subtract  $33^{\circ} 15'$  to and from  $81^{\circ} 45'$ , and you have  $115^{\circ}$  and  $48^{\circ} 30'$ , the halves of which are  $57^{\circ} 30'$  and  $24^{\circ} 15'$ , whose complements  $32^{\circ} 30'$ , and  $65^{\circ} 45'$  are the two elevations required.

The rules relating to oblique projections are seldom attended to in actual service.

In all shooting on ascents and descents, it is best to find the angle between the object and the zenith, and take the complement of half that angle for the elevation, and find by trial what charge will reach the object; for, on this elevation, a less charge of powder will do the business than on any other: And this method is commonly practised by gunners.

**PROB. 9.** The amplitude of projection with a given quantity of powder being known, to find the quantity of powder necessary to hit an object at a given distance, the elevation remaining the same.

**RULE.** When equal bodies are projected with different velocities, at the same elevation, the horizontal ranges are in proportion to one another as the weights of the charges of powder, nearly.

**EXAM.** If a 24 pounder, elevated to  $45^{\circ}$ , 16lb. of powder will throw the ball to the distance of 6750 yards; how much powder will throw the same ball 5170 yards, at the same elevation?

*Ans.*  $12\frac{1}{4}$  lb.

As the first horizontal range 6750,

Is to the second 5170;

So is the weight of the given charge 16 lb.

To the weight of the charge required 12.26 lb.

*Note.* This rule cannot be accurate.

Cannons

Cannons are either made of iron or brass, of different lengths and bores, and named from the weights of the balls suited to their capacities. Thus, a great gun whose ball weighs 12 lb. is called a 12 pounder; and one whose ball weighs 24 lb. is called a 24 pounder, &c.

The charges of powder, according to the English usage, are about the weight of the ball for proof; half its weight for service, and one-fourth of its weight for salutes.

The French allow two-thirds of the weight of the ball for service, and twice as much for proof.

Large brass guns are allowed a greater charge of powder than iron ones of the same bore.

At sea, after five or six rounds in warm service, the quantity of powder is diminished every charge, until it come to about one-third of the weight of the ball.

### *Of the COMPUTATION of SHOT.*

Cannon-balls and shells are usually piled up in a pyramidal, or prismatic form; the base being either an equilateral triangle, a square, or a rectangle. In the triangle and square, the pile is a pyramid, finished by a single ball at the top; but, when the base is a rectangle, the pile is finished by a single row of balls.

In pyramids of balls, the number of balls on each side of the base decreases from the base upwards by 1; that is, if the number of balls in one side of the base be 20, the number in the tire next above that will be 19, and in

the next 18, &c. until all the sides of the pyramid meet at the top in a single ball. Therefore, if the base be a triangle, the several tires of balls will be triangular numbers, which, reckoned from the top, are

1. 3. 6. 10. 15. 21. 28. 36. 45. 55. 66. 78. &c.

But, if the base be a square, the tires of balls will be square numbers, which, beginning at the top, are

1. 4. 9. 16. 25. 36. 49. 64. 81. 100. 121. 144. 169. &c.

1. To find the sum of a series of triangular numbers beginning at 1, the number of terms being given.

Put  $n =$  the number of the terms, and then  $\frac{n \times n + 1 \times n + 2}{6}$  is the sum of the series.

EXAM. What is the sum of 8 terms of the series of triangular numbers?

Here  $n = 8$ , and  $\frac{8 \times 9 \times 10}{6} = 120$ , is the answer.

2. To find the sum of a series of square numbers beginning with 1, the number of terms being given?

Put  $n =$  the number of terms, then  $\frac{n \times n + 1 \times 2n + 1}{6}$

is the sum of the series.

EXAM. It is required to find the sum of 10 terms of the series of square numbers?

Here  $n = 10$ , and  $\frac{10 \times 11 \times 21}{6} = 385$ , is the answer.

3. To find the number of shot in a triangular pile.

Count the number of balls in one side of the base; to that number add 1, and to the same number add 2; then multiply



multiply these three numbers, one by the other, and the product by the third, and divide the product by 6; the quotient is the number of shot in the pile.

**EXAM. 1.** How many balls are there in a finished triangular pile, one side of the base consisting of 24 balls?

By adding 1 and 2 to 24, the three numbers are 24, 25, 26, and  $\frac{24 \times 25 \times 26}{6} = 2600$  balls, the answer.

2. Required the number of balls in a finished triangular pile, one side of the lowest tire being 30 balls?

*Ans.* 4960.

*Note.* In triangular and square pyramidal piles, the number of horizontal tires, or the number of balls from top to bottom, is always equal to the number in one side of the base, or bottom tire.

4. To find the number of shot in a square pyramidal pile.

Count the number of balls in one side of the base; to the number add 1, and to its double add 1; then multiply these three numbers into each other continually, and divide the product by 6; the quotient is the number of balls in the pile.

**EXAM.** It is required to find the number of shot in a square pile, each side of its base consisting of 20 balls?

By adding 1 to 20, and 1 to the double of 20, the three numbers are 20, 21, and 41, and  $\frac{20 \times 21 \times 41}{6} = 2870$  balls, the answer.

5. In a broken pyramidal pile, either triangular or square.

Compute

Compute the number of balls in the whole pile according to its base, and then, by the number in one side of the uppermost tire, compute the number in the part that is wanting, the difference of these two is the number of balls in the broken pile.

EXAM. 1. It is required to find the number of balls in a broken triangular pile, one side of its base being 30 balls, and one side of the uppermost tire being 15 ?

*Ans.* 4280.

In the whole pile there is  $\frac{30 \times 31 \times 32}{6} = 4960$

In the part wanting there is  $\frac{15 \times 16 \times 17}{6} = 680$

Difference 4280

2. How many balls are there in a broken square pile, the number in one side of the base being 40, and the number in a side of the uppermost tire being 20 ?

*Ans.* 19270.

In the whole pile there is  $\frac{40 \times 41 \times 81}{6} = 22140$

In the part wanting there is  $\frac{20 \times 21 \times 41}{6} = 2870$

Difference 19270

In rectangular piles, each horizontal tire is a rectangle, the uppermost being one row of balls; and the number of horizontal tires is equal to the number in the breadth of the base.

6. To find the number of shot in an oblong pile.

Compute the number of shot in a square pile, each side of its base being the breadth of the rectangular base. Then take the difference between the length and breadth of the rectangular base; multiply the breadth  $+ 1$  by half of the breadth, and multiply the product by the said difference, the last product is the number of balls in the prismatic pile; and the sum of the two is the number of balls in the whole pile.

*Note.* Every oblong pile consists of a square pyramid, (each side of its base being the breadth of the oblong pile) and the triangular prism.

EXAM. There is a compleat oblong pile of 15 tires, the number of balls in the top row being 32; it is required to find the number of shot in this pile.

*Ans.* 4960.

Here, the difference between the length and breadth of the base is 31, and the breadth of the base is 15.

A square pile, the side of whose base is 15, is  $\frac{16 \times 31 \times 15}{6} = 1240$ . The breadth of the base  $+ 1$  is

16, and the half breadth is  $7\frac{1}{2}$ , and  $16 \times 7\frac{1}{2} \times 31 = 3720$ . Lastly,  $1240 + 3720 = 4960$ .

7. To find the number of shot in a broken oblong pile.

To twice the length, and to twice the breadth of the uppermost tire, add the number of tires  $- 1$ , and multiply the two sums together; also multiply the number of tires  $+ 1$ , by the same number  $- 1$ , and add one third of this product to the former: Then one fourth part of the

the sum, multiplied by the number of tires, shall give the number of shot in the oblong broken pile.

EXAM. Suppose there is a broken oblong pile of shot, the length of the uppermost tire being 25, and its breadth 16 balls, and the number of tires 11; it is required to find the number of shot in the pile?

Here twice the length is 50, and twice the breadth is 32, and the number of tires — 1 is 10.

Then  $60 \times 42 = 2520$  is the first product, also  $12 \times 10 = 120$  is the second product, one third of which is 40, and  $2520 + 40 = 2560$ .

Lastly,  $\frac{2560 \times 11}{4} = 640 \times 11 = 7040$  balls, the answer.

## S E C T. VIII.

### THE NATURE AND USE OF LOGARITHMS.

**I**F the natural numbers are considered as terms of an infinite series of proportionals, beginning at unity, and either increasing or decreasing to infinity, the logarithm of any number is its distance from unity in that series. Or,

Logarithms are artificial numbers, so related to each other, and to the natural numbers, that the sum of any two logarithms is the logarithm of the product of their two natural numbers; and the difference of any two logarithms is the logarithm of the quotient of their two natural numbers. Logarithms are a series of numbers in arithmetical progression corresponding to other numbers in

in

in geometrical progression; and their nature and properties are to be derived from the known properties of these progressions.

If the common difference of any series of numbers in arithmetical progression, whose first term is 0, be represented by  $d$ ; and the common ratio of a series of numbers in geometrical progression, whose first term is 1, be represented by  $r$ ; then shall the co-efficients of  $d$  in the arithmetical progression be equal to the indices, or exponents, of  $r$  in the geometrical progression.

EXAMPLES.

Arith. Prog.  $0 \quad d \quad 2d \quad 3d \quad 4d \quad 5d \quad 6d \quad 7d \quad 8d, \text{ \&c.}$

Geom. Prog.  $1 : r : r^2 : r^3 : r^4 : r^5 : r^6 : r^7 : r^8, \text{ \&c.}$

1. If  $d=1$ , and  $r=3$ , the above progressions will be,

Arith.  $0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$

Geom.  $1 : 3 : 9 : 27 : 81 : 243 : 729 : 2187 : 6561 : 19683$

2. If  $d=3$ , and  $r=3$ , then,

Arith.  $0 \quad 3 \quad 6 \quad 9 \quad 12 \quad 15 \quad 18 \quad 21 \quad 24 \quad 27$

Geom.  $1 : 3 : 9 : 27 : 81 : 243 : 729 : 2187 : 6561 : 19683$

3. If  $d=1$ , and  $r=10$ , then,

Arith.  $0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

Geom.  $1 : 10 : 100 : 1000 : 10000 : 100000 : 1000000$

In these progressions, it is manifest,

1. That, when  $d=1$ , every term in the arithmetical series expresses the distance of its correspondent term of the geometrical series from unity, or 1, agreeable to our definition of logarithms.

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2. That

2. That if any two or more terms of the arithmetical series be added, and their correspondent terms of the geometrical series be multiplied into each other, the sum and product shall be corresponding terms.

### EXAMPLES.

1. In the general series, the sum of  $2d$  and  $3d$  is  $5d$ , and the product of their corresponding terms, viz.  $r^2 \times r^3 = r^5$ ; which is the term corresponding to  $5d$ .

2. In the progression where  $d=1$  and  $r=3$ , we have  $2+3=5$ , and  $9 \times 27=243$ , the term corresponding to 5.

3. The sum of 2, 3, and 4, is 9; and  $9 \times 27 \times 81=19683$ , the term corresponding to 9.

4. In the series where  $d=3$ , and  $r=3$ . The sum of 6 and 18 is 24; and the product of  $9 \times 729=6561$ , the term corresponding to 24. And the same holds true every where.

3. If any term in the arithmetical series be doubled or tripled, and its corresponding term in the geometrical series be squared, or cubed, the results will be corresponding terms.

EXAM. In the series where  $d=1$  and  $r=3$ . The double of 3 is 6, and the square of its corresponding term 27 is 729, which corresponds to 6.

2. The triple of 3 is 9, and the cube of 27 is 19683, which is the term corresponding to 9.

4. If any term in the arithmetical series be subtracted from another, and their corresponding terms in the geometric series be divided one by the other, the difference and quotient shall be correspondent terms.

EXAM.

EXAM. The difference between 2 and 5 is 3; and, if 243 be divided by 9, the quotient is 27, which corresponds to 3.

5. If any term in the arithmetical series be divided by 2 or 3, and the square or cube root of the corresponding term be extracted, the quotient and root shall be corresponding terms.

EXAM. 1. The half of 8 is 4, and the square root of 6561 is 81, which corresponds to 4.

2. The third part of 9 is 3, and the cube root of 19683 is 27, which is the correspondent term to 3.

Hence, it is evident, that the terms of the arithmetical progression are logarithms of their corresponding terms of the geometrical progression. And since  $d$  and  $r$  may represent any number whatsoever; if  $d=1$ , and  $r=10$ , (as in the last progression), the arithmetical progression will be the common logarithms now in use. But these are only the logarithms of the numbers 10, 100, 1000, &c.; and, in order to find out the logarithms of the intermediate numbers 2, 3, 4, &c. it may be observed,

1. In any geometrical progression, if between any two adjacent terms, any number of mean proportionals be interpolated, a series of terms will be produced, which are also in geometrical progression.

EXAM. Let the proposed geometrical progression be

$1 \quad r^4 \quad r^8 \quad r^{12}$

Place one mean between every two terms, and the series shall be

$1 \quad r^2 \quad r^4 \quad r^6 \quad r^8 \quad r^{10} \quad r^{12}$

And if three means be placed between every two terms, the series shall be

$1 : r^2 : r^3 : r^4 : r^5 : r^6 : r^7 : r^8 : r^9 : r^{10} : r^{11} : r^{12}$

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2. III

2. In like manner, if between the terms of an arithmetical progression, there be interpolated any number of arithmetical means, there will be produced a series in arithmetical progression.

EXAM. In the arithmetical progression

$$0 : 4 : 8 : 12 : 16 : 20$$

Place one mean between every two terms, and the progression will be

$$0 : 2 : 4 : 6 : 8 : 10 : 12 : 14 : 16 : 18 : 20$$

Place three means between every two terms, and the series shall be

$$0 : 1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10 : 11 : 12 : 13 : 14 : 15 : 16 : 17 : 18 : 19 : 20$$

And if one mean be placed between every two terms of this series, we shall have

$$0 : \frac{1}{2} : 1 : 1\frac{1}{2} : 2 : 2\frac{1}{2} : 3 : 3\frac{1}{2} : 4 : 4\frac{1}{2} : 5 : 5\frac{1}{2} : 6, \&c.$$

Therefore, the number of terms in any progression may be increased to infinity, by interpolating means between every two adjacent terms. Hence,

When any system of logarithms is proposed for the natural geometrical series 1, 10, 100, 1000, 10000, &c. if a great number of mean proportionals be interpolated between the adjacent terms, and correspondent arithmetical means be interpolated between their corresponding logarithms, the logarithms of the intermediate natural numbers will be had. For example,

A mean proportional between 1 and 10 is 3.16227766, and its logarithm is .5000000, viz. the half sum of 0 and 1, and a mean proportional between 3.16227766, and 10 is 5.6234132, and its logarithm is .7500000, viz. the half sum of 1 and .5; and, if we calculate mean proportionals,



tionals, and their corresponding logarithms continually between 10 and the last found mean, until we find a mean proportional greater than 9; and then calculate mean proportionals between that mean and the next less than 9; the 25th mean will be so nearly equal to 9, that it will not want above  $\frac{1}{1000000}$  of an unit; and, by finding an arithmetical mean answering to every geometrical mean, the 25th will be 0.9542425, which is the common logarithm of the number 9.

In this manner, the first tables of logarithms were calculated; but this method being very tedious, shorter methods have been invented by means of infinite serieses, whereby logarithms may be made, or the truth of any logarithm examined, with less trouble.

The noble inventor of logarithms made choice of a system different from that in present use; but, soon after he had published his logarithms; he proposed another more commodious form, and communicated his invention to Mr. HENRY BRIGGS, Professor of Geometry at Oxford, who, upon Lord NAPIER's decease, calculated and published tables of logarithms of this kind; and hence those logarithms that were first published are called *Napier's logarithms*, and the other, which are now in common use, are called *Briggs's logarithms*, although both kinds were Lord NAPIER's invention. A specimen of each kind follows:

Numbers.

| Numbers. | Napier's<br>Logarithms. | Numbers. | Briggs's<br>Logarithms. |
|----------|-------------------------|----------|-------------------------|
| 1        | .0000000                | 1        | .0000000                |
| 10       | 2.3025850               | 10       | 1.0000000               |
| 100      | 4.6051700               | 100      | 2.0000000               |
| 1000     | 6.9077550               | 1000     | 3.0000000               |
| 10000    | 9.2103400               | 10000    | 4.0000000               |
| 100000   | 11.5129250              | 100000   | 5.0000000               |

These two systems of logarithms are so related to each other, that *Briggs's* logarithms may be said to be derived from *Napier's*; and, in order to calculate *Briggs's* logarithms, it is necessary, first of all, to calculate *Napier's* logarithm of 10, for which purpose, the following series has been invented.

If any number be represented by  $\frac{1+e}{1-e}$ , *Napier's* logarithm of that number will be

$$2 \times e + \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7} + \frac{e^9}{9} + \frac{e^{11}}{11} + \frac{e^{13}}{13}, \text{ \&c.}$$

But, in applying this series to practice, it will be proper to begin with a small number; for the less  $e$  is taken, the more swiftly will the series converge.

**EXAM.** Let it be required to find *Napier's* logarithm of 2?

$$\frac{1+e}{1-e} = 2 \quad 1+e = 2-2e \quad \therefore \quad 3e = 1 \quad e = \frac{1}{3} \text{ and } e^2 = \frac{1}{9}$$

*Ans.*

$$e = .3333333333$$

|          |           |
|----------|-----------|
| $e^3$    | 370370370 |
| $e^5$    | 41152263  |
| $e^7$    | 4572473   |
| $e^9$    | 508052    |
| $e^{11}$ | 56450     |
| $e^{13}$ | 6272      |
| $e^{15}$ | 696       |
| $e^{17}$ | 77        |

$$e = .3333333333$$

|                     |             |
|---------------------|-------------|
| $\frac{1}{3}e^3$    | .0123456790 |
| $\frac{1}{3}e^5$    | .8230452    |
| $\frac{1}{3}e^7$    | .653210     |
| $\frac{1}{3}e^9$    | .56450      |
| $\frac{1}{3}e^{11}$ | .5131       |
| $\frac{1}{3}e^{13}$ | .482        |
| $\frac{1}{3}e^{15}$ | .46         |
| $\frac{1}{3}e^{17}$ | .4          |

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$$.3465735898$$

$$\times 2$$


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*Napier's* logarithm of 2 = .6931471796

And, because the cube of 2 is 8, multiply the logarithm of 2 by 3, the product is the logarithm of 8 = 2.0794415388.

And, because  $8 \times \frac{1}{4} = 10$ , calculate the logarithm of  $\frac{1}{4}$ , and add it to the logarithm of 8, the sum shall be the logarithm of 10.

$$\frac{1+e}{1-e} = \frac{5}{4} \quad 4 + 4e = 5 - 5e \therefore 9e = 1 \quad e = \frac{1}{9}, \text{ and } ee = \frac{1}{81}$$

$$e = .1111111111$$

|       |          |
|-------|----------|
| $e^3$ | 13717421 |
| $e^5$ | 169350   |
| $e^7$ | 2090     |
| $e^9$ | 25       |

$$e = .1111111111$$

|                  |         |
|------------------|---------|
| $\frac{1}{3}e^3$ | 4572473 |
| $\frac{1}{3}e^5$ | 33870   |
| $\frac{1}{3}e^7$ | 298     |
| $\frac{1}{3}e^9$ | .2      |

---


$$.1115717754$$

$$\times 2$$


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*Napier's* logarithm of  $\frac{1}{4}$  = .2231435508

To which add the log. of 8    2.0794415388

And *Napier's* log. of 10 is = 2.3025850896

In

In calculating the logarithm of 2,  $e = \frac{1}{9}$ , and therefore the first step is to write down the decimal of  $\frac{1}{9} = .333$ , &c. To get  $e^3$ , I multiply  $e$  by  $e^2 = \frac{1}{9}$ : that is, I divide  $.333$ , &c. by 9, and write the quotient  $.037037$ , &c. under the value of  $e$ . For  $e^3$ , I multiply  $e^3$  by  $e^2$ ; that is, I divide  $.037037$ , &c. by 9, and so on, dividing every power of  $e$  by 9, I get  $e^7$ ,  $e^9$ ,  $e^{12}$ , &c.; then I divide each power by its index, and the sum of the quotients is the half of *Napier's* logarithm of 2. In the same manner, the logarithm of  $\frac{5}{4}$ , or of any other number, is found.

*Napier's* log. of 10 comes out  $\approx 2.3025850896$ ; but if in the first step of the calculation,  $e$  had been taken nearer to the truth, and made to consist of 17 places of decimals, and all the rest of the operation carried on accordingly, *Napier's* logarithm of 10 would have been  $\approx 2.302585092994045$ , and its reciprocal (that is 1 divided by it)  $\approx .434294481903251$ .

A system of logarithms being given, and one or more of another system, any logarithm of that other system may be found thus:

As any logarithm of the given system is to its correspondent logarithm in the other system, so is any other logarithm in the given system to its correspondent logarithm in the other system.

EXAM. *Napier's* logarithm of 10 being 2.3025850, and his logarithm of 2  $\approx .6931471$ , also *Briggs's* logarithm of 10 being 1.0000000; it is required to find *Briggs's* logarithm of 2?

As

As *Napier's* logarithm of 10 = 2.3025850

Is to *Briggs's* logarithm of 10 = 1.0000000

So is, *Napier's* logarithm of 2 = .6931471

To *Briggs's* logarithm of 2 = .3010299

And because the two first terms of this proportion are constant, and the second term is 1, if any of *Napier's* logarithms be divided by 2.3025850, or multiplied by its reciprocal, viz. .4342944, the quotient, or product, will be *Briggs's* logarithm of the same number. Therefore,

To calculate *Briggs's* logarithms directly, we must multiply the series  $2 \times e + \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7}$ , &c. by .43429,

&c.; or, multiply  $e \times \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7}$  &c. by the double

of .43429, &c. viz. .8685889. Put  $m = .8685889$ , &c.

and then the series expressing *Briggs's* logarithm of

$\frac{1+e}{1-e}$  will be  $me + \frac{me^3}{3} + \frac{me^5}{5} + \frac{me^7}{7} + \frac{me^9}{9} + \frac{me^{11}}{11} + \frac{me^{13}}{13}$

&c. Or,

$$m \times e + \frac{e^3}{3} + \frac{e^5}{5} + \frac{e^7}{7} + \frac{e^9}{9} + \frac{e^{11}}{11} + \frac{e^{13}}{13} + \frac{e^{15}}{15} \dots$$

EXAM. I. Let it be required to find *Briggs's* logarithm of 2?

$$\frac{1+e}{1-e} = 2 \quad 1+e = 2-2e \therefore 3e = 1 \quad e = \frac{1}{3}, \text{ and } e^2 = \frac{1}{9}.$$

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$$m = .8685889638$$


---

$$me = .2895296546$$

$$me^3 = 321.99616$$

$$me^5 = 35744401$$

$$me^7 = 3971600$$

$$me^9 = 441288$$

$$me^{11} = 49032$$

$$me^{13} = 5448$$

$$me^{15} = 605$$

$$me^{17} = 67$$

$$me = .2895296546$$

$$\frac{1}{3}me^3 = 107233205$$

$$\frac{1}{5}me^5 = 7148880$$

$$\frac{1}{7}me^7 = 567375$$

$$\frac{1}{9}me^9 = 49032$$

$$\frac{1}{11}me^{11} = 4457$$

$$\frac{1}{13}me^{13} = 419$$

$$\frac{1}{15}me^{15} = 40$$

$$\frac{1}{17}me^{17} = 4$$


---

Briggs's logarithm of 2 = .3010299954

2. To find Briggs's logarithm of 3; because  $2 \times 1\frac{1}{2} = 3$ , therefore calculate the logarithm of  $\frac{3}{2}$ , and add it to the logarithm of 2, the sum is the logarithm of 3.

$$\frac{1+e}{1-e} = \frac{3}{2}, \quad 2+2e=3-3e \therefore 5e=1 \quad e=\frac{1}{5}, \text{ and } e^2=\frac{1}{25}.$$

$$m = .8685889638$$


---

$$me = .1737177927$$

$$me^3 = 69487117$$

$$me^5 = 2779484$$

$$me^7 = 111179$$

$$me^9 = 4447$$

$$me^{11} = 177$$

$$me = .1737177927$$

$$\frac{1}{3}me^3 = 23162372$$

$$\frac{1}{5}me^5 = 555897$$

$$\frac{1}{7}me^7 = 15882$$

$$\frac{1}{9}me^9 = 494$$

$$\frac{1}{11}me^{11} = 16$$


---

Briggs's logarithm of  $\frac{3}{2}$  = 1760912588

To which add the log. of 2 .3010299954

---

The sum is the log. of 3 = .4771212542

3. To

3. To find the logarithm of 4; because the square of 2 is 4.

$$\text{Multiply the logarithm of } 2 \quad = .3010299954$$

$$\text{By } 2 \quad \underline{\hspace{1.5cm}}$$

$$\text{The product is the log. of } 4 \quad = .6020599908$$

4. To find the logarithm of 5; because 10 divided by 2 quotes 5.

$$\text{From the logarithm of } 10 \quad = 1.0000000000$$

$$\text{Subtract the logarithm of } 2 \quad .3010299954$$

$$\text{The remainder is the log. of } 5 \quad .6989700046$$

5. To find the logarithm of 6; because  $3 \times 2 = 6$ .

$$\text{To the logarithm of } 3 \quad = .4771212542$$

$$\text{Add the logarithm of } 2 \quad .3010299954$$

$$\text{The sum is the logarithm of } 6 \quad .7781512496$$

6. To find the logarithm of 7; because  $6 \times \frac{7}{6} = 7$ , therefore calculate the logarithm of  $\frac{7}{6}$ , and add it to the logarithm of 6, the sum is the logarithm of 7.

$$\frac{1+e}{1-e} = \frac{7}{6} \quad 6+6e=7-7e \quad \therefore 13e=1 \quad e=\frac{1}{13}, \text{ and } e^2=\frac{1}{169}$$

$$m = .8685889638$$

$$me = .0668145356$$

$$me^3 \quad 3953522$$

$$me^5 \quad .23393$$

$$me^7 \quad 138$$

$$me = .0668145356$$

$$\frac{1}{3}me^3 \quad 1317840$$

$$\frac{1}{5}me^5 \quad 4678$$

$$\frac{1}{7}me^7 \quad 19$$

$$\text{Logarithm of } \frac{7}{6} \quad = .0669467893$$

$$\text{Add the logarithm of } 6 \quad .7781512496$$

$$\text{The sum is the log. of } 7 \quad = .8450980389$$

7. To find the logarithm of 8; because the cube of 2 is 8.

Multiply the logarithm of 2 = .3010299954  
By 3 3

The product is the log. of  $8 = .9030899862$

8. To find the logarithm of 9; because the square of 3 is 9.

Multiply the logarithm of 3 = .4771212542  
By 2 2

The product is the log. of  $9 = .9542425084$

9. To find the logarithm of 11; because  $10 \times \frac{11}{10} = 11$ , find the logarithm of  $\frac{11}{10}$ , and add it to the logarithm of 10, the sum is the logarithm of 11.

$$\frac{1+e}{1-e} = \frac{11}{10} \quad 10+10e = 11-11e \quad \therefore 21e = 1 \quad e = \frac{1}{21}, \text{ and}$$

$$e^2 = \frac{2}{4\pi r^0}$$

$$m = .8685889638$$

$$me = .0413613792$$

me<sup>3</sup> 937899

me<sup>s</sup> 2126

$$m = .0413613792$$

$\frac{1}{3}me^3$  312633

*nie*. 425

Logarithm of  $\frac{11}{15} = .0413926850$

Add the logarithm of 10 1.0000000000

The sum is the logarithm of 11 . 1.0413926850

- 10. To



10. To find the logarithm of 12; because  $6 \times 2 = 12$ .

To the logarithm of 6 = .7781512496

Add the logarithm of 2 .3010299954

---

The sum is the logarithm of 12 = 1.0791812450

11. To find the logarithm of 13; because  $12 \times \frac{1}{12} = 13$ ; find the logarithm of  $\frac{1}{12}$ , and add it to the logarithm of 12, the sum is the logarithm of 13.

$$\frac{1+e}{1-e} = \frac{13}{12} \quad 12 + 12e = 13 - 12e \quad \therefore 24e = 1 \quad e = \frac{1}{24}, \text{ and}$$

$$e^2 = \frac{1}{576}$$

$$m = .8685889638$$

$$me = .0347435585$$

$$me^3 = 555896$$

$$me^5 = 889$$

$$me = .0347435585$$

$$\frac{2}{3}me^3 = 185298$$

$$\frac{1}{3}me^5 = 178$$

---

Logarithm of  $\frac{1}{12} = .0347621061$

Add the logarithm of 12 1.0791812450

---

The sum is the logarithm of 13 = 1.1139433511

12. To find the logarithm of 14; because  $7 \times 2 = 14$ .

To the logarithm of 7 = .8450980389

Add the logarithm of 2 .3010299954

---

The sum is the logarithm of 14 = 1.1461280343

13. To

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13. To find the logarithm of 15; because  $5 \times 3 = 15$ .

|                      |   |   |             |
|----------------------|---|---|-------------|
| To the logarithm of  | 5 | = | .6989700046 |
| Add the logarithm of | 3 |   | .4771212542 |

---

The sum is the logarithm of 15 = 1.1760912588

14. To find the logarithm of 16; because the square of 4 is 16.

|                           |   |   |             |
|---------------------------|---|---|-------------|
| Multiply the logarithm of | 4 | = | .6020599908 |
| By                        | 2 |   |             |

---

The product is the log. of 16 = 1.2041199816

15. To find the logarithm of 17; because  $16 \times \frac{17}{16} = 17$ , calculate the logarithm of  $\frac{17}{16}$ , and add it to the logarithm of 16, the sum is the logarithm of 17.

$$\frac{1+e}{1-e} = \frac{17}{16} \quad 16+16e=17-17e \therefore 33e=1 \quad e=\frac{1}{33}, \text{ and}$$

$$e^2 = \frac{1}{1089}$$

$$m = .8685889638$$


---

|        |   |             |
|--------|---|-------------|
| $me$   | = | .0263208776 |
| $me^3$ |   | 241697      |
| $me^5$ |   | 221         |

|                   |   |             |
|-------------------|---|-------------|
| $me$              | = | .0263208776 |
| $\frac{1}{3}me^3$ |   | 80565       |
| $\frac{1}{3}me^5$ |   | 44          |

---

|                      |                 |   |              |
|----------------------|-----------------|---|--------------|
| Logarithm of         | $\frac{17}{16}$ | = | .0263289385  |
| Add the logarithm of | 16              |   | 1.2041199816 |

---

The sum is the logarithm of 17 = 1.2304489201

16. To

16. To find the logarithm of 18; because  $9 \times 2 = 18$ .

|                      |   |   |             |
|----------------------|---|---|-------------|
| To the logarithm of  | 9 | = | .9542425084 |
| Add the logarithm of | 2 | = | .3010299954 |

The sum is the logarithm of 18      1.2552725038

17. To find the logarithm of 19; because  $\frac{19}{18} \times 18 = 19$ , calculate the logarithm of  $\frac{19}{18}$ , and add it to the logarithm of 18, the sum is the logarithm of 19.

$$\frac{1+e}{1-e} = \frac{19}{18} \quad 18 + 18e = 19 \quad \therefore 37e = 1 \quad e = \frac{1}{37}, \text{ and}$$

$$e^2 = \frac{1}{1369}$$

$$m = .8685889638$$

$$me = .0234753774$$

$$me^3 = .171478$$

$$me^5 = 125$$

$$me = .0234753774$$

$$\frac{1}{3}me^3 = 57159$$

$$\frac{1}{3}me^5 = 25$$

|              |                 |   |             |
|--------------|-----------------|---|-------------|
| Logarithm of | $\frac{19}{18}$ | = | .0234810958 |
|--------------|-----------------|---|-------------|

|                      |    |   |              |
|----------------------|----|---|--------------|
| Add the logarithm of | 18 | = | 1.2552725038 |
|----------------------|----|---|--------------|

The sum is the logarithm of 19 = 1.2787535996

By these calculations, we have obtained the logarithms of the first 20 numbers true to eight places of figures, which are as follows:

| N <sup>o</sup> . | Logarithms. | N <sup>o</sup> . | Logarithms. |
|------------------|-------------|------------------|-------------|
| 1                | .00000000   | 11               | 1.04139268  |
| 2                | .30102999   | 12               | 1.07918124  |
| 3                | .47712125   | 13               | 1.11394335  |
| 4                | .60205999   | 14               | 1.14612803  |
| 5                | .69897000   | 15               | 1.17609125  |
| 6                | .77815125   | 16               | 1.20411998  |
| 7                | .84509803   | 17               | 1.23044892  |
| 8                | .90308998   | 18               | 1.25527250  |
| 9                | .95424250   | 19               | 1.27875360  |
| 10               | 1.00000000  | 20               | 1.30102999  |

In the same manner, the whole table of logarithms may be made to any extent.

1. It may be observed, that this work is most tedious at the beginning, and becomes easier as we advance; for, fewer terms of the series give the logarithm of  $\frac{13}{12}$  than of  $\frac{3}{2}$ ; and, in numbers above 100 and below 1000, the first term will give the logarithm true to seven places of figures; and, in any number above 1000, the first term of the series will give the logarithm true to eight or nine places.

2. When the logarithm of any number is known, the logarithms of all its multiples by 10, 100, 1000, &c. are also known; for the logarithm of 2 is the same with the logarithm of 20, 200, 2000, 20000, or 200000, &c. except the indices which are to be prefixed according to the number of figures in the natural number. And the same is true of any other number.

3. The logarithms of all composite numbers are found by addition or subtraction, as may be seen in the above examples.

examples. It is only in calculating the logarithms of prime numbers that the series must be applied.

The logarithms of prime numbers may also be found by either of the two following rules :

1. The logarithms of the numbers next below and above any prime number being given, to find the logarithm of that prime number.

Let  $a$  and  $b$  represent the adjacent numbers, whose logarithms are known ;  $p$  = the number whose logarithm

is sought ;  $y = ab + \frac{a^2 + 2ab + b^2}{4} = 2ab + 1$  ; and  $n =$

.43429448190, &c. Then the logarithm of the ratio of the geometrical mean between  $a$  and  $b$ , viz.  $\sqrt{ab}$ , to the arithmetical mean, viz.  $p$ , shall be

$$= \frac{n}{y} + \frac{n}{3y^3} + \frac{n}{5y^5} + \frac{n}{7y^7}, \text{ \&c.}$$

In numbers between 20 and 100, the first term of this series will give the logarithm true to 9 places of figures ; and, in numbers above 100, the first term will give the logarithm true to 12 or 14 places of figures. Therefore, since the first term of the series will produce any logarithm for common use, we have this practical rule for finding the logarithm of any prime number above 20.

Multiply the adjacent numbers, one by the other, and add 1 to double of the product. Divide .43429448190 by the sum, and add the quotient to the half sum of the logarithms of the adjacent numbers, the sum is the logarithm required.

P.p.

EXAM.

EXAM. Let it be required to find the logarithm of 23?

$$a=22, b=24, \text{ and } 2ab+1=1057=y$$

$$1057) \cdot 43429448190 (.00041087462$$

The log. of 22 = 1.34242268082

Ditto of 24 = 1.38021124171

---


$$2) 2.72263392253 \text{ sum}$$


---

Half sum = 1.36131696126

To which add .00041087462

---

Log. of 23 = 1.36172783588 too little by 2 in the 10th place.

2. The logarithm of the ratio of the geometrical mean to the arithmetical mean may also be found thus :—

Let  $y$  represent the difference between the logarithms of the adjacent numbers, and  $z$  the number whose logarithm is sought. Then  $y \times \frac{1}{4z} + \frac{1}{24z^3} + \frac{7}{360z^5}$  is the lo-

garithm of the ratio of the geometrical to the arithmetical mean.

This series does not converge so quickly as the former, and therefore not so proper for small numbers. But, if the prime number is between 100 and 1000, the first term of the series will give the logarithm true to 9 places of figures; and, in numbers above 1000, to 12 or 14 places: Wherefore, to find the logarithm of any prime number above 100, divide the difference of the logarithms of the adjacent numbers by four times that

prime

prime number, and add the quotient to the half sum of the adjacent logarithms, the sum is the logarithm required.

EXAM. Let it be required to find the logarithm of 101?

$$\text{The log. of } 100 = 2.0000000000$$

$$\text{Ditto of } 102 = 2.0086001717$$

$$\text{Sum} = 4.0086001717$$

$$\text{Half sum} = 2.0043000858$$

$$\text{Diff.} = .0086001717$$

$$2=101 \times 4=404).0086001717(.0000212875$$

$$\text{Add } 2.0043000858$$

$$\text{Logarithm of } 101 = 2.0043213733$$

True to the 10th place.

The first of these rules was invented by Dr. *Halley*, the other by Dr. *John Keil*.

The use of logarithms is to shorten the operations of multiplication, division, raising powers, and extracting roots, in common arithmetic.

To answer this end, it is necessary to have the logarithms of all numbers, ready to be used as occasion requires. But because, in the common problems where logarithms are most useful, the numbers seldom exceed five or six figures, tables of logarithms have been made for all numbers under 10 or 100 thousand only, from which the logarithm of any number, and the number

answering to any logarithm, may be had true to six or seven figures, according to the extent of the tables.

Since the logarithm of 1 is 0, and of 10 is 1, the logarithms of numbers between 1 and 10 are greater than 0, and less than 1; that is, they are decimal fractions. *Vide p. 281, &c.*

- And, since the logarithm of 10 is 1, and of 100 is 2, the logarithms of numbers between 10 and 100 will be greater than 1, and less than 2, or mixed numbers, having 1 in the place of integers, and the rest decimal fractions.

The integral part of a logarithm is called its *index*, or *characteristic*; because it denotes the number of figures in the natural number answering to the logarithm.

The index of a logarithm is always an unit less than the number of figures in the integer number, whose logarithm it is. Thus, the index of the logarithm of a number consisting of one figure is 0, of 2 figures is 1, of 3 figures 2, and 4 figures, 3, &c. as in these examples.

| Numb. | Logarithms. | Numb.      | Logarithms. |
|-------|-------------|------------|-------------|
| 7     | 0.8450980   | 325675     | 5.5127845   |
| 17    | 1.2304489   | 4897255    | 6.6899527   |
| 139   | 2.1430148   | 92675000   | 7.9669626   |
| 5262  | 3.7211508   | 687540000  | 8.8372980   |
| 56990 | 4.7557987   | 4997560000 | 9.6987580   |

Therefore, the indices of logarithms (being known by the number of figures in their natural numbers) need not be printed in the tables.

When



When a logarithm is proposed, its index shows how many integer figures the corresponding number consists of, being always one figure more than there are units in the index of the logarithm.

Numbers which consist of the same significant figures, have the fractional parts of their logarithms always the same, and differ only in their indices, as in the following

EXAMPLES.

| Numb. | Logarithms. |
|-------|-------------|
| 47500 | 4,6766936   |
| 4750  | 3,6766936   |
| 475   | 2,6766936   |
| 47.5  | 1,6766936   |
| 4.75  | 0,6766936   |
| .475  | 9,6766936   |
| .0475 | 8,6766936   |

The index of the logarithm of a decimal fraction is negative; thus the index of .475 is properly  $-1$ , of .0475  $-2$ , and of .00475  $-3$ ; but, to avoid negative numbers in addition and subtraction, their complements to 10 or 100 are made use of; and, instead of  $-1$ ,  $-2$ ,  $-3$ , the indices are made 9, 8, 7, or 99, 98, 97.

The following table contains the logarithms of numbers from 1 to 10000; their fractional parts consisting of 7 figures, from which the logarithm of any number may be had from 1 to 10000000 by the following rules:

1. To find the logarithm of any number below 1000.

Find the number near the beginning of the table in the column marked N°. at the head; and next it on the right

right hand is the fractional part of the logarithm, to which prefix its proper index.

|           | Numb. | Logarith. |
|-----------|-------|-----------|
| EXAMPLES. | 19    | 1.2787536 |
|           | 364   | 2.5611014 |
|           | 795   | 2.9003671 |

2. To find the logarithm of any number consisting of four figures.

Find the first three figures of the number in the column marked N°. and the fourth or last figure on the head of the page; below which, opposite to the first three figures, you have the four or five last figures of the logarithm; to which prefix the common figures in the second column marked with 0, or 5, and the index 3.

|           | Numb. | Logarith. |
|-----------|-------|-----------|
| EXAMPLES. | 1974  | 3.2953471 |
|           | 3785  | 3.5780659 |
|           | 4977  | 3.6969676 |
|           | 6748  | 3.8291751 |

N. B. The common figures in the second column answer to those in every column after it, in the same and following lines, until the figures in the following column begins with 0, and then you are to take the common figures next below, as in the fourth example above.

3. To find the logarithm of any number consisting of five, six, or seven figures.

Find the logarithm of the first four figures as before, to which prefix the index according to the number of figures

figures in the given number ; and then take the difference between the logarithm found, and the next greater in the table ; which multiply by the remaining figure or figures in the given number ; and strike off the number of figures from the product by which the difference was multiplied. Add the remaining part of the product to the logarithm before found ; the sum is the logarithm required.

EXAM. 1. What is the logarithm of 25768 ?

|                           |     |                         |
|---------------------------|-----|-------------------------|
| The logarithm of 25760 is |     | 4.4109459               |
| Difference 1685           | add | 1348                    |
| Multiply by 8             |     | <u>          </u>       |
| 1348 0                    |     | Log. of 25768=4.4110807 |

2. What is the logarithm of 498797 ?

|                            |     |                          |
|----------------------------|-----|--------------------------|
| The logarithm of 498700 is |     | 5.6978374                |
|                            | add | 845                      |
| Difference 872             |     | <u>          </u>        |
| Multiply 97                |     | Log. of 498797=5.6979239 |
| 6104                       |     |                          |
| 7848                       |     |                          |
| <u>          </u>          |     |                          |
| 845 84                     |     |                          |

3. What is the logarithm of 5678979 ?

Ans. 6.7542703.

4. To find the logarithm of a proper fraction, subtract the logarithm of the denominator from the logarithm of the numerator, the remainder is the logarithm of the fraction.

EXAM.

EXAM. What is the logarithm of  $\frac{179}{479}$ ?

From the log. of 179 (add 10 to its index) 12.2528530

Subtract the log. of 479 2.6803355

*Ans.* 9.5725175

To find the logarithm of a mixt number, such as  $1\frac{1}{2}$ ,  $2\frac{1}{4}$ ,  $23\frac{1}{47}$ , &c.

Reduce the mixt number to an improper fraction, and subtract the logarithm of the denominator from the logarithm of the numerator; the remainder is the answer.

EXAM. 1. What is the logarithm of  $1\frac{1}{2}$ ?

$1\frac{1}{2} = \frac{3}{2}$  From the log. of 3 .4771213

Subtr. the log. of 2 — .3010300

There remains the log. of  $1\frac{1}{2} = .1760913$

2. What is the logarithm of  $2\frac{1}{4}$ ?

$2\frac{1}{4} = \frac{9}{4}$  log. of 9 .9542425

ditto of 4 .6020600

log. of  $2\frac{1}{4} = .3521825$

3. Required the logarithm of  $23\frac{1}{47}$ ?

$23\frac{1}{47} = \frac{1092}{47}$

$\frac{47}{1092}$  log. of 1092 3.0382226

$\frac{1092}{47}$  ditto of 47 — 1.6720979

log. of  $23\frac{1}{47} = 1.3661247$

4. What is the logarithm of  $197\frac{5}{9}$ ?

$197\frac{5}{9} = \frac{3748}{9}$  log. of 3748 3.5737996

ditto of 9 — 1.2787536

log. of  $197\frac{5}{9} = 2.2950460$

5. To

5. To find the logarithm of a decimal fraction, proceed as if it were an integer, and prefix the proper index.

| N <sup>o</sup> . | Logarithms. | N <sup>o</sup> . | Logarithms. |
|------------------|-------------|------------------|-------------|
| .5               | 9.6989700   | .075             | 8.8750613   |
| .25              | 9.3979400   | .0015            | 7.1760913   |
| .125             | 9.0969100   | .000325          | 6.5118834   |
| .4795            | 9.6807886   | .00007925        | 5.8989993   |

6. To find the number answering to any given logarithm.

Look for the fractional part of the given logarithm in the different columns until you find it, either exactly, or the next less. And, in a line with the logarithm found, in the column marked N<sup>o</sup>. you have three figures of the number sought, and on the top of the page you have one figure more, which write down, and point it according to the index of the given logarithm.

If the logarithm is not found exactly in the tables, and more than four figures are required, subtract the logarithm found in the table from the given one, and divide the remainder by the difference between the logarithm found and the next greater; the quotient joined to the four figures already found, gives the number answering to the logarithm.

#### EXAMPLES.

To find the number answering to the logarithm 2.7342957. Seek in the second column for the three figures next the index (.734), and, having found them, look among the columns on the right hand for the last four figures, and in that under 3, you find 2396; there-

Q q

fore

fore the number is 5423 ; but, because the index of the logarithm is 2, the number answering hath only three integer figures, and the last is a decimal, viz. 542.3.

If more figures are desired, divide the difference between 2957 and 2396, viz. 561, by 801, the difference of the logarithms (annexing 0 to 561), the quotient 7, joined to the former, makes the answer 542.37.

Required the numbers answering to these logarithms.

|           |           |
|-----------|-----------|
| 0.7369425 | 1.2786536 |
| 1.2345678 | 2.9003671 |
| 2.6549764 | 3.7654321 |
| 3.7496543 | 4.6849776 |
| 5.9769423 | 1.2497894 |

## LOGARITHMICAL ARITHMETIC.

7. To perform multiplication by logarithms, add the logarithm of the multiplier to the logarithm of the multiplicand ; the sum is the logarithm of the product.

### EXAMPLE.

|             |       |           |
|-------------|-------|-----------|
| 1. Multiply | 379   | 2.5786392 |
| By          | 47    | 1.6720979 |
|             |       | <hr/>     |
| Product     | 17813 | 4.2507371 |

2. Multiply 79.8 by 2.79.

8. To perform division by logarithms.

Subtract the logarithm of the divisor from the logarithm of the dividend, the remainder is the logarithm of the quotient.

EXAM.

EXAM. 1. Divide 25768                      4.4110807  
                     By                      364                      2.5611014

Quotient 70.7912                      1.8499793

2. Divide 476954 by 89.5.

9. To find a fourth proportional to three given numbers, or to work the rule of three by logarithms :

Add the logarithms of the second and third terms, and subtract the logarithm of the first term from the sum; the remainder is the logarithm of the fourth term, or answer.

EXAM. If 497 yards cost L. 287; what will 389 yards cost?                      *Ans.* L. 224 : 12 : 8.

|               |           |
|---------------|-----------|
| As 497        | 2.6963564 |
| Is to 287     | 2.4578819 |
| So is 389     | 2.5899496 |
|               | <hr/>     |
|               | 5.0478315 |
|               | 2.6963564 |
|               | <hr/>     |
| To L. 224.633 | 2.3514751 |

N. B. Instead of subtracting the logarithm of the first term, some chuse to add its arithmetical complement.

10. To find the arithmetical complement of a logarithm; begin at the left hand, and write down what each figure wants of 9, and what the last significant figure wants of 10; so the arith. comp. of 2.6963564 is 7.3036436.

11. To raise a number to any power, by logarithms.

Multiply the logarithm of the given number by the index of the power; the product is the logarithm of the power required.

Q. 9. 2

EXAM.

EXAM. Required the third power or cube of 81?

81 its log. is 1.9084850

Multiply by 3

*Ans.* 531441 5.7254550

What is the 3d power of  $37\frac{1}{8}$ ? *Ans.* 53529.34

$$37\frac{1}{8} = \frac{603}{16}$$

603 2.7803173

16 — 1.2041200

Log. of  $\frac{603}{16} = 1.5761973$

Multiply by 3

4.7285919

The 3d power of  $37\frac{1}{8}$  is 53529.34

*Note.* In large numbers, the logarithms give the answer true only to seven figures; and even the 7th figure is sometimes doubtful, as in this example, where it is too little by 1.

2. Required the fourth power of 1.05?

1.05 its logarithm is 0.0211893

Multiply by 4

*Ans.* 1.215506 0.0847572

It is required to find the 7th power of 1.05?

*Ans.* 1.4071.

1.05 its logarithm is 0.0211893

Multiply by 7

0.1483251

The 7th power of 1.05 = 1.4071.

3. Required the sixth power of .05?

Its logarithm is 8.6989700

Multiply by 6

*Ans.* .000000015625 52.1938200

Here



Here the tens are thrown away from the index 52, and the 2 remaining shews that the first significant figure must be in the eighth place from unity, and therefore seven cyphers must be prefixed.

12. To extract roots by logarithms.

Divide the logarithm of the given number by the index of the power; the quotient is the logarithm of the root; that is, divide by 2 for the square root, by 3 for the cube root, and by 4 for the biquadrate, &c.

1. What is the square root of 2? *Ans* 1.414213.

The logarithm of 2. = 0.3010300

$\frac{1}{2}$  = 0.1505150

The square root is 1.414213.

2. Required the square root of 19110?

*Ans*. 138.2389.

19110 its log. is. 4.2812607

$\frac{1}{2}$  = 2.1406303

The square root is 138.2389.

3. What is the square root of  $99\frac{1}{2}$ ? *Ans*. 9.99583.

$99\frac{1}{2} = 199$  log. 3.2788192

12 log. 1.0791812

2) 1.9999980

0 9998190

The square root of  $99\frac{1}{2}$  is 9.99583.

To find the side of a square which shall contain a given square any number of times.

To the logarithm of the given square add the logarithm of the number of times the square sought is to contain

contain the given one, and divide the sum by 2, the quotient is the logarithm of the side of the square required.

EXAM. Suppose there is a square which contains an acre, it is required to find the side of a square which shall contain 12 acres? *Ans.* 1095.445 links.

1 Acre is 100000 square links. Log. 5.0000000  
add the log. of 12 1.0791812

---

2)6.0791812

---

1095.445 links.

3.0395906

To find the side of a square which shall contain an area equal to a given rectangle.

Take the logarithms of the length and breadth of the given rectangle; and half their sum is the logarithm of the side of the square.

EXAM. Let there be a right angled parallelogram, its length 600 feet, and breadth 430; it is required to find the side of a square which shall contain the same area?

*Ans.* 532.728 feet.

Length 660 log. 2.8195439

Breadth 430 2.6334685

---

2)5.4530124

---

side of the square 532.728 F. 2.7265062

To find the side of a square, equal to a triangle; the base and perpendicular height of the triangle being given in numbers.

To the logarithm of the base add the logarithm of half the perpendicular, and divide the product by 2, the quotient is the logarithm of the side of the square.

EXAM.

EXAM. Let the base of a triangle be one mile, and the perpendicular height of it three quarters of a mile; it is required to find the side of a square which shall contain the same area as the triangle? *Ans.* 3233.326 feet.

Feet.

$$\begin{array}{rcl} 1 \text{ mile} & = & 5280 \text{ log. } 3.7226339 \\ \frac{3}{4} \text{ the perpend. } & = & 1980 \quad 3.2966652 \end{array}$$

$$\begin{array}{r} \text{Feet} \quad \cdot \quad 2)7.0192991 \\ \hline \end{array}$$

$$\text{Side of the sqr. } 3233.326 \quad 3.5696495$$

To find the side of a square equal to a trapezium, its diagonal and sides being given.

The diagonal divides the trapezium into two triangles; therefore, calculate the area of each triangle by its three sides, the sum of these is the area of the trapezium in numbers: The half of the logarithm of the area is the logarithm of the side of the square, equal to the trapezium.

EXAM. Let the side AB=46, BC=37, CD=75, and DA=63 feet, also the diagonal AC=73 feet: required the side of a square which shall contain the same area as the trapezium? *Ans.* 53.136 feet.

For the area of the triangle ACD.

|       |                 |                 |                 |                 |       |  |                 |
|-------|-----------------|-----------------|-----------------|-----------------|-------|--|-----------------|
| 1st,  | 73              | 105.5           | 105.5           | 105.5           | 105.5 |  | 2.0232525       |
|       | 75              | 73.             | 75.             | 63.             | 32.5  |  | 1.5118834       |
|       | 63              | <u>        </u> | <u>        </u> | <u>        </u> | 30.5  |  | 1.4842998       |
|       | <u>        </u> | 32.5            | 30.5            | 42.5            | 42.5  |  | 1.6283889       |
| 2)211 |                 |                 |                 |                 |       |  | <u>        </u> |
|       |                 |                 |                 |                 |       |  | 2)6.6478246     |
|       | 105.5           |                 |                 |                 |       |  | <u>        </u> |

$$2108.2 \quad 3.3239123$$

For

For the area of the triangle ACB.

|     |               |    |    |    |    |                             |
|-----|---------------|----|----|----|----|-----------------------------|
| 2d, | 73            | 78 | 78 | 78 | 78 | 1.8920946                   |
|     | 46            | 73 | 46 | 37 | 41 | 1.6127839                   |
|     | 37            | —  | —  | —  | 32 | 1.5051500                   |
|     | <u>      </u> | 5  | 32 | 41 | 5  | 0.6989700                   |
|     | 2)156         |    |    |    |    | <u>                    </u> |
|     | <u>      </u> |    |    |    |    | 2)5.7089985                 |
|     | 78            |    |    |    |    | <u>                    </u> |

715.3 2.8544992

3d, Sq<sup>r</sup>. feet.  
triangle ACD = 2108.2  
ACB = 715.3

Sum = 2823.5 log.  $\frac{2}{3.4507878}$

*Ans.* 53.136 F. 1.7253939

In the same manner, the side of a square may be found, whereof the area is equal to the area of any given rectilinear figure.

The diameter of a circle being given, to find the side of a square of the same area.

Multiply the logarithm of the given diameter by 2, and to the product add the logarithm of .7854, the sum is the logarithm of the area of the circle; and its half is the logarithm of the side of the square.

EXAM. Let the diameter of a circle be 112.837 feet, it is required to find the side of a square of equal area with the circle? *Ans.* 99.9992 feet.

Diam. 112.837. its log. 2.0524516  
Mult.                     2

4.1049032  
.7854 log. 9.8950909

2)3.9999941 log. area of the circle  
                      
1.9999970

Side of the square 99.9992 F.

What

1. What is the cube root of 208? *Ans.* 5.92499  
 208 its log. 2.3180633

The root 5.92499  $\div$  0.7726877

2. Required the cube root of 11976? *Ans.* 22.879  
 11976 log. 4.0783117

The root 22.879  $\div$  1.3594372

3. What is the cube root of 312908547069?  
*Ans.* 6789.

To find the logarithm of this large number, take the four first figures on the left hand 3129, and find their logarithm; then take the difference between this logarithm and the next greater in the table (1387): Multiply the remaining figures of the given number 08547069 by 1387: strike off eight figures from the product; add the remaining figures to the logarithm formerly found, and the sum is the logarithm of 312908547069.

|                     |            |          |
|---------------------|------------|----------|
| 31290000000000 log. | 11.4954056 | 08547069 |
| For 08547069 add    | 118        | 1387     |

|                   |            |          |
|-------------------|------------|----------|
| 312908547069 log. | 11.4954174 | 59829483 |
|                   |            | 68376552 |

|                |                  |          |
|----------------|------------------|----------|
| Cube root 6789 | $\div$ 3.8318058 | 25641207 |
|                |                  | 8547069  |

118.54784703

4. Extract the cube root of 53529  $\frac{481}{1180} = \frac{68517601}{1180}$ ?  
*Ans.* 37.6875.

From the log. of 68517601 = 7.8358022  
 Subtr. the log. of 1280 = 3.1072100

3)4.7285922

The cube root 37.6875

1.5761974

R r

5. Extract

5. Extract the cube root, and the biquadrate root of  $\frac{15}{768}$ ?

$$\begin{array}{r}
 15 \log. 11.1760913 \\
 768 \log. 2.8853612 \\
 \hline
 3)28.2907301 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 4)38.2907301 \\
 \hline
 .3738 \quad 9.5726825 \\
 \text{biquadrate root.}
 \end{array}$$

Cube root .2693 9.4302433

In this example. From the logarithm of the numerator 15, (10 being added to its index,) subtract the logarithm of the denominator 768; the remainder 8.2907301 is the logarithm of the given fraction: then prefix 2 to the index of this logarithm, and divide the whole 28.2907301 by 3, the quotient 9.4302433 is the logarithm of the cube root. And for the biquadrate root, prefix 3 to the index, which makes it 38.2907301 and divide the whole by 4, the quotient is the logarithm of the biquadrate root.

N. B. In extracting the roots of proper fractions by logarithms, prefix to the index of the logarithm a number less by 1 than the index of the root, and then divide by the index of the root.

6. Suppose there are three cubic vessels, their sides being 11.5 17.25 and 20.2 inches: It is required to find the side of a cubic vessel which shall contain as much as all the three?

$$\begin{array}{r}
 11.5 \log. 1.0606978 \\
 \hline
 3 \\
 \hline
 1520.874 = 3.1820934
 \end{array}
 \qquad
 \begin{array}{r}
 17.25 \log. 1.2367891 \\
 \hline
 3 \\
 \hline
 5132.952 = 3.7103673
 \end{array}$$

$$\begin{array}{r}
 20.2 \log. 1.3053514 \\
 \hline
 3 \\
 \hline
 8242.409 = 3.9160542
 \end{array}$$

1520.874

$$\begin{array}{r}
 1520.874 \\
 5132.952 \\
 8242.409 \\
 \hline
 \text{Sum of the cubes} \quad 14896.235 \quad \log. \quad 4.1730765 \\
 \text{Ans. } 24.605 \quad \frac{1}{3} \quad 1.3910255
 \end{array}$$

Logarithms are of great use in the solution of problems which require tedious calculations, such as,

In the mensuration of surfaces.

When the four sides and the diagonal of any quadrilateral field are measured, to find its area; such as, in Q. 12, p. 143, here are two triangles, and in each of them the three sides are given to find its area; (the rule for performing this by logarithms, is given in p. 119;) and the sum of these areas is the area of the field.

EXAM. 1. What is the area of the quadrilateral field ABCD, its diagonal BD=3096 links, the side AB=1976, the side BC=1760 links, CD=2016 links and DA=2340 links? Fig. 58.

|   |   |
|---|---|
| <p>1. For the area of the triangle ABD.</p> $  \begin{array}{r}  2340 \quad 3706 \quad 3706 \quad 3706 \\  1976 \quad 2340 \quad 1976 \quad 3096 \\  3096 \quad \text{---} \quad \text{---} \quad \text{---} \\  \text{---} \quad 1366 \quad 1730 \quad 610 \\  2)7412 \\  \hline  3706  \end{array}  $ | <p>2. For the area of the triangle CBD.</p> $  \begin{array}{r}  3096 \quad 3436 \quad 3436 \quad 3436 \\  2016 \quad 3096 \quad 2016 \quad 1760 \\  1760 \quad \text{---} \quad \text{---} \quad \text{---} \\  \text{---} \quad 340 \quad 1420 \quad 1676 \\  2)6872 \\  \hline  3436  \end{array}  $ |
|---|---|

|            | Logarithms.        |            | Logarithms.  |
|------------|--------------------|------------|--------------|
| 3706       | 3.5689054          | 3436       | 3.5360532    |
| 1366       | 3.1354507          | 1676       | 3.2242740    |
| 1730       | 3.2380461          | 1420       | 3.1522883    |
| 610        | 2.7853298          | 340        | 2.5314789    |
|            | <hr/>              |            | <hr/>        |
|            | 2)12.7277320       |            | 2)12.4440944 |
| Sq. links. | <hr/>              | Sq. links. | <hr/>        |
| 2311351    | 6.3638660          | 1667428    | 6.2220472    |
| 1667428    |                    |            |              |
| <hr/>      | <i>A. R. Fall.</i> |            |              |

3978779 = 39 3 6 the answer.

1. To find the area of any regular polygon by logarithms; its side and the perpendicular falling from the center on it, being given. Multiply the length of the given side by the number of sides, and take half of the product, and then, to the logarithm of half the sum of the sides, add the logarithm of the perpendicular, the sum is the logarithm of the area of the polygon.

EXAM. What is the area of a heptagon, its side being 21 yards 2 feet, and the perpendicular falling from the center on that side = 67.486 feet?

*Ans.* 1705 square yards, 8 feet 8 inches.

|                 |               |
|-----------------|---------------|
|                 | <i>Yd. F.</i> |
| Side            | 21 2          |
|                 | 3             |
|                 | <hr/>         |
|                 | 65 feet.      |
|                 | 7             |
|                 | <hr/>         |
| Sum of the side | 455           |
|                 | <hr/>         |
| half sum        | 227.5         |

Half



Half sum of the sides 227.5 log. 2.3569814  
 perpendicular 67.486 1.8292137

Square feet 15353.06 4.1861951  
 or 1705 yards 8 feet 8 inches.

2. The diameter of a circle being given to find its area by logarithms.

Multiply the logarithm of the diameter by 2, and to the product, add the logarithm of .7854, the sum is the logarithm of the area.

EXAM. Let the diameter of a circle be 1760 yards, what is its area? *Ans.* 2432855 square yards.

1760 log. 3.2455127  
 2

6.4910254

.7854 log. 9.8950909

6.3861163

Area sq. yds. 2432855.

*Mensuration of Solids by Logarithms.*

1. The side of a cube being given, to find its solid content, find the logarithm of the given side, and multiply it by 3, the product is the logarithm of the solid content.

EXAM. 1. The side of a cube is 19 feet, what is its solid content? *Ans.* 6859 cubic feet.

19 its log. 1.2787536

3

3.8362608

cubic feet 6859.

2. What

2. What is the solid content of a cube, its side being 9 feet 1 inch? *Ans.* 749 cubic feet 757 inches.

*F. I.*

9 1 = 109 inches. log. 2.0374265

3

cubic inches 1295028.8      6.1122795

Subtract 1728 log. 3.2375437

2.8747358

cubic feet 749.438

or 749 feet 757 inches.

3. The length, breadth, and thickness of any solid body having equal rectangular bases being given, to find its solid content by logarithms.

Reduce the given dimensions to the same denomination; take their logarithms from the table and add them, the sum is the logarithm of the solid content.

*EXAM.* Let the length of a log of wood be 17 feet 6 inches, the breadth 2 feet 3 inches, and the thickness 1 foot 9 inches? What is its solid content?

*Ans.* 68 cubic feet 1566 inches.

*Feet.*

Length      17.5      1.2430380

Breadth      2.25      0.3521825

Thickness      1.75      0.2430380

68.9062 feet = 1.8382585

or 68 cubic feet 1566 inches.

4. To find the solid content of a prism, its bases and length being given, by logarithms.

Calculate the logarithm of the area of one of its bases, by some of the former rules; to which add the logarithm of

of the given length, the sum is the logarithm of the solid content : If it be in inches subtract the logarithm of 1728 from it, and the remainder is the logarithm of the solid content in feet.

EXAM. What is the solid content of a prism, whose bases are regular pentagons, one side of each pentagon being 2 feet  $9\frac{1}{2}$  inches, and the perpendicular falling from the centre on that side 23.05 inches : also the length of the prism 6 feet 8 inches ?

F. I.

Side = 2  $9\frac{1}{2}$

12

33.5

5

2)167.5

$\frac{1}{2}$  sum of the sides 83.75

Half sum, sides of the base 83.75 1.9229848

Perpendicular 23.05 1.3626709

Log. area of the base in inches 3.2856557

Length in inches 80 + 1.9030900

Log. of the solid cont. in inches 5.1887457

Subtr. log. of 1728 — 3.2375437

Log. of solid content, feet, 89.372 = 1.9512020

or 89 cubic feet 643 inches.

5. To find the solid content of a pyramid, its base and height being given, by logarithms.

Calculate the solid content of a prism of the same base and altitude, and one third part of it is solidity of the pyramid.

EXAM.

**EXAM.** What is the solid content of a pyramid, the side of its square base being 9 feet 6 inches, and its height 25 feet 7 inches? *Ans.* 769.632 cubic feet.

|   |   |
|---|---|
| $\begin{array}{r} 25\cancel{7}^{\frac{7}{12}} \\ 12 \\ \hline 307 \\ \hline 12 \end{array}$ | $\begin{array}{r} 9.5 \log. 0.9777236 \\ \hline \text{Log. area of the base } 1.9554472 \\ \text{Log. of the height } 1.4079572 \\ \hline 3)2308.896 \end{array}$ |
|   | $\begin{array}{r} 2 \\ \hline 3.3634044 \end{array}$  |

Solidity 769.632 Feet.

*Note.* The rules for calculating the solid contents of the Cylinder and Cone, are the same with those for the prism and pyramid.

6. What is the solid content of a Cylinder, the diameter of the base being 5 feet 6 inches, and its length 21 feet 10 inches? *Ans.* 518.724 cubic feet.

|   |   |
|---|---|
| $\begin{array}{r} P. I. \\ 21-10 \\ 12 \\ \hline 262 \end{array}$ | $\begin{array}{r} \text{Diam. of the base } 5.5 \log. 0.7403627 \\ \hline 1.4807254 \\ .7854 \log. 9.8950909 \\ \text{length } 2\frac{6}{12} \log. 1.3391201 \\ \hline \text{Solid content } 518.724 \end{array}$ |
|   | $\begin{array}{r} 2 \\ \hline F. 2.7149364 \end{array}$   |

7. What is the solid content of a Cone, the diameter of its base being 36 inches, and its axis, or height, 48 inches?

*Ans.* 9 cubic feet 734 inches.

Diam.

$$\begin{array}{r} \text{Diam. 36 log. } 1.5563025 \\ \times 2 \\ \hline \end{array}$$

$$3.1126050$$

$$7.854 \text{ log. } 9.8950909$$

$$\text{axis 48 log. } 1.6812412$$

$$3)48858.15 \text{ inches, } 4.6889371$$

Solid content 16286.05 inches,

or 9 cubic feet 734 inches.

8. The length of a round tree being given in feet, and its mean girt in inches, to find its solid content, by logarithms.

Multiply the logarithm of the mean girt by 2, and to the product add the logarithms of .0795775 and of the length, and from the sum subtract the logarithm of 144; the remainder is the logarithm of the solid content of the tree in cubic feet.

EXAM. What is the solid content of a round tree, 25 feet 9 inches long, and its mean girt  $57\frac{1}{2}$  inches: what is its solid content? *Ans.* 47.0479 cubic feet.

$$\begin{array}{r} \text{Mean girt } 57.5 \text{ log. } 1.7596678 \\ \times 2 \\ \hline \end{array}$$

$$3.5193356$$

$$.0795775 \text{ log. } 8.9007902$$

$$\text{length } 25.75 \text{ log. } 1.4107772$$

$$3.8309030$$

$$144 \text{ log. } -2.1583625$$

$$47.0479 \text{ feet. } 1.6725405$$

S. f.

2. The

2. The length of a round tree is 45 feet 6 inches, and its mean girth is  $61\frac{1}{4}$  inches; what is its solid content?

*Ans.* 94.33 cubic feet.

$$\begin{array}{r}
 \text{Mean girth } 61.25 \text{ log. } 1.7871061 \\
 \times 2 \\
 \hline
 3.5742122 \\
 .0795775 \text{ log. } 8.9007902 \\
 \text{length } 45.5 \quad 1.6580114 \\
 \hline
 4.1330138 \\
 144 \text{ log. } -2.1583625 \\
 \hline
 94.33 \quad 1.9746513
 \end{array}$$

9. To find the solid content of a tetrahedron by logarithms, the side of one triangular face being given.

Multiply the logarithm of the given side by 3, and to the product add half the logarithm of 2, and from the sum subtract the logarithm of 12, the remainder is the logarithm of the solid content.

**EXAM.** Let the side of one of the containing triangles be 12: required the solid content of the tetrahedron?

*Ans.* 203.646 cubic inches, or feet, according to the name of the given side.

$$\begin{array}{r}
 12 \text{ its log. } 1.0791812 \\
 \times 3 \\
 \hline
 3.2375436 \\
 \frac{1}{2} \text{ log. of } 2 \quad + 0.1505150 \\
 \hline
 3.3880586 \\
 \text{log. of } 12 \quad - 1.0791812 \\
 \hline
 \text{solid cont. } 203.646 \quad 2.3088774
 \end{array}$$

2. Let

2. Let the side of one of the triangles be 20 inches; required the solid content of the tetrahedron?

*Ans.* 942.8 cubic inches.

$$\begin{array}{r}
 20 \text{ log. } 1.3010300 \\
 \times 3 \\
 \hline
 3.9030900 \\
 \frac{1}{2} \text{ log. of } 2 = 0.1505150 \\
 \hline
 4.0536050 \\
 \text{log. of } 12 = 1.0791812 \\
 \hline
 942.8 \quad 2.9744238
 \end{array}$$

10. To find the solid content of an octahedron by logarithms; the side of one triangular face being given.

Multiply the logarithm of the given side by 3, and from the product subtract the logarithm of 3; add half the logarithm of 2 to the remainder; the sum is the logarithm of the solid content.

EXAM. Let the side of one of the containing triangles be 12 inches; what is the solid content of the octahedron? *Ans.* 814.586 cubic inches.

$$\begin{array}{r}
 \text{side } 12 \text{ log. } 1.0791812 \\
 \times 3 \\
 \hline
 3.2375436 \\
 \text{Subt. the log. } 3 \quad 0.4771213 \\
 \hline
 2.7604223 \\
 \frac{1}{2} \text{ log. of } 2 \quad + 0.1505150 \\
 \hline
 \text{Solid cont. } 814.586. \quad 2.9109373
 \end{array}$$

11. To find the solid content of a dodecahedron by logarithms; the area of one of the containing pentagons,

gons, and the diameter, or height of the solid being given.

To the logarithm of the area of the pentagonal face, add the logarithm of twice the diameter; the sum is the logarithm of the solid content.

EXAM. Let the area of one of the pentagons be 247.7487 square inches, and the diameter of the solid 26.72424 inches, what is the solid content of the dodecahedron; *Ans.* 13241.78 cubic inches.

|  |           |
|--|-----------|
| given area 247.7487 log.               | 2.3940113 |
| twice the diam <sup>r</sup> . 53.44848 | 1.7279352 |

---

4.1219465

Solid cont. 13241.78 inches, or 7 cubic feet, 1145 cubic inches.

12. To find the solid content of an icosaedron by logarithms; the side of one of the containing triangles, and the diameter of the solid being given.

To twice the logarithm of the given side add half the logarithm of 3, and from the sum subtract the logarithm of 4; the remainder is the area of one of the containing triangles; to which add the logarithm of the diameter and the logarithm of 20, and from the sum subtract the logarithm of 6; the remainder is the logarithm of the solid content of the icosaedron.

EXAM. What is the solid content of an icosaedron; its diameter being 18.138264 inches, and the side of each containing triangle 12 inches?

*Ans.* 3769.96 cubic inches.

*Given*



|                         |                 |
|-------------------------|-----------------|
| Given side 12, its log. | 1.0791812       |
|                         | $\times 2$      |
|                         | <hr/> 2.1583624 |
| half, the log. of 3     | $+ 0.2385606$   |
|                         | <hr/> 2.3969230 |
| Subtr. the log. 4       | $0.6020600$     |
|                         | <hr/> 1.7948630 |
| 18.138264 log.          | 1.2585932       |
| 20. log.                | 1.3010300       |
|                         | <hr/> 4.3544862 |
| 6 log. subtr.           | $0.7781513$     |
|                         | <hr/> 3.5763349 |
| Solid content 3769.96   |                 |

## *Of Logarithm Sines and Tangents.*

The tables of sines and tangents has the 90° of the quadrant disposed thus. At the head of the two first pages, there is 0 degrees, and in the left hand columns are the minutes of a degree. At the head of the next two pages, there is 1°, and so on to 44° on the head of the two last pages, at the foot of which is 45, from whence they proceed gradually backward unto the first two, where there is 89°; and on each page, the right hand columns contain the minutes of a degree increasing upwards.

13. To find the logarithm sine and tangent of any number of degrees and minutes below 45°. Look for the degrees on the head of the page, and the minutes in the left hand column; and, opposite to the minutes, you have their sines and tangents in their respective columns.

Sines

|           |        | Sines.    | Tan.      |
|-----------|--------|-----------|-----------|
| EXAMPLES. | 19° 0' | 9.5126419 | 9.5369718 |
|           | 19 24  | 9.5213488 | 9.5467346 |
|           | 19 56  | 9.5326608 | 9.5594914 |

14. To find the logarithm sine and tangent of any number of degrees and minutes above  $45^\circ$ .

Look for the degrees at the foot of the page, and the minutes in the right hand column, and opposite to the minutes you have the sine and tangent, each in its proper column, marked at the foot with sine or tangent.

|           |         | Sines.    | Tan.       |
|-----------|---------|-----------|------------|
| EXAMPLES. | 53° 00' | 9.9023486 | 10.1228856 |
|           | 53 23   | 9.9045230 | 10.1289428 |
|           | 53 49   | 9.9069446 | 10.1358197 |
|           | 53 57   | 9.9076820 | 10.1379422 |

15. To find the sine or tangent of a number of degrees, minutes, and seconds.

Find the sine or tangent of the degrees as before; then find the difference between the sine or tangent found and the next greater, and say, as 60 is to that difference, so is the number of seconds to the part to be added for the seconds.

EXAM. Require the sine of  $19^\circ 24' 36''$ ?

The sine of  $19^\circ 24'$  is 9.5213488, which subtract from 9.5217074, the sine of  $19^\circ 25'$ ; the diff. is 3586.

Then, as  $60'' : 3586 :: 36'' : 2151$ , which add to 9.5213488, and the sum 9.5215639 is the sine of  $19^\circ 24' 36''$ . Proceed in the same manner when a tangent is wanted.

16. To find the co-sine or co-tangent of any number of degrees and minutes.

Look

Look for the sine or tangent of the given number of degrees and minutes, and next it in the column of the same name you have the co-sine or co-tangent required.

Thus the co-sine of  $15^{\circ} 24'$  is 9.9841200, which is the sine of  $74^{\circ} 36'$ , and the co-tangent of  $73^{\circ} 36'$  is 9.4400363, or the tangent of  $15^{\circ} 24'$ .

17. To find the number of degrees and minutes answering to a given logarithm sine.

Seek the given logarithm sine in the columns of sines; and, if you find it, or the next less in the first column on the left hand, you have the degrees on the head of the page, and the minutes on the left, counting downwards. But, if you find it in the second column, you have the degrees at the foot of the page, and the minutes on the right hand, counting upwards. Proceed in the same manner with tangents.

EXAMPLES.

Sines.

Tangents.

$$9.6843280 = 28^{\circ} 54'$$

$$9.8764564 = 36^{\circ} 57'$$

$$9.9532976 = 63^{\circ} 54'$$

$$10.1097654 = 52^{\circ} 9'$$

18. When the given logarithm sine is not found exactly in the table, and it is required to find seconds, subtract the logarithm sine found in the table from the given one, also subtract the same from the next greater in the table, and say,

As the difference of the two tabular sines is to  $60''$ , so is the difference between the given one, and the next less than it, found in the table, to the seconds required.

EXAM-

**Exam.** Required the degrees, minutes, and seconds answering to the log. sine

9.9533476

Next less in the tables is  $63^{\circ} 54'$  9.9532897

Difference 579

Next greater 9.9533516

Subtract 9.9532897

619

As 619 : 60" :: 579 : 56"

Therefore the answer is  $63^{\circ} 54' 56''$ .

19. To find the natural sine or tangent of any arc by these tables.

Find the logarithm, sine, or tangent of the given arc in the table; cancel its index, and find the number answering to it in the table of logarithms to seven figures; it is the answer.

20. To find the log. secant of any arc. Because the radius is a mean proportional between the co-sine of an arc and its secant; therefore, subtract the co-sine of the given arc from 20.0000000, the remainder is the logarithm secant.

21. To find the logarithm versed sine of any arc. Multiply the logarithm sine of half the arc by 2, add .3010300 to the product, and subtract the radius from the sum; the remainder is the logarithm versed sine of the arc.

THE END.

---

A  
T A B L E  
OF THE  
LOGARITHMS  
OF ALL NUMBERS,  
From 1 to 10,000.

---

| N <sup>o</sup> | Log.    | N <sup>o</sup> | Log.    | N <sup>o</sup> | Log.    | N <sup>o</sup> | Log.    |
|----------------|---------|----------------|---------|----------------|---------|----------------|---------|
| 0              |         | 22             | 3424227 | 44             | 6434527 | 66             | 8195439 |
| 1              | 0000000 | 23             | 3617278 | 45             | 6532125 | 67             | 8260748 |
| 2              | 3010300 | 24             | 3802112 | 46             | 6627578 | 68             | 8325089 |
| 3              | 4771213 | 25             | 3979400 | 47             | 6720979 | 69             | 8388491 |
| 4              | 6020600 | 26             | 4149733 | 48             | 6812412 | 70             | 8450980 |
| 5              | 6989700 | 27             | 4313638 | 49             | 6901961 | 71             | 8512583 |
| 6              | 7781513 | 28             | 4471580 | 50             | 6989700 | 72             | 8573325 |
| 7              | 8450980 | 29             | 4623980 | 51             | 7075702 | 73             | 8633229 |
| 8              | 9030900 | 30             | 4771213 | 52             | 7160033 | 74             | 8692317 |
| 9              | 9542425 | 31             | 4913617 | 53             | 7242759 | 75             | 8750613 |
| 10             | 0000000 | 32             | 5051500 | 54             | 7323938 | 76             | 8808136 |
| 11             | 0413927 | 33             | 5185139 | 55             | 7403627 | 77             | 8864907 |
| 12             | 0791812 | 34             | 5314789 | 56             | 7481880 | 78             | 8920946 |
| 13             | 1139434 | 35             | 5440680 | 57             | 7558749 | 79             | 8976271 |
| 14             | 1461280 | 36             | 5563025 | 58             | 7634280 | 80             | 9030900 |
| 15             | 1760913 | 37             | 5682017 | 59             | 7708520 | 81             | 9084850 |
| 16             | 2041200 | 38             | 5797836 | 60             | 7781513 | 82             | 9138139 |
| 17             | 2304489 | 39             | 5910646 | 61             | 7853298 | 83             | 9190781 |
| 18             | 2552725 | 40             | 6020600 | 62             | 7923917 | 84             | 9242793 |
| 19             | 2787536 | 41             | 6127839 | 63             | 7993405 | 85             | 9294189 |
| 20             | 3010300 | 42             | 6232493 | 64             | 8061800 | 86             | 9344985 |
| 21             | 3222193 | 43             | 6334685 | 65             | 8129134 | 87             | 9395193 |

| Nº  | Log.    | Nº  | Log.    | Nº  | Log.    | Nº  | Log.    |
|-----|---------|-----|---------|-----|---------|-----|---------|
| 88  | 9444827 | 126 | 1003705 | 164 | 2148438 | 202 | 3053514 |
| 89  | 9493900 | 127 | 1038037 | 165 | 2174839 | 203 | 3074960 |
| 90  | 9542425 | 128 | 1072100 | 166 | 2201081 | 204 | 3096302 |
| 91  | 9590414 | 129 | 1105897 | 167 | 2227165 | 205 | 3117539 |
| 92  | 9637878 | 130 | 1139434 | 168 | 2253093 | 206 | 3138672 |
| 93  | 9684829 | 131 | 1172713 | 169 | 2278867 | 207 | 3159703 |
| 94  | 9731279 | 132 | 1205739 | 170 | 2304489 | 208 | 3180633 |
| 95  | 9777236 | 133 | 1238516 | 171 | 2329961 | 209 | 3201463 |
| 96  | 9822712 | 134 | 1271048 | 172 | 2355284 | 210 | 3222193 |
| 97  | 9867717 | 135 | 1303338 | 173 | 2380461 | 211 | 3242825 |
| 98  | 9912261 | 136 | 1335389 | 174 | 2405492 | 212 | 3263359 |
| 99  | 9956352 | 137 | 1367206 | 175 | 2430380 | 213 | 3283796 |
| 100 | 0000000 | 138 | 1398791 | 176 | 2455127 | 214 | 3304138 |
| 101 | 0043214 | 139 | 1430148 | 177 | 2479733 | 215 | 3324385 |
| 102 | 0086002 | 140 | 1461280 | 178 | 2504200 | 216 | 3344538 |
| 103 | 0128372 | 141 | 1492191 | 179 | 2528530 | 217 | 3364597 |
| 104 | 0170333 | 142 | 1522883 | 180 | 2552725 | 218 | 3384565 |
| 105 | 0211893 | 143 | 1553360 | 181 | 2576786 | 219 | 3404441 |
| 106 | 0253059 | 144 | 1583625 | 182 | 2600714 | 220 | 3424227 |
| 107 | 0293838 | 145 | 1613680 | 183 | 2624511 | 221 | 3443923 |
| 108 | 0334238 | 146 | 1643529 | 184 | 2648178 | 222 | 3463530 |
| 109 | 0374265 | 147 | 1673173 | 185 | 2671717 | 223 | 3483049 |
| 110 | 0413927 | 148 | 1702617 | 186 | 2695129 | 224 | 3502480 |
| 111 | 0453230 | 149 | 1731863 | 187 | 2718416 | 225 | 3521825 |
| 112 | 0492180 | 150 | 1760913 | 188 | 2741578 | 226 | 3541084 |
| 113 | 0530784 | 151 | 1789769 | 189 | 2764618 | 227 | 3560259 |
| 114 | 0569049 | 152 | 1818436 | 190 | 2787536 | 228 | 3579348 |
| 115 | 0606978 | 153 | 1846914 | 191 | 2810334 | 229 | 3598355 |
| 116 | 0644580 | 154 | 1875207 | 192 | 2833012 | 230 | 3617278 |
| 117 | 0681859 | 155 | 1903317 | 193 | 2855573 | 231 | 3636120 |
| 118 | 0718820 | 156 | 1931246 | 194 | 2878017 | 232 | 3654880 |
| 119 | 0755470 | 157 | 1958997 | 195 | 2900346 | 233 | 3673559 |
| 120 | 0791812 | 158 | 1986571 | 196 | 2922561 | 234 | 3692159 |
| 121 | 0827854 | 159 | 2013971 | 197 | 2944662 | 235 | 3710679 |
| 122 | 0863598 | 160 | 2041200 | 198 | 2966652 | 236 | 3729120 |
| 123 | 0899051 | 161 | 2068259 | 199 | 2988531 | 237 | 3747483 |
| 124 | 0934217 | 162 | 2095150 | 200 | 3010300 | 238 | 3765770 |
| 125 | 0969100 | 163 | 2121876 | 201 | 3031951 | 239 | 3783979 |

| N°  | Log.    | N°  | Log.    | N°  | Log.    | N°  | Log.    |
|-----|---------|-----|---------|-----|---------|-----|---------|
| 240 | 3801112 | 278 | 4440448 | 316 | 4996871 | 354 | 5490033 |
| 241 | 3820170 | 279 | 4456042 | 317 | 5010593 | 355 | 5502284 |
| 242 | 3838154 | 280 | 4471580 | 318 | 5024271 | 356 | 5514500 |
| 243 | 3856063 | 281 | 4487063 | 319 | 5037907 | 357 | 5526682 |
| 244 | 3873878 | 282 | 4502491 | 320 | 5051500 | 358 | 5538830 |
| 245 | 3891665 | 283 | 4517864 | 321 | 5065050 | 359 | 5550944 |
| 246 | 3909351 | 284 | 4533183 | 322 | 5078559 | 360 | 5563025 |
| 247 | 3926970 | 285 | 4548449 | 323 | 5092025 | 361 | 5575072 |
| 248 | 3944517 | 286 | 4563660 | 324 | 5105450 | 362 | 5587086 |
| 249 | 3961993 | 287 | 4578819 | 325 | 5118834 | 363 | 5599066 |
| 250 | 3979400 | 288 | 4593925 | 326 | 5132176 | 364 | 5611014 |
| 251 | 3996737 | 289 | 4608978 | 327 | 5145478 | 365 |         |
| 252 | 4014005 | 290 | 4623981 | 328 | 5158738 | 366 |         |
| 253 | 4031205 | 291 | 4638930 | 329 | 5171959 | 367 |         |
| 254 | 4048337 | 292 | 4653829 | 330 | 5185139 | 368 | 5658478 |
| 255 | 4065402 | 293 | 4668676 | 331 | 5198280 | 369 | 5670264 |
| 256 | 4082400 | 294 | 4683473 | 332 | 5211381 | 370 | 5682017 |
| 257 | 4099331 | 295 | 4698220 | 333 | 5224442 | 371 | 5693739 |
| 258 | 4116197 | 296 | 4712917 | 334 | 5237465 | 372 | 5705429 |
| 259 | 4132998 | 297 | 4727564 | 335 | 5250448 | 373 | 5717088 |
| 260 | 4149733 | 298 | 4742163 | 336 | 5263393 | 374 | 5728716 |
| 261 | 4166405 | 299 | 4756712 | 337 | 5276299 | 375 | 5740313 |
| 262 | 4183013 | 300 | 4771213 | 338 | 5289167 | 376 | 5751878 |
| 263 | 4199557 | 301 | 4785665 | 339 | 5301997 | 377 | 5763414 |
| 264 | 4216039 | 302 | 4800069 | 340 | 5314789 | 378 | 5774918 |
| 265 | 4232459 | 303 | 4814426 | 341 | 5327544 | 379 | 5786392 |
| 266 | 4248816 | 304 | 4828736 | 342 | 5340261 | 380 | 5797836 |
| 267 | 4265113 | 305 | 4842998 | 343 | 5352941 | 381 | 5809250 |
| 268 | 4281348 | 306 | 4857214 | 344 | 5365584 | 382 | 5820634 |
| 269 | 4297523 | 307 | 4871384 | 345 | 5378191 | 383 | 5831988 |
| 270 | 4313638 | 308 | 4885500 | 346 | 5390761 | 384 | 5843312 |
| 271 | 4329693 | 309 | 4899585 | 347 | 5403295 | 385 | 5854607 |
| 272 | 4345689 | 310 | 4913617 | 348 | 5415792 | 386 | 5865873 |
| 273 | 4361626 | 311 | 4927604 | 349 | 5428254 | 387 | 5877110 |
| 274 | 4377506 | 312 | 4941546 | 350 | 5440680 | 388 | 5888317 |
| 275 | 4393327 | 313 | 4955443 | 351 | 5453071 | 389 | 5899496 |
| 276 | 4409091 | 314 | 4969496 | 352 | 5465427 | 390 | 5910646 |
| 277 | 4424798 | 315 | 4983106 | 353 | 5477747 | 391 | 5921768 |

| N°  | Log.    | N°  | Log.    | N°  | Log.    | N°  | Log.    |
|-----|---------|-----|---------|-----|---------|-----|---------|
| 392 | 5932861 | 430 | 6334685 | 468 | 6702459 | 506 | 7041505 |
| 393 | 5943926 | 431 | 6344773 | 469 | 6711728 | 507 | 7050080 |
| 394 | 5954962 | 432 | 6354837 | 470 | 6720979 | 508 | 7058637 |
| 395 | 5965971 | 433 | 6364879 | 471 | 6730209 | 509 | 7067178 |
| 396 | 5976952 | 434 | 6374897 | 472 | 6739420 | 510 | 7075702 |
| 397 | 5987905 | 435 | 6384893 | 473 | 6748611 | 511 | 7084209 |
| 398 | 5998831 | 436 | 6394865 | 474 | 6757783 | 512 | 7092700 |
| 399 | 6009729 | 437 | 6404814 | 475 | 6766936 | 513 | 7101174 |
| 400 | 6020600 | 438 | 6414741 | 476 | 6776070 | 514 | 7109631 |
| 401 | 6031444 | 439 | 6424645 | 477 | 6785184 | 515 | 7118072 |
| 402 | 6042261 | 440 | 6434527 | 478 | 6794279 | 516 | 7126497 |
| 403 | 6053050 | 441 | 6444386 | 479 | 6803355 | 517 | 7134905 |
| 404 | 6063814 | 442 | 6454223 | 480 | 6812412 | 518 | 7143298 |
| 405 | 6074550 | 443 | 6464037 | 481 | 6821451 | 519 | 7151674 |
| 406 | 6085260 | 444 | 6473830 | 482 | 6830470 | 520 | 7160033 |
| 407 | 6095944 | 445 | 6483600 | 483 | 6839471 | 521 | 7168377 |
| 408 | 6106602 | 446 | 6493349 | 484 | 6848454 | 522 | 7176705 |
| 409 | 6117233 | 447 | 6503075 | 485 | 6857417 | 523 | 7185017 |
| 410 | 6127839 | 448 | 6512780 | 486 | 6866363 | 524 | 7193313 |
| 411 | 6138418 | 449 | 6522463 | 487 | 6875290 | 525 | 7201593 |
| 412 | 6148972 | 450 | 6532125 | 488 | 6884198 | 526 | 7209857 |
| 413 | 6159501 | 451 | 6541765 | 489 | 6893089 | 527 | 7218106 |
| 414 | 6170003 | 452 | 6551384 | 490 | 6901961 | 528 | 7226339 |
| 415 | 6180481 | 453 | 6560982 | 491 | 6910815 | 529 | 7234557 |
| 416 | 6190933 | 454 | 6570559 | 492 | 6919651 | 530 | 7242759 |
| 417 | 6201361 | 455 | 6580114 | 493 | 6928469 | 531 | 7250945 |
| 418 | 6211763 | 456 | 6589648 | 494 | 6937269 | 532 | 7259116 |
| 419 | 6222140 | 457 | 6599162 | 495 | 6946052 | 533 | 7267272 |
| 420 | 6232493 | 458 | 6608655 | 496 | 6954814 | 534 | 7275413 |
| 421 | 6242821 | 459 | 6618127 | 497 | 6963564 | 535 | 7283538 |
| 422 | 6253125 | 460 | 6627578 | 498 | 6972293 | 536 | 7291648 |
| 423 | 6263404 | 461 | 6637009 | 499 | 6981005 | 537 | 7299743 |
| 424 | 6273659 | 462 | 6646420 | 500 | 6989700 | 538 | 7307823 |
| 425 | 6283889 | 463 | 6655810 | 501 | 6998377 | 539 | 7315888 |
| 426 | 6294096 | 464 | 6665180 | 502 | 7007037 | 540 | 7323983 |
| 427 | 6304279 | 465 | 6674530 | 503 | 7015680 | 541 | 7331973 |
| 428 | 6314438 | 466 | 6683859 | 504 | 7024305 | 542 | 7339993 |
| 429 | 6324573 | 467 | 6693169 | 505 | 7032914 | 543 | 7347998 |



| N°  | Log.    | N°  | Log.    | N°  | Log.    | N°  | Log.    |
|-----|---------|-----|---------|-----|---------|-----|---------|
| 544 | 7355980 | 582 | 7649230 | 620 | 7923917 | 658 | 8182259 |
| 545 | 7363905 | 583 | 7656685 | 621 | 7930916 | 659 | 8188854 |
| 546 | 7371926 | 584 | 7664328 | 622 | 7937904 | 660 | 8195439 |
| 547 | 7379873 | 585 | 7671559 | 623 | 7944880 | 661 | 8202015 |
| 548 | 7387806 | 586 | 7678976 | 624 | 7951846 | 662 | 8208589 |
| 549 | 7395723 | 587 | 7686381 | 625 | 7958800 | 663 | 8215135 |
| 550 | 7403627 | 588 | 7693773 | 626 | 7965743 | 664 | 8221681 |
| 551 | 7411516 | 589 | 7701153 | 627 | 7972675 | 665 | 8228216 |
| 552 | 7419391 | 590 | 7708520 | 628 | 7979596 | 666 | 8234742 |
| 553 | 7427251 | 591 | 7715875 | 629 |         | 667 | 8241258 |
| 554 | 7435098 | 592 | 7723217 | 630 |         | 668 | 8247765 |
| 555 | 7442930 | 593 | 7730547 | 631 |         | 669 | 8254261 |
| 556 | 7450748 | 594 | 7737864 | 632 |         | 670 | 8260748 |
| 557 | 7458552 | 595 | 7745170 | 633 |         | 671 | 8267225 |
| 558 | 7466342 | 596 | 7752463 | 634 |         | 672 | 8273693 |
| 559 | 7474118 | 597 | 7759743 | 635 |         | 673 | 8280151 |
| 560 | 7481880 | 598 | 7767012 | 636 |         | 674 | 8286599 |
| 561 | 7489629 | 599 | 7774268 | 637 |         | 675 | 8293038 |
| 562 | 7497363 | 600 | 7781513 | 638 |         | 676 | 8299467 |
| 563 | 7505084 | 601 | 7788745 | 639 |         | 677 | 8305887 |
| 564 | 7512791 | 602 | 7795965 | 640 | 8061800 | 678 | 8312297 |
| 565 | 7520484 | 603 | 7803173 | 641 | 8068580 | 679 | 8318698 |
| 566 | 7528164 | 604 | 7810369 | 642 | 8075350 | 680 | 8325089 |
| 567 | 7535831 | 605 | 7817554 | 643 | 8082110 | 681 | 8331471 |
| 568 | 7543483 | 606 | 7824726 | 644 | 8088859 | 682 | 8337844 |
| 569 | 7551123 | 607 | 7831887 | 645 | 8095597 | 683 | 8344207 |
| 570 | 7558749 | 608 | 7839036 | 646 | 8102325 | 684 | 8350561 |
| 571 | 7566361 | 609 | 7846173 | 647 | 8109043 | 685 | 8356906 |
| 572 | 7573960 | 610 | 7853298 | 648 | 8115750 | 686 | 8363241 |
| 573 | 7581546 | 611 | 7860412 | 649 | 8122447 | 687 | 8369567 |
| 574 | 7589119 | 612 | 7867514 | 650 | 8129134 | 688 | 8375884 |
| 575 | 7596678 | 613 | 7874605 | 651 | 8135810 | 689 | 8382192 |
| 576 | 7604225 | 614 | 7881684 | 652 | 8142476 | 690 | 8388491 |
| 577 | 7611758 | 615 | 7888751 | 653 | 8149132 | 691 | 8394780 |
| 578 | 7619278 | 616 | 7895807 | 654 | 8155777 | 692 | 8401061 |
| 579 | 7626786 | 617 | 7902852 | 655 | 8162413 | 693 | 8407332 |
| 580 | 7634280 | 618 | 7909885 | 656 | 8169038 | 694 | 8413595 |
| 581 | 7641761 | 619 | 7916906 | 657 | 8175654 | 695 | 8419848 |

| N <sup>o</sup> | Log.    | N <sup>o</sup> | Log.    | N <sup>o</sup> | Log.    | N <sup>o</sup> | Log.    |
|----------------|---------|----------------|---------|----------------|---------|----------------|---------|
| 696            | 8426092 | 734            | 8656961 | 772            | 8876173 | 810            | 9084850 |
| 697            | 8432328 | 735            | 8662873 | 773            | 8881795 | 811            | 9090209 |
| 698            | 8438554 | 736            | 8668778 | 774            | 8887410 | 812            | 9095560 |
| 699            | 8444772 | 737            | 8674675 | 775            | 8893017 | 813            | 9100905 |
| 700            | 8450980 | 738            | 8680564 | 776            | 8898617 | 814            | 9106244 |
| 701            | 8457180 | 739            | 8686444 | 777            | 8904210 | 815            | 9111576 |
| 702            | 8463371 | 740            | 8692317 | 778            | 8909796 | 816            | 9116901 |
| 703            | 8469553 | 741            | 8698182 | 779            | 8915375 | 817            | 9122221 |
| 704            | 8475727 | 742            | 8704039 | 780            | 8920946 | 818            | 9127533 |
| 705            | 8481891 | 743            | 8709888 | 781            | 8926510 | 819            | 9132839 |
| 706            | 8488047 | 744            | 8715729 | 782            | 8932068 | 820            | 9138139 |
| 707            | 8494194 | 745            | 8721563 | 783            | 8937618 | 821            | 9143432 |
| 708            | 8500333 | 746            | 8727388 | 784            | 8943161 | 822            | 9148718 |
| 709            | 8506462 | 747            | 8733206 | 785            | 8948696 | 823            | 9153998 |
| 710            | 8512583 | 748            | 8739016 | 786            | 8954225 | 824            | 9159272 |
| 711            | 8518696 | 749            | 8744818 | 787            | 8959747 | 825            | 9164539 |
| 712            | 8524800 | 750            | 8750613 | 788            | 8965262 | 826            | 9169800 |
| 713            | 8530895 | 751            | 8756399 | 789            | 8970770 | 827            | 9175055 |
| 714            | 8536982 | 752            | 8762178 | 790            | 8976271 | 828            | 9180303 |
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| 717            | 8555192 | 755            | 8779470 | 793            | 8992732 | 831            | 9196010 |
| 718            | 8561244 | 756            | 8785218 | 794            | 8998205 | 832            | 9201233 |
| 719            | 8567289 | 757            | 8790959 | 795            | 9003671 | 833            | 9206450 |
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| 108 | 0334238 | 38257 | 42273 | 46285 | 50293 |
| 109 | 74265   | 78248 | 82226 | 86202 | 90173 |
| 110 | 0413927 | 17873 | 21816 | 25755 | 29691 |
| 111 | 53230   | 57141 | 61048 | 64952 | 68852 |
| 112 | 92180   | 96056 | 99929 | 03798 | 07663 |
| 113 | 0530784 | 34626 | 38464 | 42299 | 46131 |
| 114 | 69049   | 72856 | 76661 | 80462 | 84260 |
| 115 | 0606978 | 10753 | 14525 | 18293 | 22058 |
| 116 | 44580   | 48322 | 52061 | 55797 | 59530 |
| 117 | 81859   | 85569 | 89276 | 92980 | 96681 |
| 118 | 0718820 | 22499 | 26175 | 29847 | 33517 |
| 119 | 55470   | 59118 | 62763 | 66404 | 70043 |
| 120 | 91812   | 95430 | 99045 | 02656 | 06265 |
| 121 | 0827854 | 41441 | 35026 | 38608 | 42187 |
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| 130 | 39434   | 42773 | 46110 | 49444 | 52776 |
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| 135 | 1303338 | 06553 | 09767 | 12978 | 16187 |
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| 103 | 49403   | 53598 | 57788 | 61974 | 66155 |
| 104 | 91163   | 95317 | 99467 | 03613 | 07755 |
| 105 | 0432525 | 36639 | 40750 | 44857 | 48960 |
| 106 | 73496   | 77572 | 81644 | 85713 | 89777 |
| 107 | 0314085 | 18123 | 22157 | 26188 | 30214 |
| 108 | 54297   | 58298 | 62295 | 66289 | 70279 |
| 109 | 94141   | 98106 | 02066 | 06023 | 09977 |
| 110 | 0433623 | 37551 | 41476 | 45398 | 49315 |
| 111 | 72749   | 76642 | 80532 | 84418 | 88301 |
| 112 | 0511525 | 15384 | 19239 | 23091 | 26939 |
| 113 | 49959   | 53783 | 57605 | 61423 | 65237 |
| 114 | 88055   | 91846 | 95634 | 99419 | 03200 |
| 115 | 0625820 | 29578 | 33334 | 37086 | 40834 |
| 116 | 63259   | 66986 | 70709 | 74428 | 78145 |
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| 118 | 37184   | 40847 | 44507 | 48164 | 51819 |
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| 120 | 0809770 | 13473 | 17073 | 20669 | 24263 |
| 121 | 45763   | 49336 | 52906 | 56473 | 60037 |
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| 123 | 0916670 | 20185 | 23697 | 27206 | 30713 |
| 124 | 51694   | 55180 | 58665 | 62146 | 65624 |
| 125 | 86437   | 89896 | 93353 | 96806 | 00257 |
| 126 | 1020905 | 24337 | 27766 | 31193 | 34616 |
| 127 | 55102   | 58507 | 61909 | 65309 | 68705 |
| 128 | 89031   | 92410 | 95785 | 99159 | 02529 |
| 129 | 1122698 | 26050 | 29400 | 32747 | 36092 |
| 130 | 56105   | 59432 | 62756 | 66077 | 69396 |
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| 133 | 54813   | 58065 | 61314 | 64561 | 67806 |
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| 135 | 1319393 | 22597 | 25798 | 28998 | 32195 |
| 136 | 51327   | 54507 | 57685 | 60861 | 64034 |
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B

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| 139 | 1430148 | 33271 | 36392 | 39511 | 42628 |
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| 141 | 92191   | 95270 | 98347 | 01422 | 04494 |
| 142 | 1522883 | 25941 | 28996 | 32049 | 35109 |
| 143 | 53360   | 56396 | 59430 | 62462 | 65492 |
| 144 | 83625   | 86640 | 89653 | 92663 | 95672 |
| 145 | 1613680 | 16674 | 19666 | 22656 | 25644 |
| 146 | 43529   | 46502 | 49474 | 52443 | 55411 |
| 147 | 73173   | 76127 | 79078 | 82027 | 84975 |
| 148 | 1702617 | 05551 | 08482 | 11412 | 14339 |
| 149 | 31863   | 34776 | 37688 | 40598 | 43506 |
| 150 | 60913   | 63807 | 66699 | 69590 | 72478 |
| 151 | 89769   | 92645 | 95518 | 98389 | 01259 |
| 152 | 1818436 | 21292 | 24147 | 26999 | 29850 |
| 153 | 46914   | 49752 | 52588 | 55422 | 58254 |
| 154 | 75207   | 78026 | 80844 | 83659 | 86473 |
| 155 | 1903317 | 06118 | 08917 | 11715 | 14510 |
| 156 | 31246   | 34029 | 36810 | 39590 | 42367 |
| 157 | 58997   | 61762 | 64525 | 67287 | 70047 |
| 158 | 86571   | 89319 | 92065 | 94804 | 97552 |
| 159 | 2013971 | 16702 | 19431 | 22158 | 24883 |
| 160 | 41200   | 43913 | 46625 | 49335 | 52044 |
| 161 | 68259   | 70955 | 73650 | 76344 | 79035 |
| 162 | 95150   | 97830 | 00508 | 03185 | 05860 |
| 163 | 2121876 | 24540 | 27202 | 29862 | 32521 |
| 164 | 48438   | 51086 | 53732 | 56376 | 59018 |
| 165 | 74839   | 77471 | 80100 | 82729 | 85355 |
| 166 | 2201081 | 03696 | 06310 | 08922 | 11533 |
| 167 | 27165   | 29764 | 32363 | 34959 | 37555 |
| 168 | 53093   | 55677 | 58260 | 60841 | 63421 |
| 169 | 78867   | 81436 | 84004 | 86570 | 89134 |
| 170 | 2304489 | 07043 | 09596 | 12146 | 14696 |
| 171 | 29961   | 32500 | 35038 | 37574 | 40108 |
| 172 | 55284   | 57809 | 60331 | 62853 | 65373 |
| 173 | 80461   | 82971 | 85479 | 87986 | 90491 |
| 174 | 2405492 | 07988 | 10482 | 12974 | 15465 |
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| 138            | 1414498 | 17632 | 20765 | 23895 | 27022 |
| 139            | 45742   | 48854 | 51964 | 55072 | 58177 |
| 140            | 76763   | 79853 | 82941 | 86027 | 89110 |
| 141            | 1507564 | 10633 | 13699 | 16762 | 19824 |
| 142            | 38149   | 41195 | 44240 | 47282 | 50322 |
| 143            | 68519   | 71544 | 74568 | 77589 | 80608 |
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| 145            | 1628639 | 31614 | 34596 | 37575 | 40553 |
| 146            | 58376   | 61340 | 64301 | 67261 | 70218 |
| 147            | 87920   | 90864 | 93805 | 96744 | 99682 |
| 148            | 1717265 | 20188 | 23110 | 26029 | 28947 |
| 149            | 46412   | 49316 | 52218 | 55118 | 58016 |
| 150            | 75365   | 78250 | 81132 | 84013 | 86892 |
| 151            | 1804126 | 06992 | 09856 | 12718 | 15578 |
| 152            | 32698   | 35545 | 38390 | 41234 | 44075 |
| 153            | 61084   | 63912 | 66739 | 69563 | 72386 |
| 154            | 89285   | 92095 | 94993 | 97710 | 00514 |
| 155            | 1917304 | 20096 | 22886 | 25675 | 28461 |
| 156            | 45143   | 47918 | 50690 | 53461 | 56229 |
| 157            | 72806   | 75562 | 78317 | 81070 | 83821 |
| 158            | 2000293 | 03032 | 05769 | 08505 | 11239 |
| 159            | 27607   | 30329 | 33049 | 35768 | 38485 |
| 160            | 54750   | 57455 | 60159 | 62860 | 65560 |
| 161            | 81725   | 84414 | 87100 | 89785 | 92468 |
| 162            | 2108534 | 11205 | 13876 | 16544 | 19211 |
| 163            | 35178   | 37833 | 40487 | 43139 | 45790 |
| 164            | 61659   | 64298 | 66936 | 69572 | 72207 |
| 165            | 87980   | 90603 | 93225 | 95845 | 98464 |
| 166            | 2214142 | 16750 | 19356 | 21960 | 24563 |
| 167            | 40148   | 42740 | 45331 | 47920 | 50507 |
| 168            | 65999   | 68576 | 71151 | 73724 | 76296 |
| 169            | 91697   | 94258 | 96818 | 99377 | 01934 |
| 170            | 2317244 | 19790 | 22335 | 24879 | 27421 |
| 171            | 42641   | 45173 | 47703 | 50232 | 52759 |
| 172            | 67891   | 70408 | 72923 | 75437 | 77950 |
| 173            | 92995   | 95497 | 97998 | 00498 | 02996 |
| 174            | 2417954 | 20442 | 22929 | 25414 | 27898 |
| 175            | 42771   | 45245 | 47718 | 50189 | 52658 |

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| 177 | 79733   | 82186 | 84637 | 87087 | 89536 |
| 178 | 2504200 | 06639 | 09077 | 11513 | 13949 |
| 179 | 28530   | 30956 | 33380 | 35803 | 38224 |
| 180 | 52725   | 55137 | 57548 | 59957 | 62365 |
| 181 | 76786   | 79185 | 81582 | 83978 | 86373 |
| 182 | 2600714 | 03099 | 05484 | 07867 | 10248 |
| 183 | 24511   | 26883 | 29255 | 31625 | 33993 |
| 184 | 48178   | 50538 | 52896 | 55253 | 57609 |
| 185 | 71717   | 74064 | 76410 | 78754 | 81097 |
| 186 | 95129   | 97464 | 99797 | 02129 | 04459 |
| 187 | 2718416 | 20738 | 23058 | 25378 | 27696 |
| 188 | 41578   | 43888 | 46196 | 48503 | 50809 |
| 189 | 64618   | 66915 | 69211 | 71506 | 73800 |
| 190 | 87536   | 89821 | 92105 | 94388 | 96669 |
| 191 | 2810334 | 12607 | 14879 | 17150 | 19419 |
| 192 | 33012   | 35274 | 37534 | 39793 | 42051 |
| 193 | 55573   | 57823 | 60071 | 62319 | 64565 |
| 194 | 78017   | 80255 | 82492 | 84728 | 86963 |
| 195 | 2900346 | 02573 | 04798 | 07022 | 09246 |
| 196 | 22561   | 24776 | 26990 | 29203 | 31415 |
| 197 | 44662   | 46866 | 49069 | 51271 | 53471 |
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| 199 | 88531   | 90713 | 92893 | 95073 | 97252 |
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| 202 | 53514   | 55663 | 57812 | 59959 | 62105 |
| 203 | 74960   | 77099 | 79237 | 81374 | 83509 |
| 204 | 96302   | 98430 | 00557 | 02684 | 04809 |
| 205 | 3117539 | 19657 | 21774 | 23889 | 26094 |
| 206 | 38672   | 40780 | 42887 | 44992 | 47097 |
| 207 | 59703   | 61801 | 63898 | 65993 | 68088 |
| 208 | 80633   | 82721 | 84807 | 86893 | 88977 |
| 209 | 3201463 | 03540 | 05617 | 07692 | 09767 |
| 210 | 22193   | 24261 | 26327 | 28393 | 30457 |
| 211 | 42825   | 44882 | 46939 | 48995 | 51050 |
| 212 | 63359   | 65407 | 67454 | 69500 | 71545 |
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| 178 | 2516382 | 18815 | 21246 | 23675 | 26103 |
| 179 | 40645   | 43063 | 45481 | 47897 | 50312 |
| 180 | 64772   | 67177 | 69582 | 71984 | 74386 |
| 181 | 88766   | 91158 | 93549 | 95939 | 98327 |
| 182 | 2612629 | 15008 | 17385 | 19762 | 22137 |
| 183 | 36361   | 38727 | 41092 | 43455 | 45817 |
| 184 | 59964   | 62317 | 64669 | 67020 | 69369 |
| 185 | 83439   | 85780 | 88119 | 90457 | 92794 |
| 186 | 2706788 | 09116 | 11443 | 13769 | 16093 |
| 187 | 30013   | 32328 | 34643 | 36956 | 39268 |
| 188 | 53114   | 55417 | 57719 | 60020 | 62320 |
| 189 | 76092   | 78383 | 80673 | 82962 | 85250 |
| 190 | 98950   | 01229 | 03507 | 05784 | 08059 |
| 191 | 2821688 | 23955 | 26221 | 28486 | 30750 |
| 192 | 44307   | 46563 | 48817 | 51170 | 53322 |
| 193 | 66810   | 69054 | 71296 | 73538 | 75778 |
| 194 | 89196   | 91428 | 93660 | 95890 | 98118 |
| 195 | 2911468 | 13689 | 15908 | 18127 | 20344 |
| 196 | 33626   | 35835 | 38044 | 40251 | 42457 |
| 197 | 55671   | 57869 | 60067 | 62263 | 64458 |
| 198 | 77605   | 79792 | 81979 | 84164 | 86348 |
| 199 | 99429   | 01605 | 03781 | 05955 | 08128 |
| 200 | 3921144 | 23309 | 25474 | 27637 | 29799 |
| 201 | 42751   | 44905 | 47059 | 49212 | 51363 |
| 202 | 64250   | 66394 | 68537 | 70680 | 72820 |
| 203 | 85644   | 87778 | 89910 | 92042 | 94172 |
| 204 | 3106933 | 09056 | 11178 | 13300 | 15420 |
| 205 | 3128118 | 30231 | 32343 | 34454 | 36563 |
| 206 | 49201   | 51303 | 53405 | 55505 | 57605 |
| 207 | 70181   | 72273 | 74365 | 76455 | 78545 |
| 208 | 91061   | 93143 | 95224 | 97305 | 99384 |
| 209 | 3211840 | 13913 | 15984 | 18055 | 20124 |
| 210 | 32521   | 34584 | 36645 | 38706 | 40766 |
| 211 | 53104   | 55157 | 57209 | 59260 | 61310 |
| 212 | 73589   | 75633 | 77675 | 79716 | 81757 |
| 213 | 93979   | 96012 | 98045 | 00077 | 02198 |

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| 217 | 64597   | 66598 | 68598 | 70597 | 72595 |
| 218 | 84565   | 86557 | 88547 | 90537 | 92526 |
| 219 | 3404441 | 06424 | 08405 | 10386 | 12366 |
| 220 | 24227   | 26200 | 28173 | 30145 | 32116 |
| 221 | 43923   | 45887 | 47851 | 49814 | 51776 |
| 222 | 63530   | 65486 | 67441 | 69395 | 71348 |
| 223 | 83049   | 84996 | 86942 | 88887 | 90832 |
| 224 | 3502480 | 04419 | 06356 | 08293 | 10229 |
| 225 | 21825   | 23755 | 25684 | 27612 | 29539 |
| 226 | 41084   | 43006 | 44926 | 46846 | 48764 |
| 227 | 60259   | 62171 | 64083 | 65994 | 67905 |
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| 231 | 36120   | 37999 | 39878 | 41756 | 43634 |
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| 234 | 92159   | 94014 | 95869 | 97723 | 99576 |
| 235 | 3710679 | 12526 | 14373 | 16219 | 18065 |
| 236 | 29120   | 30960 | 32799 | 34637 | 36475 |
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| 241 | 20170   | 21972 | 23773 | 25573 | 27373 |
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| 243 | 56063   | 57850 | 59636 | 61421 | 63206 |
| 244 | 73898   | 75678 | 77457 | 79235 | 81012 |
| 245 | 91661   | 93433 | 95205 | 96975 | 98746 |
| 246 | 390935  | 11116 | 12880 | 14644 | 16407 |
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| 248 | 44517   | 46268 | 48018 | 49767 | 51516 |
| 249 | 61993   | 63737 | 65480 | 67223 | 68964 |
| 250 | 79400   | 81137 | 82873 | 84608 | 86343 |
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| 217 | 74593   | 76589 | 78584 | 80579 | 82572 |
| 218 | 94514   | 96502 | 98488 | 00473 | 02458 |
| 219 | 3414345 | 16323 | 18301 | 20277 | 22252 |
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| 227 | 69814   | 71723 | 73630 | 75537 | 77443 |
| 228 | 88862   | 90762 | 92662 | 94560 | 96458 |
| 229 | 3607827 | 09719 | 11610 | 13500 | 15390 |
| 230 | 26709   | 28593 | 30476 | 32358 | 34239 |
| 231 | 45510   | 47386 | 49260 | 51134 | 53007 |
| 232 | 64230   | 66097 | 67964 | 69830 | 71695 |
| 233 | 82869   | 84728 | 86587 | 88445 | 90302 |
| 234 | 3701428 | 03280 | 05131 | 06981 | 08830 |
| 235 | 19909   | 21753 | 23596 | 25438 | 27279 |
| 236 | 38311   | 40147 | 41983 | 43817 | 45651 |
| 237 | 56636   | 58464 | 60292 | 62119 | 63944 |
| 238 | 74884   | 76704 | 78524 | 80343 | 82161 |
| 239 | 93055   | 94868 | 96680 | 98492 | 00302 |
| 240 | 3811151 | 12956 | 14761 | 16565 | 18368 |
| 241 | 29171   | 30969 | 32767 | 34563 | 36359 |
| 242 | 47117   | 48908 | 50698 | 52487 | 54275 |
| 243 | 64990   | 66773 | 68555 | 70337 | 72118 |
| 244 | 82789   | 84565 | 86340 | 88114 | 89888 |
| 245 | 3900515 | 02284 | 04052 | 05819 | 07585 |
| 246 | 18169   | 19931 | 21691 | 23452 | 25211 |
| 247 | 35752   | 37506 | 39260 | 41013 | 42765 |
| 248 | 53264   | 55011 | 56758 | 58504 | 60249 |
| 249 | 70705   | 72446 | 74185 | 75924 | 77663 |
| 250 | 88077   | 89811 | 91543 | 93275 | 95007 |
| 251 | 4005380 | 07106 | 08832 | 10557 | 12282 |

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| 252 | 4014005 | 15728 | 17451 | 19173 | 20894 |
| 253 | 31205   | 32921 | 34637 | 36352 | 38066 |
| 254 | 48337   | 50047 | 51755 | 53464 | 55171 |
| 255 | 65402   | 67105 | 68807 | 70508 | 72209 |
| 256 | 82400   | 84096 | 85791 | 87486 | 89180 |
| 257 | 99331   | 01021 | 02710 | 04398 | 06085 |
| 258 | 4116197 | 17880 | 19562 | 21244 | 22925 |
| 259 | 32998   | 34674 | 36350 | 38025 | 39700 |
| 260 | 49733   | 51404 | 53073 | 54742 | 56410 |
| 261 | 66405   | 68069 | 69732 | 71394 | 73056 |
| 262 | 83013   | 84670 | 86327 | 87983 | 89638 |
| 263 | 99557   | 01208 | 02859 | 04509 | 06158 |
| 264 | 4216039 | 17684 | 19328 | 20972 | 22615 |
| 265 | 32459   | 34097 | 35735 | 37372 | 39009 |
| 266 | 48816   | 50449 | 52081 | 53712 | 55342 |
| 267 | 65113   | 66739 | 68365 | 69990 | 71614 |
| 268 | 81348   | 82968 | 84588 | 86207 | 87825 |
| 269 | 97523   | 99137 | 00751 | 02364 | 03976 |
| 270 | 4313638 | 15246 | 16853 | 18460 | 20067 |
| 271 | 29693   | 31295 | 32897 | 34498 | 36098 |
| 272 | 45689   | 47285 | 48881 | 50476 | 52071 |
| 273 | 61626   | 63217 | 64807 | 66396 | 67985 |
| 274 | 77506   | 79090 | 80675 | 82258 | 83841 |
| 275 | 93327   | 94906 | 96484 | 98062 | 99639 |
| 276 | 4409091 | 10664 | 12237 | 13809 | 15380 |
| 277 | 24798   | 26365 | 27932 | 29499 | 31065 |
| 278 | 40448   | 42010 | 43571 | 45132 | 46692 |
| 279 | 56042   | 57598 | 59154 | 60709 | 62264 |
| 280 | 71580   | 73131 | 74681 | 76231 | 77780 |
| 281 | 87063   | 88608 | 90153 | 91697 | 93241 |
| 282 | 4502491 | 04031 | 05570 | 07109 | 08647 |
| 283 | 17864   | 19399 | 20932 | 22466 | 23998 |
| 284 | 33183   | 34712 | 36241 | 37769 | 39296 |
| 285 | 48449   | 49972 | 51495 | 53018 | 54540 |
| 286 | 63660   | 65179 | 66696 | 68213 | 69730 |
| 287 | 78819   | 80332 | 81844 | 83356 | 84868 |
| 288 | 93925   | 95433 | 96940 | 98446 | 99953 |
| 289 | 4608978 | 10481 | 11983 | 13484 | 14985 |

| N <sup>o</sup> | 5       | 6     | 7     | 8     | 9     |
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| 252            | 4022614 | 24333 | 26052 | 27771 | 29488 |
| 253            | 39780   | 41492 | 43205 | 44916 | 46627 |
| 254            | 56878   | 58584 | 60289 | 61994 | 63698 |
| 255            | 73909   | 75608 | 77307 | 79005 | 80703 |
| 256            | 90874   | 92567 | 94259 | 95950 | 97641 |
| 257            | 4107772 | 09459 | 11144 | 12829 | 14513 |
| 258            | 24605   | 26285 | 27964 | 29643 | 31321 |
| 259            | 41374   | 43047 | 44719 | 46391 | 48063 |
| 260            | 58077   | 59744 | 61410 | 63076 | 64741 |
| 261            | 74717   | 76377 | 78037 | 79696 | 81355 |
| 262            | 91293   | 92947 | 94601 | 96254 | 97906 |
| 263            | 4207806 | 09454 | 11101 | 12748 | 14394 |
| 264            | 24257   | 25898 | 27539 | 29180 | 30820 |
| 265            | 40645   | 42281 | 43916 | 45550 | 47183 |
| 266            | 56972   | 58601 | 60230 | 61858 | 63486 |
| 267            | 73238   | 74861 | 76484 | 78106 | 79727 |
| 268            | 89443   | 91060 | 92677 | 94293 | 95908 |
| 269            | 4305588 | 07199 | 08809 | 10419 | 12029 |
| 270            | 21672   | 23278 | 24883 | 26487 | 28090 |
| 271            | 37698   | 39298 | 40896 | 42495 | 44092 |
| 272            | 53665   | 55259 | 56851 | 58444 | 60035 |
| 273            | 69573   | 71161 | 72748 | 74334 | 75920 |
| 274            | 85423   | 87005 | 88587 | 90167 | 91747 |
| 275            | 4401216 | 02792 | 84368 | 05943 | 07517 |
| 276            | 16951   | 18522 | 20092 | 21661 | 23230 |
| 277            | 32630   | 34195 | 35759 | 37322 | 38885 |
| 278            | 48251   | 49811 | 51370 | 52928 | 54485 |
| 279            | 63818   | 65372 | 66925 | 68477 | 70029 |
| 280            | 79329   | 80877 | 82424 | 83971 | 85517 |
| 281            | 94784   | 96327 | 97868 | 99410 | 00951 |
| 282            | 4510185 | 11722 | 13258 | 14794 | 16329 |
| 283            | 25531   | 27062 | 28593 | 30124 | 31654 |
| 284            | 40823   | 42349 | 43875 | 45400 | 46924 |
| 285            | 56061   | 57582 | 59102 | 60622 | 62142 |
| 286            | 71246   | 72762 | 74277 | 75791 | 77305 |
| 287            | 86378   | 87889 | 89399 | 90908 | 92417 |
| 288            | 4601458 | 02963 | 04468 | 05972 | 07475 |
| 289            | 16486   | 17986 | 19485 | 20984 | 22482 |

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| 291 | 38930   | 40422 | 41914 | 43405 | 44895 |
| 292 | 53829   | 55316 | 56802 | 58288 | 59774 |
| 293 | 68676   | 70158 | 71640 | 73121 | 74601 |
| 294 | 83473   | 84950 | 86427 | 87903 | 89378 |
| 295 | 98220   | 99692 | 01164 | 02634 | 04105 |
| 296 | 4712917 | 14384 | 15851 | 17317 | 18782 |
| 297 | 27564   | 29027 | 30488 | 31949 | 33410 |
| 298 | 42163   | 43620 | 45076 | 46533 | 47988 |
| 299 | 56712   | 58164 | 59616 | 61067 | 62518 |
| 300 | 71213   | 72660 | 74107 | 75553 | 76999 |
| 301 | 85665   | 87108 | 88550 | 89991 | 91432 |
| 302 | 4800069 | 01507 | 02945 | 04381 | 05818 |
| 303 | 14426   | 15859 | 17292 | 18724 | 20156 |
| 304 | 28736   | 30164 | 31592 | 33020 | 34446 |
| 305 | 42998   | 44422 | 45845 | 47268 | 48690 |
| 306 | 57214   | 58633 | 60052 | 61470 | 62888 |
| 307 | 71384   | 72798 | 74212 | 75626 | 77839 |
| 308 | 85507   | 86917 | 88326 | 89735 | 91144 |
| 309 | 99585   | 00990 | 02395 | 03799 | 05203 |
| 310 | 4913617 | 15018 | 16418 | 17818 | 19217 |
| 311 | 27604   | 29000 | 30396 | 31791 | 33186 |
| 312 | 41546   | 42938 | 44329 | 45720 | 47110 |
| 313 | 55443   | 56831 | 58218 | 59604 | 60990 |
| 314 | 69296   | 70679 | 72062 | 73444 | 74825 |
| 315 | 83106   | 84484 | 85862 | 87240 | 88617 |
| 316 | 96871   | 98245 | 99619 | 00992 | 02365 |
| 317 | 5010593 | 11962 | 13332 | 14701 | 16069 |
| 318 | 24271   | 25637 | 27002 | 28366 | 29731 |
| 319 | 37907   | 39268 | 40629 | 41989 | 43349 |
| 320 | 51500   | 52857 | 54213 | 55569 | 56925 |
| 321 | 65050   | 66403 | 67755 | 69107 | 70459 |
| 322 | 78559   | 79907 | 81255 | 82603 | 83950 |
| 323 | 92025   | 93370 | 94714 | 96057 | 97400 |
| 324 | 5105450 | 06790 | 08130 | 09469 | 10808 |
| 325 | 18834   | 20170 | 21505 | 22841 | 24175 |
| 326 | 32176   | 33508 | 34840 | 36171 | 37502 |
| 327 | 45478   | 46805 | 48133 | 49460 | 50787 |

| N°  | 5       | 6     | 7     | 8     | 9     |
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| 290 | 4631461 | 32956 | 34450 | 35944 | 37437 |
| 291 | 46386   | 47875 | 49364 | 50853 | 52341 |
| 292 | 61259   | 62743 | 64227 | 65711 | 67194 |
| 293 | 76081   | 77561 | 79039 | 80518 | 81996 |
| 294 | 90853   | 92327 | 93801 | 95275 | 96748 |
| 295 | 4705575 | 07047 | 08513 | 09982 | 11450 |
| 296 | 20247   | 21711 | 23175 | 24639 | 26102 |
| 297 | 34870   | 36329 | 37788 | 39247 | 40705 |
| 298 | 49443   | 50898 | 52352 | 53806 | 55259 |
| 299 | 63968   | 65418 | 66867 | 68316 | 69765 |
| 300 | 78445   | 79890 | 81334 | 82778 | 84222 |
| 301 | 92873   | 94313 | 95753 | 97192 | 98631 |
| 302 | 4807254 | 08689 | 10124 | 11559 | 12993 |
| 303 | 21587   | 23018 | 24448 | 25878 | 27307 |
| 304 | 35873   | 37299 | 38725 | 40150 | 41574 |
| 305 | 50112   | 51533 | 52954 | 54375 | 55795 |
| 306 | 64305   | 65722 | 67138 | 68554 | 69969 |
| 307 | 78451   | 79863 | 81275 | 82686 | 84097 |
| 308 | 92552   | 93959 | 95366 | 96773 | 98179 |
| 309 | 4906607 | 08010 | 09412 | 10814 | 12216 |
| 310 | 20616   | 22015 | 23413 | 24810 | 26207 |
| 311 | 34581   | 35974 | 37368 | 38761 | 40154 |
| 312 | 48500   | 49890 | 51279 | 52667 | 54056 |
| 313 | 62375   | 63761 | 65145 | 66529 | 67913 |
| 314 | 76206   | 77587 | 78967 | 80347 | 81727 |
| 315 | 89994   | 91370 | 92746 | 94121 | 95496 |
| 216 | 5003737 | 05109 | 06481 | 07852 | 09222 |
| 317 | 17437   | 18805 | 20172 | 21539 | 22905 |
| 318 | 31094   | 32458 | 33821 | 35183 | 36545 |
| 319 | 44709   | 46068 | 47426 | 48785 | 50142 |
| 320 | 58280   | 59635 | 60990 | 62344 | 63697 |
| 321 | 71810   | 73160 | 74511 | 75860 | 77210 |
| 322 | 85297   | 86644 | 87990 | 89335 | 90680 |
| 323 | 98743   | 00085 | 01427 | 02768 | 04109 |
| 324 | 5112147 | 13485 | 14823 | 16160 | 17497 |
| 325 | 25510   | 26844 | 28178 | 29511 | 30844 |
| 326 | 38832   | 40162 | 41491 | 42820 | 44149 |
| 327 | 52113   | 53439 | 54764 | 56089 | 57414 |

| N°  | 0       | 1     | 2     | 3     | 4     |
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| 328 | 5158738 | 60062 | 61386 | 62709 | 64031 |
| 329 | 71959   | 73279 | 74598 | 75917 | 77236 |
| 330 | 85139   | 86455 | 87771 | 89086 | 90400 |
| 331 | 98280   | 99592 | 00903 | 02214 | 03525 |
| 332 | 5211381 | 12689 | 13996 | 15303 | 16610 |
| 333 | 24442   | 25746 | 27050 | 28353 | 29656 |
| 334 | 37465   | 38765 | 40064 | 41364 | 42663 |
| 335 | 50448   | 51744 | 53040 | 54336 | 55631 |
| 336 | 63393   | 64685 | 65977 | 67269 | 68560 |
| 337 | 76299   | 77588 | 78876 | 80163 | 81451 |
| 338 | 89167   | 90452 | 91736 | 93020 | 94304 |
| 339 | 5301997 | 03278 | 04558 | 05839 | 07118 |
| 340 | 14789   | 16066 | 17343 | 18619 | 19896 |
| 341 | 27544   | 28817 | 30090 | 31363 | 32635 |
| 342 | 40261   | 41531 | 42800 | 44069 | 45338 |
| 343 | 52941   | 54207 | 55473 | 56738 | 58003 |
| 344 | 65584   | 66847 | 68109 | 69370 | 70631 |
| 345 | 78191   | 79450 | 80708 | 81966 | 83223 |
| 346 | 90761   | 92016 | 93271 | 94525 | 95779 |
| 347 | 5403295 | 04546 | 05797 | 07048 | 08298 |
| 348 | 15792   | 17040 | 18288 | 19535 | 20781 |
| 349 | 28254   | 29498 | 30742 | 31986 | 33229 |
| 350 | 40680   | 41921 | 43161 | 44401 | 45641 |
| 351 | 53071   | 54308 | 55545 | 56781 | 58018 |
| 352 | 65427   | 66660 | 67894 | 69126 | 70359 |
| 353 | 77747   | 78977 | 80207 | 81436 | 82665 |
| 354 | 90033   | 91259 | 92486 | 93712 | 94937 |
| 355 | 5502284 | 03507 | 04730 | 05952 | 07174 |
| 356 | 14500   | 15720 | 16939 | 18158 | 19377 |
| 357 | 26682   | 27899 | 29115 | 30330 | 31545 |
| 358 | 38830   | 40043 | 41256 | 42468 | 43680 |
| 359 | 50944   | 52154 | 53363 | 54572 | 55781 |
| 360 | 63025   | 64231 | 65437 | 66643 | 67848 |
| 361 | 75072   | 76275 | 77477 | 78680 | 79881 |
| 362 | 87086   | 88285 | 89484 | 90683 | 91882 |
| 363 | 99066   | 00262 | 01458 | 02654 | 03849 |
| 364 | 5611014 | 12207 | 13399 | 14592 | 15784 |
| 365 | 22929   | 24118 | 25308 | 26497 | 27685 |



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| 328 | 5165354 | 66676 | 67997 | 69318 | 70699 |
| 329 | 78554   | 79872 | 81189 | 82507 | 83823 |
| 330 | 91715   | 93028 | 94342 | 95655 | 96968 |
| 331 | 5204835 | 06145 | 07455 | 08764 | 10073 |
| 332 | 17916   | 19222 | 20528 | 21833 | 23138 |
| 333 | 30958   | 32260 | 33562 | 34863 | 36164 |
| 334 | 43961   | 45259 | 46557 | 47854 | 49151 |
| 335 | 56925   | 58220 | 59513 | 60807 | 62106 |
| 336 | 69851   | 71141 | 72431 | 73721 | 75016 |
| 337 | 82738   | 84024 | 85311 | 86596 | 87882 |
| 338 | 95587   | 96870 | 98152 | 99434 | 00716 |
| 339 | 5308398 | 09677 | 10955 | 12234 | 13512 |
| 340 | 21171   | 22446 | 23721 | 24996 | 26270 |
| 341 | 33907   | 35179 | 36450 | 37721 | 38991 |
| 342 | 46606   | 47874 | 49141 | 50408 | 51675 |
| 343 | 59267   | 60532 | 61795 | 63059 | 64322 |
| 344 | 71892   | 73153 | 74413 | 75673 | 76932 |
| 345 | 84481   | 85737 | 86994 | 88250 | 89506 |
| 346 | 97032   | 98286 | 99538 | 00791 | 02043 |
| 347 | 5409548 | 10798 | 12047 | 13296 | 14544 |
| 348 | 22028   | 23274 | 24519 | 25765 | 27010 |
| 349 | 34472   | 35714 | 36956 | 38198 | 39439 |
| 350 | 46880   | 48119 | 49358 | 50596 | 51834 |
| 351 | 59253   | 60489 | 61724 | 62958 | 64193 |
| 352 | 71591   | 72823 | 74055 | 75286 | 76517 |
| 353 | 83894   | 85123 | 86351 | 87578 | 88806 |
| 354 | 96162   | 97387 | 98612 | 99836 | 01060 |
| 355 | 5508396 | 09618 | 10839 | 12059 | 13280 |
| 356 | 20595   | 21813 | 23031 | 24248 | 25465 |
| 357 | 32760   | 33975 | 35189 | 36403 | 37617 |
| 358 | 44892   | 46103 | 47314 | 48524 | 49735 |
| 359 | 56989   | 58197 | 59404 | 60612 | 61818 |
| 360 | 69053   | 70257 | 71461 | 72665 | 73869 |
| 361 | 81083   | 82284 | 83485 | 84686 | 85886 |
| 362 | 93080   | 94278 | 95476 | 96673 | 97870 |
| 363 | 5605044 | 06239 | 07433 | 08627 | 09821 |
| 364 | 16975   | 18167 | 19358 | 20548 | 21739 |
| 365 | 28874   | 30062 | 31250 | 32437 | 33624 |

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| 366 | 5634811 | 35997 | 37183 | 38369 | 39555 |
| 367 | 46661   | 47844 | 49027 | 50209 | 51392 |
| 368 | 58478   | 59658 | 60838 | 62017 | 63196 |
| 369 | 70264   | 71440 | 72617 | 73793 | 74969 |
| 370 | 82017   | 83191 | 84364 | 85537 | 86710 |
| 371 | 93739   | 94910 | 96080 | 97249 | 98419 |
| 372 | 5705429 | 06597 | 07764 | 08930 | 10097 |
| 373 | 17088   | 18252 | 19416 | 20580 | 21743 |
| 374 | 28716   | 29877 | 31038 | 32198 | 33358 |
| 375 | 40313   | 41471 | 42628 | 43786 | 44943 |
| 376 | 51878   | 53033 | 54188 | 55342 | 56496 |
| 377 | 63414   | 64565 | 65717 | 66868 | 68019 |
| 378 | 74918   | 76067 | 77215 | 78363 | 79511 |
| 379 | 86392   | 87538 | 88683 | 89828 | 90973 |
| 380 | 97836   | 98979 | 00121 | 01263 | 02405 |
| 381 | 5809250 | 10389 | 11529 | 12668 | 13807 |
| 382 | 20634   | 21770 | 22907 | 24043 | 25179 |
| 383 | 31988   | 33122 | 34255 | 35388 | 36521 |
| 384 | 43312   | 44443 | 45574 | 46704 | 47834 |
| 385 | 54607   | 55735 | 56863 | 57990 | 59117 |
| 386 | 65873   | 66998 | 68123 | 69247 | 70371 |
| 387 | 77110   | 78232 | 79353 | 80475 | 81596 |
| 388 | 88317   | 89436 | 90555 | 91674 | 92792 |
| 389 | 99496   | 00612 | 01728 | 02844 | 03959 |
| 390 | 5910646 | 11760 | 12873 | 13986 | 15098 |
| 391 | 21768   | 22878 | 23988 | 25098 | 26208 |
| 392 | 32861   | 33968 | 35076 | 36183 | 37290 |
| 393 | 43926   | 45030 | 46135 | 47239 | 48344 |
| 394 | 54962   | 56064 | 57166 | 58268 | 59369 |
| 395 | 65971   | 67070 | 68169 | 69268 | 70367 |
| 396 | 76952   | 78048 | 79145 | 80241 | 81336 |
| 397 | 87905   | 88999 | 90092 | 91186 | 92279 |
| 398 | 98831   | 99922 | 01013 | 02103 | 03193 |
| 399 | 6009729 | 10817 | 11905 | 12993 | 14081 |
| 400 | 20600   | 21686 | 22771 | 23856 | 24941 |
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| 403 | 53050   | 54128 | 55205 | 56282 | 57359 |

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| 368 | 64375   | 65553 | 66731 | 67909 | 69087 |
| 369 | 76144   | 77320 | 78495 | 79669 | 80843 |
| 370 | 87882   | 89054 | 90226 | 91397 | 92568 |
| 371 | 99588   | 00757 | 01926 | 03094 | 04262 |
| 372 | 5711263 | 12429 | 13594 | 14759 | 15924 |
| 373 | 22906   | 24069 | 25231 | 26393 | 27555 |
| 374 | 34518   | 35678 | 36837 | 37996 | 39154 |
| 375 | 46099   | 47256 | 48412 | 49568 | 50723 |
| 376 | 57650   | 58803 | 59956 | 61109 | 62261 |
| 377 | 69170   | 70320 | 71470 | 72620 | 73769 |
| 378 | 80659   | 81806 | 82953 | 84100 | 85246 |
| 379 | 92118   | 93262 | 94406 | 95550 | 96693 |
| 380 | 5803547 | 04688 | 05829 | 06969 | 08110 |
| 381 | 14945   | 16084 | 17222 | 18359 | 19497 |
| 382 | 26314   | 27450 | 28585 | 29719 | 30854 |
| 383 | 37654   | 38786 | 39918 | 41050 | 42181 |
| 384 | 48963   | 50093 | 51222 | 52351 | 53479 |
| 385 | 60244   | 61370 | 62496 | 63622 | 64748 |
| 386 | 71495   | 72618 | 73742 | 74865 | 75987 |
| 387 | 82717   | 83838 | 84958 | 86078 | 87198 |
| 388 | 93910   | 95028 | 96145 | 97263 | 98379 |
| 389 | 5905075 | 06189 | 07304 | 08418 | 09532 |
| 390 | 16210   | 17322 | 18434 | 19546 | 20657 |
| 391 | 27318   | 28427 | 29536 | 30644 | 31753 |
| 392 | 38397   | 39503 | 40609 | 41715 | 42820 |
| 393 | 49447   | 50551 | 51654 | 52757 | 53860 |
| 394 | 60470   | 61571 | 62671 | 63771 | 64871 |
| 395 | 71465   | 72563 | 73661 | 74758 | 75855 |
| 396 | 82432   | 83527 | 84622 | 85717 | 86811 |
| 397 | 93371   | 94464 | 95556 | 96648 | 97739 |
| 398 | 6004283 | 05373 | 06462 | 07551 | 08640 |
| 399 | 15168   | 16255 | 17341 | 18428 | 19514 |
| 400 | 26025   | 27109 | 28193 | 29277 | 30361 |
| 401 | 36855   | 37937 | 39018 | 40099 | 41180 |
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| 403 | 58435   | 59512 | 60587 | 61663 | 62739 |

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| 405 | 74550   | 75622 | 76694 | 77766 | 78837 |
| 406 | 85260   | 86330 | 87399 | 88468 | 89537 |
| 407 | 95944   | 97011 | 98078 | 99144 | 00210 |
| 408 | 6106602 | 07666 | 08730 | 09794 | 10857 |
| 409 | 17233   | 18295 | 19356 | 20417 | 21478 |
| 410 | 27839   | 28898 | 29957 | 31015 | 32074 |
| 411 | 38418   | 39475 | 40531 | 41587 | 42643 |
| 412 | 48972   | 50026 | 51080 | 52133 | 53187 |
| 413 | 59501   | 60552 | 61603 | 62654 | 63705 |
| 414 | 70003   | 71052 | 72101 | 73149 | 74197 |
| 415 | 80481   | 81527 | 82573 | 83619 | 84665 |
| 416 | 90933   | 91977 | 93021 | 94064 | 95107 |
| 417 | 6201361 | 02402 | 03443 | 04484 | 05524 |
| 418 | 11763   | 12802 | 13840 | 14879 | 15917 |
| 419 | 22140   | 23177 | 24213 | 25249 | 26284 |
| 420 | 32493   | 33527 | 34560 | 35594 | 36627 |
| 421 | 42821   | 43852 | 44884 | 45915 | 46945 |
| 422 | 53125   | 54154 | 55182 | 56211 | 57239 |
| 423 | 63404   | 64430 | 65457 | 66483 | 67509 |
| 424 | 73659   | 74683 | 75707 | 76730 | 77754 |
| 425 | 83889   | 84911 | 85933 | 86954 | 87975 |
| 426 | 94096   | 95115 | 96134 | 97153 | 98172 |
| 427 | 6304279 | 05296 | 06312 | 07329 | 08345 |
| 428 | 14438   | 15452 | 16467 | 17481 | 18495 |
| 429 | 24573   | 25585 | 26597 | 27609 | 28620 |
| 430 | 34685   | 35694 | 36704 | 37713 | 38723 |
| 431 | 44773   | 45780 | 46788 | 47795 | 48801 |
| 432 | 54837   | 55843 | 56848 | 57852 | 58857 |
| 433 | 64879   | 65882 | 66884 | 67887 | 68889 |
| 434 | 74897   | 75898 | 76898 | 77898 | 78898 |

| Nº  | 5       | 6     | 7     | 8     | 9     |
|-----|---------|-------|-------|-------|-------|
| 404 | 6069185 | 70259 | 71332 | 72405 | 73478 |
| 405 | 79909   | 80979 | 82050 | 83120 | 84191 |
| 406 | 90605   | 91674 | 92742 | 93809 | 94877 |
| 407 | 6101276 | 02342 | 03407 | 04472 | 05537 |
| 408 | 11921   | 12984 | 14046 | 15109 | 16171 |
| 409 | 22539   | 23599 | 24660 | 25720 | 26779 |
| 410 | 33132   | 34189 | 35247 | 36304 | 37361 |
| 411 | 43698   | 44754 | 45809 | 46863 | 47918 |
| 412 | 54240   | 55292 | 56345 | 57397 | 58449 |
| 413 | 64755   | 65805 | 66855 | 67905 | 68954 |
| 414 | 75245   | 76293 | 77340 | 78387 | 79434 |
| 415 | 6185710 | 86755 | 87800 | 88845 | 89889 |
| 416 | 96150   | 97193 | 98235 | 99277 | 00319 |
| 417 | 6206565 | 07605 | 08645 | 09684 | 10724 |
| 418 | 16955   | 17992 | 19030 | 20067 | 21104 |
| 419 | 27320   | 28355 | 29390 | 30424 | 31459 |
| 420 | 37660   | 38693 | 39725 | 40757 | 41789 |
| 421 | 47976   | 49006 | 50036 | 51066 | 52095 |
| 422 | 58267   | 59295 | 60322 | 61350 | 62377 |
| 423 | 68534   | 69560 | 70585 | 71610 | 72634 |
| 424 | 78777   | 79800 | 80823 | 81845 | 82867 |
| 425 | 88996   | 90016 | 91037 | 92057 | 93076 |
| 426 | 99190   | 00209 | 01226 | 02244 | 03262 |
| 427 | 6309361 | 10377 | 11393 | 12408 | 13423 |
| 428 | 19508   | 20522 | 21535 | 22548 | 23560 |
| 429 | 29632   | 30643 | 31654 | 32664 | 33674 |
| 430 | 39732   | 40740 | 41749 | 42757 | 43765 |
| 431 | 49808   | 50814 | 51820 | 52826 | 53832 |
| 432 | 59861   | 60865 | 61869 | 62873 | 63876 |
| 433 | 69891   | 70893 | 71894 | 72895 | 73897 |
| 434 | 79898   | 80897 | 81896 | 82895 | 83894 |

D.

| N <sup>o</sup> | o        | r    | 2    | 3    | 4    | Dif |
|----------------|----------|------|------|------|------|-----|
| 435            | 638.4893 | 5891 | 6889 | 7887 | 8884 | 998 |
| 436            | 639.4865 | 5861 | 6857 | 7852 | 8847 | 995 |
| 437            | 640.4814 | 5808 | 6802 | 7795 | 8788 | 993 |
| 438            | 641.4741 | 5733 | 6724 | 7715 | 8705 | 991 |
| 439            | 642.4645 | 5634 | 6623 | 7612 | 8601 | 989 |
| 440            | 643.4527 | 5514 | 6500 | 7487 | 8473 | 986 |
| 441            | 644.4386 | 5371 | 6355 | 7339 | 8323 | 984 |
| 442            | 645.4223 | 5205 | 6187 | 7169 | 8151 | 982 |
| 443            | 646.4037 | 5018 | 5998 | 6977 | 7957 | 980 |
| 444            | 647.3839 | 4808 | 5786 | 6763 | 7741 | 978 |
| 445            | 648.3600 | 4576 | 5552 | 6527 | 7502 | 975 |
| 446            | 649.3349 | 4322 | 5296 | 6269 | 7242 | 973 |
| 447            | 650.3075 | 4047 | 5018 | 5989 | 6960 | 971 |
| 448            | 651.2780 | 3749 | 4719 | 5687 | 6656 | 969 |
| 449            | 652.2463 | 3431 | 4397 | 5364 | 6331 | 966 |
| 450            | 653.2125 | 3090 | 4055 | 5019 | 5984 | 964 |
| 451            | 654.1765 | 2728 | 3691 | 4653 | 5616 | 962 |
| 452            | 655.1384 | 2345 | 3306 | 4266 | 5226 | 960 |
| 453            | 656.0982 | 1941 | 2899 | 3857 | 4815 | 958 |
| 454            | 657.0559 | 1515 | 2471 | 3427 | 4383 | 956 |
| 455            | 658.0114 | 1068 | 2023 | 2977 | 3930 | 954 |
| 456            | 9648     | 0601 | 1553 | 2505 | 3456 | 952 |
| 457            | 659.9162 | 0112 | 1062 | 2012 | 2962 | 950 |
| 458            | 660.8655 | 9603 | 0551 | 1499 | 2446 | 947 |
| 459            | 661.8127 | 9073 | 0019 | 0964 | 1910 | 945 |
| 460            | 662.7578 | 8522 | 9466 | 0410 | 1353 | 943 |
| 461            | 663.7009 | 7951 | 8893 | 9835 | 0776 | 941 |
| 462            | 664.6420 | 7360 | 8299 | 9239 | 0178 | 939 |
| 463            | 665.5810 | 6748 | 7686 | 8623 | 9560 | 937 |
| 464            | 666.5180 | 6116 | 7051 | 7987 | 8922 | 935 |
| 465            | 667.4530 | 5463 | 6397 | 7331 | 8264 | 933 |
| 466            | 668.3859 | 4791 | 5723 | 6654 | 7585 | 931 |
| 467            | 669.3169 | 4099 | 5028 | 5958 | 6887 | 929 |
| 468            | 670.2459 | 3386 | 4314 | 5242 | 6169 | 927 |
| 469            | 671.1728 | 2654 | 3580 | 4506 | 5431 | 925 |
| 470            | 672.0979 | 1903 | 2826 | 3750 | 4673 | 923 |
| 471            | 673.0209 | 1131 | 2053 | 2974 | 3896 | 921 |
| 472            | 9420     | 0340 | 1260 | 2179 | 3099 | 919 |

| N <sup>o</sup> | 5        | 6    | 7    | 8    | 9    | Dif |
|----------------|----------|------|------|------|------|-----|
| 435            | 638.9882 | 0879 | 1876 | 2872 | 3869 | 998 |
| 436            | 639.9842 | 0837 | 1832 | 2826 | 3820 | 995 |
| 437            | 640.9781 | 0773 | 1765 | 2758 | 3749 | 993 |
| 438            | 641.9696 | 0686 | 1676 | 2666 | 3656 | 991 |
| 439            | 642.9589 | 0577 | 1565 | 2552 | 3540 | 989 |
| 440            | 643.9459 | 0445 | 1431 | 2416 | 3401 | 986 |
| 441            | 644.9307 | 0291 | 1274 | 2257 | 3240 | 984 |
| 442            | 645.9133 | 0114 | 1095 | 2076 | 3057 | 982 |
| 443            | 646.8936 | 9915 | 0894 | 1873 | 2851 | 980 |
| 444            | 647.8718 | 9695 | 0671 | 1648 | 2624 | 978 |
| 445            | 648.8477 | 9452 | 0426 | 1401 | 2375 | 975 |
| 446            | 649.8215 | 9187 | 0160 | 1132 | 2104 | 973 |
| 447            | 650.7930 | 8901 | 9871 | 0841 | 1811 | 971 |
| 448            | 651.7624 | 8593 | 9561 | 0528 | 1496 | 969 |
| 449            | 652.7297 | 8263 | 9229 | 0195 | 1160 | 966 |
| 450            | 653.6948 | 7912 | 8876 | 9839 | 0802 | 964 |
| 451            | 654.6578 | 7539 | 8501 | 9462 | 0423 | 962 |
| 452            | 655.6186 | 7145 | 8105 | 9064 | 0023 | 960 |
| 453            | 656.5773 | 6730 | 7688 | 8645 | 9602 | 958 |
| 454            | 657.5339 | 6294 | 7250 | 8205 | 9159 | 956 |
| 455            | 658.4884 | 5837 | 6790 | 7743 | 8696 | 954 |
| 456            | 659.4408 | 5359 | 6310 | 7261 | 8212 | 952 |
| 457            | 660.3911 | 4860 | 5809 | 6758 | 7706 | 950 |
| 458            | 661.3393 | 4341 | 5287 | 6234 | 7181 | 947 |
| 459            | 662.2855 | 3800 | 4745 | 5690 | 6634 | 945 |
| 460            | 663.2296 | 3239 | 4182 | 5125 | 6067 | 943 |
| 461            | 664.1717 | 2658 | 3599 | 4539 | 5480 | 941 |
| 462            | 665.1117 | 2056 | 2995 | 3934 | 4872 | 939 |
| 463            | 666.0497 | 1434 | 2371 | 3307 | 4244 | 937 |
| 464            | 667.9857 | 0792 | 1727 | 2661 | 3595 | 935 |
| 465            | 667.9197 | 0130 | 1062 | 1995 | 2927 | 933 |
| 466            | 668.8516 | 9447 | 0378 | 1308 | 2239 | 931 |
| 467            | 669.7816 | 8745 | 9674 | 0602 | 1530 | 929 |
| 468            | 670.7096 | 8023 | 8950 | 9876 | 0802 | 927 |
| 469            | 671.6356 | 7281 | 8206 | 9130 | 0054 | 925 |
| 470            | 672.5596 | 6519 | 7442 | 8365 | 9287 | 923 |
| 471            | 673.4817 | 5738 | 6659 | 7579 | 8500 | 921 |
| 472            | 674.4018 | 4937 | 5856 | 6775 | 7693 | 919 |

| N <sup>o</sup> | o        | 1    | 2    | 3    | 4    | Dif |
|----------------|----------|------|------|------|------|-----|
| 473            | 674.8611 | 9529 | 0447 | 1365 | 2283 | 917 |
| 474            | 675.7783 | 8700 | 9615 | 0531 | 1447 | 915 |
| 475            | 676.6936 | 7850 | 8764 | 9678 | 0592 | 913 |
| 476            | 677.6070 | 6982 | 7894 | 8806 | 9718 | 911 |
| 477            | 678.5184 | 6094 | 7004 | 7914 | 8824 | 910 |
| 478            | 679.4279 | 5187 | 6096 | 7004 | 7912 | 908 |
| 479            | 680.3355 | 4262 | 5168 | 6074 | 6980 | 906 |
| 480            | 681.2412 | 3317 | 4222 | 5126 | 6030 | 904 |
| 481            | 682.1451 | 2354 | 3256 | 4159 | 5061 | 902 |
| 482            | 683.0470 | 1371 | 2272 | 3173 | 4073 | 900 |
| 483            | 9471     | 0370 | 1269 | 2168 | 3066 | 898 |
| 484            | 684.8454 | 9351 | 0248 | 1145 | 2041 | 896 |
| 485            | 685.7417 | 8313 | 9208 | 0103 | 0998 | 895 |
| 486            | 686.6363 | 7256 | 8150 | 9043 | 9936 | 893 |
| 487            | 687.5290 | 6181 | 7073 | 7964 | 8855 | 891 |
| 488            | 688.4198 | 5088 | 5978 | 6867 | 7757 | 889 |
| 489            | 689.3089 | 3977 | 4864 | 5752 | 6640 | 887 |
| 490            | 690.1961 | 2847 | 3733 | 4619 | 5505 | 885 |
| 491            | 691.0815 | 1699 | 2584 | 3468 | 4352 | 884 |
| 492            | 9651     | 0534 | 1416 | 2298 | 3180 | 882 |
| 493            | 692.8469 | 9350 | 0231 | 1111 | 1991 | 880 |
| 494            | 693.7269 | 8149 | 9027 | 9906 | 0785 | 878 |
| 495            | 694.6052 | 6929 | 7806 | 8683 | 9560 | 877 |
| 496            | 695.4817 | 5692 | 6568 | 7443 | 8318 | 875 |
| 497            | 696.3564 | 4438 | 5311 | 6185 | 7058 | 873 |
| 498            | 697.2293 | 3165 | 4037 | 4909 | 5780 | 872 |
| 499            | 698.1005 | 1876 | 2746 | 3616 | 4485 | 870 |
| 500            | 9700     | 0569 | 1437 | 2305 | 3173 | 868 |
| 501            | 699.8377 | 9244 | 0111 | 0977 | 1843 | 866 |
| 502            | 700.7037 | 7902 | 8767 | 9632 | 0496 | 864 |
| 503            | 701.5680 | 6543 | 7406 | 8269 | 9132 | 863 |
| 504            | 702.4305 | 5167 | 6028 | 6890 | 7751 | 861 |
| 505            | 703.2914 | 3774 | 4633 | 5493 | 6352 | 860 |
| 506            | 704.1505 | 2363 | 3221 | 4079 | 4937 | 858 |
| 507            | 705.0080 | 0936 | 1792 | 2649 | 3505 | 856 |
| 508            | 8637     | 9492 | 0347 | 1201 | 2055 | 854 |
| 509            | 706.7178 | 8031 | 8884 | 9737 | 0589 | 852 |
| 510            | 707.5702 | 6553 | 7405 | 8256 | 9107 | 851 |



| N°  | 5        | 6    | 7    | 8    | 9     | Dif |
|-----|----------|------|------|------|-------|-----|
| 473 | 675.3200 | 4117 | 5034 | 5951 | 6867  | 917 |
| 474 | 676.2362 | 3277 | 4192 | 5107 | 6022  | 915 |
| 475 | 677.1505 | 2418 | 3332 | 4244 | 5157. | 913 |
| 476 | 678.0629 | 1540 | 2452 | 3362 | 4273  | 911 |
| 477 | 9734     | 0643 | 1552 | 2461 | 3370  | 910 |
| 478 | 679.8819 | 9727 | 0634 | 1541 | 2448  | 908 |
| 479 | 680.7886 | 8792 | 9697 | 0602 | 1507  | 906 |
| 480 | 681.6934 | 7838 | 8741 | 9645 | 0548  | 904 |
| 481 | 682.5963 | 6865 | 7766 | 8668 | 9569  | 902 |
| 482 | 683.4973 | 5873 | 6773 | 7673 | 8572  | 900 |
| 483 | 684.3965 | 4863 | 5761 | 6659 | 7556  | 898 |
| 484 | 685.2938 | 3834 | 4730 | 5626 | 6522  | 896 |
| 485 | 686.1892 | 2787 | 3681 | 4575 | 5469  | 895 |
| 486 | 687.0828 | 1721 | 2613 | 3506 | 4398  | 893 |
| 487 | 9746     | 0637 | 1528 | 2418 | 3308  | 891 |
| 488 | 688.8646 | 9535 | 0423 | 1312 | 2200  | 889 |
| 489 | 689.7527 | 8414 | 9301 | 0188 | 1074  | 887 |
| 490 | 690.6390 | 7275 | 8161 | 9046 | 9930  | 885 |
| 491 | 691.5235 | 6119 | 7002 | 7885 | 8768  | 884 |
| 492 | 692.4062 | 4944 | 5826 | 6707 | 7588  | 882 |
| 493 | 693.2872 | 3752 | 4631 | 5511 | 6390  | 880 |
| 494 | 694.1663 | 2541 | 3419 | 4297 | 5175  | 878 |
| 495 | 695.0437 | 1312 | 2189 | 3065 | 3941  | 877 |
| 496 | 9193     | 0067 | 0942 | 1816 | 2690  | 875 |
| 497 | 696.7931 | 8804 | 9676 | 0549 | 1421  | 873 |
| 498 | 697.6652 | 7523 | 8394 | 9264 | 0135  | 872 |
| 499 | 698.5355 | 6224 | 7093 | 7963 | 8831  | 870 |
| 500 | 699.4041 | 4908 | 5776 | 6643 | 7510  | 868 |
| 501 | 700.2709 | 3575 | 4441 | 5307 | 6172  | 866 |
| 502 | 701.1361 | 2225 | 3089 | 3953 | 4816  | 864 |
| 503 | 9995     | 0857 | 1720 | 2582 | 3444  | 863 |
| 504 | 702.8612 | 9472 | 0333 | 1193 | 2054  | 861 |
| 505 | 703.7212 | 8071 | 8930 | 9788 | 0647  | 860 |
| 506 | 704.5794 | 6652 | 7509 | 8366 | 9223  | 858 |
| 507 | 705.4360 | 5216 | 6072 | 6927 | 7782  | 856 |
| 508 | 706.2910 | 3764 | 4617 | 5471 | 6325  | 854 |
| 509 | 707.1442 | 2294 | 3146 | 3998 | 4850  | 852 |
| 510 | 9957     | 0808 | 1659 | 2509 | 3359  | 851 |

| Nº  | 0        | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 511 | 708.4209 | 5059 | 5908 | 6758 | 7607 | 849 |
| 512 | 709.2700 | 3548 | 4396 | 5244 | 6091 | 847 |
| 513 | 710.1174 | 2020 | 2866 | 3714 | 4559 | 846 |
| 514 | 9631     | 0476 | 1321 | 2165 | 3010 | 844 |
| 515 | 711.8072 | 8915 | 9759 | 0601 | 1444 | 843 |
| 516 | 712.6497 | 7339 | 8180 | 9021 | 9862 | 841 |
| 517 | 713.4905 | 5745 | 6585 | 7425 | 8264 | 839 |
| 518 | 714.3298 | 4136 | 4974 | 5812 | 6650 | 838 |
| 519 | 715.1674 | 2510 | 3347 | 4183 | 5019 | 836 |
| 520 | 716.0033 | 0869 | 1703 | 2538 | 3373 | 835 |
| 521 | 8377     | 9211 | 0044 | 0877 | 1710 | 833 |
| 522 | 717.6705 | 7537 | 8369 | 9200 | 0032 | 831 |
| 523 | 718.5017 | 5847 | 6677 | 7507 | 8337 | 830 |
| 524 | 719.3313 | 4142 | 4970 | 5799 | 6627 | 828 |
| 525 | 720.1593 | 2420 | 3247 | 4074 | 4901 | 826 |
| 526 | 9857     | 0683 | 1508 | 2334 | 3159 | 825 |
| 527 | 721.8106 | 8930 | 9754 | 0578 | 1401 | 823 |
| 528 | 722.6339 | 7162 | 7984 | 8806 | 9628 | 822 |
| 529 | 723.4557 | 5378 | 6198 | 7019 | 7839 | 820 |
| 530 | 724.2759 | 3578 | 4397 | 5216 | 6035 | 818 |
| 531 | 725.0945 | 1763 | 2581 | 3398 | 4216 | 817 |
| 532 | 9116     | 9933 | 0749 | 1565 | 2380 | 815 |
| 533 | 726.7272 | 8087 | 8901 | 9716 | 0530 | 814 |
| 534 | 727.5413 | 6226 | 7039 | 7852 | 8664 | 812 |
| 535 | 728.3538 | 4350 | 5161 | 5972 | 6784 | 811 |
| 536 | 729.1648 | 2458 | 3268 | 4078 | 4888 | 809 |
| 537 | 9743     | 0552 | 1360 | 2168 | 2977 | 808 |
| 538 | 730.7823 | 8630 | 9437 | 0244 | 1051 | 806 |
| 539 | 731.5888 | 6693 | 7499 | 8304 | 9109 | 805 |
| 540 | 732.3938 | 4742 | 5546 | 6350 | 7153 | 804 |
| 541 | 733.1973 | 2775 | 3578 | 4380 | 5183 | 802 |
| 542 | 9993     | 0794 | 1595 | 2396 | 3197 | 801 |
| 543 | 734.7998 | 8798 | 9598 | 0397 | 1196 | 799 |
| 544 | 735.5989 | 6787 | 7585 | 8383 | 9181 | 798 |
| 545 | 736.3965 | 4762 | 5558 | 6355 | 7151 | 796 |
| 546 | 737.1926 | 2722 | 3517 | 4312 | 5107 | 795 |
| 547 | 9873     | 0667 | 1461 | 2254 | 3048 | 793 |
| 548 | 738.7806 | 8598 | 9390 | 0182 | 0974 | 792 |

| N°  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 511 | 708.8456 | 9305 | 0154 | 1003 | 1851 | 849 |
| 512 | 709.6939 | 7786 | 8633 | 9480 | 0327 | 847 |
| 513 | 710.5404 | 6250 | 7096 | 7941 | 8788 | 846 |
| 514 | 711.3854 | 4698 | 5542 | 6385 | 7229 | 844 |
| 515 | 712.2287 | 3129 | 3971 | 4813 | 5655 | 843 |
| 516 | 713.0703 | 1544 | 2385 | 3225 | 4065 | 841 |
| 517 | 9104     | 9943 | 0782 | 1620 | 2459 | 839 |
| 518 | 714.7488 | 8325 | 9162 | 0000 | 0837 | 838 |
| 519 | 715.5856 | 6691 | 7527 | 8363 | 9198 | 836 |
| 520 | 716.4207 | 5042 | 5876 | 6710 | 7544 | 835 |
| 521 | 717.2543 | 3376 | 4208 | 5041 | 5873 | 833 |
| 522 | 718.0863 | 1694 | 2525 | 3356 | 4186 | 831 |
| 523 | 9167     | 9996 | 0826 | 1655 | 2484 | 830 |
| 524 | 719.7455 | 8283 | 9111 | 9938 | 0766 | 828 |
| 525 | 720.5727 | 6554 | 7380 | 8206 | 9032 | 826 |
| 526 | 721.3984 | 4809 | 5633 | 6458 | 7282 | 825 |
| 527 | 722.2225 | 3048 | 3871 | 4694 | 5517 | 823 |
| 528 | 723.0450 | 1272 | 2093 | 2914 | 3736 | 822 |
| 529 | 8660     | 9480 | 0300 | 1120 | 1939 | 820 |
| 530 | 724.6854 | 7672 | 8491 | 9309 | 0127 | 818 |
| 531 | 725.5033 | 5850 | 6667 | 7483 | 8300 | 817 |
| 532 | 726.3196 | 4012 | 4827 | 5642 | 6457 | 815 |
| 533 | 727.1344 | 2158 | 2972 | 3786 | 4599 | 814 |
| 534 | 9477     | 0290 | 1102 | 1914 | 2726 | 812 |
| 535 | 728.7595 | 8406 | 9216 | 0027 | 0838 | 811 |
| 536 | 729.5697 | 6507 | 7316 | 8125 | 8934 | 809 |
| 537 | 730.3785 | 4593 | 5400 | 6208 | 7015 | 808 |
| 538 | 731.1857 | 2663 | 3470 | 4276 | 5082 | 806 |
| 539 | 9914     | 0719 | 1524 | 2329 | 3133 | 805 |
| 540 | 732.7957 | 8760 | 9564 | 0367 | 1170 | 804 |
| 541 | 733.5985 | 6787 | 7588 | 8390 | 9192 | 802 |
| 542 | 734.3997 | 4798 | 5598 | 6398 | 7198 | 801 |
| 543 | 735.1995 | 2794 | 3593 | 4392 | 5191 | 799 |
| 544 | 9979     | 0776 | 1574 | 2371 | 3168 | 798 |
| 545 | 736.7948 | 8744 | 9540 | 0335 | 1131 | 796 |
| 546 | 737.5902 | 6696 | 7491 | 8285 | 9079 | 795 |
| 547 | 738.3841 | 4634 | 5427 | 6220 | 7013 | 793 |
| 548 | 739.1766 | 2558 | 3350 | 4141 | 4932 | 792 |

| N°  | o        | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 587 | 768.6381 | 7121 | 7860 | 8600 | 9339 | 739 |
| 588 | 769.3773 | 4512 | 5250 | 5988 | 6727 | 738 |
| 589 | 770.1153 | 1890 | 2627 | 3364 | 4101 | 737 |
| 590 | 8520     | 9256 | 9992 | 0728 | 1463 | 736 |
| 591 | 771.5875 | 6610 | 7344 | 8079 | 8813 | 735 |
| 592 | 772.3217 | 3951 | 4684 | 5417 | 6150 | 733 |
| 593 | 773.0547 | 1279 | 2011 | 2743 | 3475 | 732 |
| 594 | 7864     | 8595 | 9326 | 0057 | 0788 | 731 |
| 595 | 774.5170 | 5899 | 6629 | 7359 | 8088 | 729 |
| 596 | 775.2463 | 3191 | 3920 | 4648 | 5376 | 728 |
| 597 | 9743     | 0471 | 1198 | 1925 | 2652 | 727 |
| 598 | 776.7012 | 7738 | 8464 | 9190 | 9916 | 726 |
| 599 | 777.4268 | 4993 | 5718 | 6443 | 7167 | 724 |
| 600 | 778.1512 | 2236 | 2960 | 3683 | 4407 | 723 |
| 601 | 8745     | 9467 | 0190 | 0912 | 1634 | 722 |
| 602 | 779.5965 | 6686 | 7407 | 8129 | 8850 | 721 |
| 603 | 780.3173 | 3893 | 4613 | 5333 | 6053 | 720 |
| 604 | 781.0369 | 1088 | 1807 | 2526 | 3245 | 719 |
| 605 | 7554     | 8271 | 8989 | 9707 | 0424 | 717 |
| 606 | 782.4726 | 5443 | 6159 | 6876 | 7592 | 716 |
| 607 | 783.1887 | 2602 | 3318 | 4033 | 4748 | 715 |
| 608 | 9036     | 9750 | 0464 | 1178 | 1892 | 714 |
| 609 | 784.6173 | 6886 | 7599 | 8312 | 9024 | 713 |
| 610 | 785.3298 | 4010 | 4722 | 5434 | 6145 | 712 |
| 611 | 786.0412 | 1123 | 1833 | 2544 | 3254 | 710 |
| 612 | 7514     | 8224 | 8933 | 9643 | 0352 | 709 |
| 613 | 787.4605 | 5313 | 6021 | 6730 | 7438 | 708 |
| 614 | 788.1684 | 2391 | 3098 | 3805 | 4512 | 707 |
| 615 | 8751     | 9457 | 0163 | 0869 | 1575 | 706 |
| 616 | 789.5807 | 6512 | 7217 | 7922 | 8626 | 705 |
| 617 | 790.2853 | 3555 | 4259 | 4963 | 5666 | 703 |
| 618 | 9885     | 0587 | 1290 | 1992 | 2695 | 702 |
| 619 | 791.6906 | 7608 | 8309 | 9011 | 9712 | 701 |
| 620 | 792.3917 | 4617 | 5318 | 6018 | 6718 | 700 |
| 621 | 793.0916 | 1615 | 2313 | 3013 | 3712 | 699 |
| 622 | 7904     | 8602 | 9300 | 9998 | 0696 | 698 |
| 623 | 794.4880 | 5577 | 6274 | 6971 | 7668 | 696 |
| 624 | 795.1846 | 2542 | 3238 | 3933 | 4629 | 695 |

| N°  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 587 | 769.0079 | 6818 | 1557 | 2296 | 3035 | 739 |
| 588 | 7465     | 8203 | 8940 | 9678 | 0415 | 738 |
| 589 | 770.4838 | 5575 | 6311 | 7048 | 7784 | 737 |
| 590 | 771.2199 | 2934 | 3670 | 4405 | 5140 | 736 |
| 591 | 9547     | 0282 | 1016 | 1750 | 2483 | 735 |
| 592 | 772.6883 | 7616 | 8349 | 9082 | 9814 | 733 |
| 593 | 773.4207 | 4939 | 5670 | 6402 | 7133 | 732 |
| 594 | 774.1519 | 2249 | 2979 | 3710 | 4440 | 731 |
| 595 | 8818     | 9547 | 0276 | 1005 | 1734 | 729 |
| 596 | 775.6104 | 6832 | 7560 | 8288 | 9016 | 728 |
| 597 | 776.3379 | 4106 | 4833 | 5559 | 6285 | 727 |
| 598 | 777.0641 | 1367 | 2093 | 2818 | 3543 | 726 |
| 599 | 7892     | 8616 | 9340 | 0065 | 6789 | 724 |
| 600 | 778.5130 | 5853 | 6576 | 7299 | 8022 | 723 |
| 601 | 779.2356 | 3078 | 3800 | 4522 | 5243 | 722 |
| 602 | 9570     | 0291 | 1012 | 1732 | 2453 | 721 |
| 603 | 780.6773 | 7492 | 8212 | 8931 | 9650 | 720 |
| 604 | 781.3963 | 4681 | 5400 | 6118 | 6836 | 719 |
| 605 | 782.1141 | 1859 | 2576 | 3293 | 4009 | 717 |
| 606 | 8308     | 9024 | 9740 | 0456 | 1171 | 716 |
| 607 | 783.5463 | 6178 | 6892 | 7607 | 8321 | 715 |
| 608 | 784.2606 | 3319 | 4033 | 4746 | 5460 | 714 |
| 609 | 9737     | 0450 | 1162 | 1874 | 2586 | 713 |
| 610 | 785.6857 | 7568 | 8279 | 8990 | 9701 | 712 |
| 611 | 786.3965 | 4675 | 5385 | 6095 | 6804 | 710 |
| 612 | 787.1061 | 1770 | 2479 | 3188 | 3896 | 709 |
| 613 | 8146     | 8853 | 9561 | 0269 | 0976 | 708 |
| 614 | 788.5219 | 5926 | 6632 | 7339 | 8045 | 707 |
| 615 | 789.2281 | 2986 | 3691 | 4397 | 5102 | 706 |
| 616 | 9331     | 0035 | 0739 | 1444 | 2148 | 705 |
| 617 | 790.6370 | 7073 | 7776 | 8479 | 9182 | 703 |
| 618 | 791.3397 | 4099 | 4801 | 5503 | 6205 | 702 |
| 619 | 792.0413 | 1114 | 1815 | 2516 | 3216 | 701 |
| 620 | 7418     | 8118 | 8817 | 9517 | 0217 | 700 |
| 621 | 793.4411 | 5110 | 5809 | 6507 | 7206 | 699 |
| 622 | 794.1394 | 2091 | 2789 | 3486 | 4183 | 698 |
| 623 | 8365     | 9061 | 9757 | 0454 | 1150 | 696 |
| 624 | 795.5324 | 6020 | 6715 | 7410 | 8105 | 695 |

| N°  | 0        | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 625 | 795.8800 | 9495 | 0190 | 0884 | 1579 | 694 |
| 626 | 796.5743 | 6437 | 7131 | 7824 | 8517 | 693 |
| 627 | 797.2675 | 3368 | 4060 | 1753 | 5445 | 692 |
| 628 | 9596     | 0288 | 0970 | 1671 | 2362 | 691 |
| 629 | 798.6506 | 7197 | 7887 | 8577 | 9267 | 690 |
| 630 | 799.3405 | 4095 | 4784 | 5473 | 6162 | 689 |
| 631 | 800.0294 | 0982 | 1670 | 2358 | 3046 | 688 |
| 632 | 7171     | 7858 | 8545 | 9232 | 9919 | 687 |
| 633 | 801.4037 | 4723 | 5409 | 6095 | 6781 | 686 |
| 634 | 802.0893 | 1577 | 2262 | 2947 | 3632 | 684 |
| 635 | 7737     | 8421 | 9105 | 9789 | 0472 | 683 |
| 636 | 803.4571 | 5254 | 5937 | 6619 | 7302 | 682 |
| 637 | 804.1394 | 2076 | 2758 | 3439 | 4121 | 681 |
| 638 | 8207     | 8887 | 9568 | 0248 | 0929 | 680 |
| 639 | 805.5009 | 5688 | 6368 | 7047 | 7726 | 679 |
| 640 | 806.1800 | 2478 | 3157 | 3835 | 4513 | 678 |
| 641 | 8580     | 9258 | 9935 | 0612 | 1290 | 677 |
| 642 | 807.5350 | 6027 | 6703 | 7379 | 8055 | 676 |
| 643 | 808.2110 | 2785 | 3460 | 4135 | 4811 | 675 |
| 644 | 8859     | 9533 | 0207 | 0881 | 1555 | 674 |
| 645 | 809.5597 | 6270 | 6944 | 7617 | 8290 | 673 |
| 646 | 810.2325 | 2997 | 3669 | 4342 | 5013 | 672 |
| 647 | 9043     | 9714 | 0385 | 1056 | 1727 | 671 |
| 648 | 811.5750 | 6420 | 7090 | 7700 | 8430 | 670 |
| 649 | 812.2447 | 3116 | 3785 | 4454 | 5123 | 669 |
| 650 | 9134     | 9802 | 0470 | 1137 | 1805 | 668 |
| 651 | 813.5810 | 6477 | 7144 | 7811 | 8477 | 667 |
| 652 | 814.2476 | 3142 | 3808 | 4474 | 5139 | 666 |
| 653 | 9132     | 9797 | 0462 | 1127 | 1791 | 665 |
| 654 | 815.5777 | 6441 | 7105 | 7769 | 8433 | 664 |
| 655 | 816.2413 | 3076 | 3739 | 4402 | 5064 | 663 |
| 656 | 9038     | 9700 | 0362 | 1024 | 1686 | 662 |
| 657 | 817.5654 | 6315 | 6975 | 7636 | 8297 | 660 |
| 658 | 818.2259 | 299  | 3579 | 4239 | 4898 | 659 |
| 659 | 8854     | 9513 | 0172 | 0831 | 1489 | 658 |
| 660 | 819.5439 | 6097 | 6755 | 7413 | 8071 | 658 |
| 661 | 820.2015 | 2672 | 3328 | 3985 | 4642 | 657 |
| 662 | 8580     | 9236 | 9892 | 0548 | 1203 | 656 |

| Nº  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 625 | 796.2273 | 2967 | 3662 | 4356 | 5049 | 694 |
| 626 | 9211     | 9904 | 0597 | 1290 | 1983 | 693 |
| 627 | 797.6137 | 6829 | 7521 | 8213 | 8905 | 692 |
| 628 | 798.3053 | 3744 | 4435 | 5125 | 5816 | 691 |
| 629 | 9957     | 0647 | 1337 | 2027 | 2716 | 690 |
| 630 | 799.6851 | 7540 | 8228 | 8917 | 9605 | 689 |
| 631 | 800.3731 | 4421 | 5109 | 5796 | 6484 | 688 |
| 632 | 801.0605 | 1292 | 1978 | 2665 | 3351 | 687 |
| 633 | 7466     | 8152 | 8837 | 9522 | 0207 | 686 |
| 634 | 802.4316 | 5001 | 5685 | 6369 | 7053 | 684 |
| 635 | 803.1155 | 1839 | 2522 | 3205 | 3888 | 683 |
| 636 | 7984     | 8666 | 9348 | 0030 | 0712 | 682 |
| 637 | 804.4802 | 5483 | 6164 | 6845 | 7526 | 681 |
| 638 | 805.1609 | 2289 | 2969 | 3649 | 4329 | 680 |
| 639 | 8405     | 9085 | 9763 | 0442 | 1121 | 679 |
| 640 | 806.5191 | 5869 | 6547 | 7225 | 7903 | 678 |
| 641 | 807.1967 | 2644 | 3320 | 3997 | 4674 | 677 |
| 642 | 8731     | 9407 | 0083 | 0759 | 1434 | 676 |
| 643 | 808.5485 | 6160 | 6835 | 7510 | 8184 | 675 |
| 644 | 809.2229 | 2903 | 3577 | 4250 | 4924 | 674 |
| 645 | 8962     | 9635 | 0308 | 0980 | 1653 | 673 |
| 646 | 810.5685 | 6357 | 7029 | 7700 | 8371 | 672 |
| 647 | 811.2398 | 3068 | 3739 | 4409 | 5080 | 671 |
| 648 | 9100     | 9769 | 0439 | 1108 | 1778 | 670 |
| 649 | 812.5792 | 6460 | 7129 | 7797 | 8465 | 669 |
| 650 | 813.2473 | 3141 | 3808 | 4475 | 5143 | 668 |
| 651 | 9144     | 9811 | 0477 | 1144 | 1810 | 667 |
| 652 | 814.5805 | 6471 | 7136 | 7801 | 8467 | 666 |
| 653 | 815.2456 | 3120 | 3785 | 4449 | 5113 | 665 |
| 654 | 9096     | 9760 | 0423 | 1087 | 1750 | 664 |
| 655 | 816.5727 | 6389 | 7052 | 7714 | 8376 | 663 |
| 656 | 817.2347 | 3009 | 3670 | 4331 | 4993 | 662 |
| 657 | 8958     | 9618 | 0278 | 0939 | 1599 | 660 |
| 658 | 818.5558 | 6217 | 6877 | 7536 | 8195 | 659 |
| 659 | 819.2148 | 2806 | 3465 | 4123 | 4781 | 658 |
| 660 | 8728     | 9386 | 0043 | 0700 | 1358 | 658 |
| 661 | 820.5298 | 5955 | 6611 | 7268 | 7924 | 657 |
| 662 | 821.1859 | 2514 | 3170 | 3825 | 4480 | 656 |

| Nº  | 0        | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 663 | 821.5135 | 5790 | 6445 | 7100 | 7755 | 655 |
| 664 | 822.1581 | 2335 | 2980 | 3642 | 4296 | 654 |
| 665 | 8216     | 8869 | 9522 | 0175 | 0828 | 653 |
| 666 | 823.4742 | 5394 | 6046 | 6693 | 7350 | 652 |
| 667 | 824.1258 | 1909 | 2560 | 3211 | 3862 | 651 |
| 668 | 7765     | 8415 | 9065 | 9715 | 0364 | 650 |
| 669 | 825.4261 | 4910 | 5559 | 6208 | 6857 | 649 |
| 670 | 826.0748 | 1395 | 2044 | 2692 | 3340 | 648 |
| 671 | 7225     | 7872 | 8519 | 9166 | 9813 | 647 |
| 672 | 827.3693 | 4339 | 4985 | 5631 | 6277 | 646 |
| 673 | 828.0151 | 0796 | 1441 | 2086 | 2731 | 645 |
| 674 | 6599     | 7243 | 7887 | 8532 | 9176 | 644 |
| 675 | 829.3038 | 3681 | 4324 | 4967 | 5611 | 643 |
| 676 | 9467     | 0109 | 0752 | 1394 | 2036 | 642 |
| 677 | 830.5887 | 6528 | 7169 | 7811 | 8452 | 641 |
| 678 | 831.2297 | 2937 | 3578 | 4218 | 4858 | 640 |
| 679 | 8698     | 9337 | 9977 | 0616 | 1255 | 639 |
| 680 | 832.5089 | 5728 | 6366 | 7005 | 7643 | 638 |
| 681 | 833.1471 | 2109 | 2746 | 3384 | 4021 | 637 |
| 682 | 7844     | 8480 | 9117 | 9754 | 0390 | 636 |
| 683 | 834.4207 | 4843 | 5479 | 6114 | 6750 | 635 |
| 684 | 835.0561 | 1196 | 1831 | 2465 | 3100 | 635 |
| 685 | 6906     | 7540 | 8174 | 8807 | 9441 | 634 |
| 686 | 836.3241 | 3874 | 4507 | 5140 | 5773 | 633 |
| 687 | 9567     | 0199 | 0832 | 1463 | 2095 | 632 |
| 688 | 837.5884 | 6516 | 7147 | 7778 | 8409 | 631 |
| 689 | 838.2192 | 2822 | 3453 | 4083 | 4713 | 630 |
| 690 | 8491     | 9120 | 9750 | 0379 | 1008 | 629 |
| 691 | 839.4780 | 5409 | 6037 | 6666 | 7294 | 628 |
| 692 | 840.1061 | 1688 | 2316 | 2943 | 3571 | 627 |
| 693 | 7332     | 7959 | 8586 | 9212 | 9838 | 626 |
| 694 | 841.3595 | 4220 | 4846 | 5472 | 6097 | 625 |
| 695 | 9848     | 0473 | 1098 | 1722 | 2347 | 624 |
| 696 | 842.6092 | 6716 | 7340 | 7964 | 8588 | 624 |
| 697 | 843.2328 | 2951 | 3574 | 4197 | 4819 | 623 |
| 698 | 8554     | 9176 | 9798 | 0420 | 1042 | 622 |
| 699 | 844.4772 | 5393 | 6014 | 6635 | 7256 | 621 |
| 700 | 845.0980 | 1601 | 2221 | 2841 | 3461 | 620 |



| N°  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 663 | 821.8409 | 9064 | 9718 | 0372 | 1027 | 655 |
| 664 | 822.4950 | 5603 | 6257 | 6910 | 7563 | 654 |
| 665 | 823.1481 | 2133 | 2786 | 3438 | 4090 | 653 |
| 666 | 8002     | 8653 | 9305 | 9956 | 0607 | 652 |
| 667 | 824.4513 | 5163 | 5814 | 6464 | 7114 | 651 |
| 668 | 825.1014 | 1664 | 2313 | 2963 | 3612 | 650 |
| 669 | 7506     | 8154 | 8803 | 9451 | 0100 | 649 |
| 670 | 826.3988 | 4635 | 5283 | 5931 | 6578 | 648 |
| 671 | 827.0460 | 1107 | 1753 | 2400 | 3046 | 647 |
| 672 | 6923     | 7569 | 8214 | 8860 | 9505 | 646 |
| 673 | 828.3376 | 4021 | 4665 | 5310 | 5955 | 645 |
| 674 | 9820     | 0463 | 1107 | 1751 | 2394 | 644 |
| 675 | 829.6254 | 6896 | 7539 | 8182 | 8824 | 643 |
| 676 | 830.2678 | 3320 | 3962 | 4604 | 5245 | 642 |
| 677 | 9093     | 9734 | 0375 | 1016 | 1656 | 641 |
| 678 | 831.5499 | 6139 | 6778 | 7418 | 8058 | 640 |
| 679 | 832.1895 | 2534 | 3173 | 3812 | 4450 | 639 |
| 680 | 8281     | 8919 | 9558 | 0195 | 0833 | 638 |
| 681 | 833.4659 | 5296 | 5933 | 6570 | 7207 | 637 |
| 682 | 834.1027 | 1663 | 2299 | 2935 | 3571 | 636 |
| 683 | 7385     | 8021 | 8656 | 9291 | 9926 | 635 |
| 684 | 835.3735 | 4369 | 5003 | 5638 | 6272 | 635 |
| 685 | 836.0075 | 0708 | 1341 | 1975 | 2608 | 634 |
| 686 | 6405     | 7038 | 7670 | 8303 | 8935 | 633 |
| 687 | 837.2727 | 3359 | 3990 | 4622 | 5253 | 632 |
| 688 | 9039     | 9670 | 0391 | 0931 | 1562 | 631 |
| 689 | 838.5343 | 5973 | 6602 | 7232 | 7861 | 630 |
| 690 | 039.1637 | 2266 | 2895 | 3523 | 4152 | 629 |
| 691 | 7922     | 8550 | 9178 | 9806 | 0433 | 628 |
| 692 | 840.4198 | 4825 | 5452 | 6079 | 6706 | 627 |
| 693 | 841.0465 | 1091 | 1717 | 2343 | 2969 | 626 |
| 694 | 6723     | 7348 | 7973 | 8598 | 9223 | 625 |
| 695 | 842.2971 | 3596 | 4220 | 4844 | 5468 | 624 |
| 696 | 9211     | 9835 | 0458 | 1081 | 1705 | 624 |
| 697 | 843.5442 | 6065 | 6687 | 7310 | 7932 | 623 |
| 698 | 844.1664 | 2286 | 2907 | 3529 | 4150 | 622 |
| 699 | 7877     | 8498 | 9119 | 9739 | 0360 | 621 |
| 700 | 845.4081 | 4701 | 5521 | 5941 | 6561 | 620 |

| N°  | ●        | I    | 2    | 3    | 4    | Diff |
|-----|----------|------|------|------|------|------|
| 701 | 845.7180 | 7800 | 8419 | 9038 | 9658 | 619  |
| 702 | 846.3371 | 3990 | 4608 | 5227 | 5845 | 618  |
| 703 | 9553     | 0171 | 0789 | 1406 | 2024 | 618  |
| 704 | 847.5727 | 6343 | 6960 | 7577 | 8193 | 617  |
| 705 | 848.1891 | 2507 | 3123 | 3739 | 4355 | 616  |
| 706 | 8047     | 8662 | 9277 | 9892 | 0507 | 615  |
| 707 | 849.4194 | 4868 | 5423 | 6037 | 6651 | 614  |
| 708 | 850.0333 | 0946 | 1559 | 2172 | 2786 | 613  |
| 709 | 6462     | 7075 | 7687 | 8300 | 8912 | 612  |
| 710 | 851.2583 | 3195 | 3807 | 4418 | 5030 | 611  |
| 711 | 8696     | 9307 | 9917 | 0528 | 1139 | 611  |
| 712 | 852.4800 | 5410 | 6020 | 6629 | 7239 | 610  |
| 713 | 853.0895 | 1504 | 2113 | 2722 | 3331 | 609  |
| 714 | 6982     | 7590 | 8198 | 8807 | 9414 | 608  |
| 715 | 854 3060 | 3668 | 4275 | 4882 | 5489 | 607  |
| 716 | 9130     | 9737 | 0343 | 0950 | 1556 | 606  |
| 717 | 855.5192 | 5797 | 6403 | 7008 | 7614 | 605  |
| 718 | 856.1244 | 1849 | 2454 | 3059 | 3663 | 604  |
| 719 | 7289     | 7893 | 8497 | 9101 | 9704 | 604  |
| 720 | 857.3325 | 3928 | 4531 | 5134 | 5737 | 603  |
| 721 | 9353     | 9955 | 0557 | 1159 | 1761 | 602  |
| 722 | 858.5372 | 5973 | 6575 | 7176 | 7777 | 601  |
| 723 | 859.1383 | 1984 | 2584 | 3185 | 3785 | 600  |
| 724 | 7386     | 7985 | 8585 | 9185 | 9784 | 600  |
| 725 | 860.3380 | 3979 | 4578 | 5177 | 5776 | 599  |
| 726 | 9366     | 9964 | 0562 | 1160 | 1758 | 598  |
| 727 | 861.5344 | 5941 | 6539 | 7136 | 7733 | 597  |
| 728 | 862.1314 | 1910 | 2507 | 3103 | 3699 | 596  |
| 729 | 7275     | 7871 | 8467 | 9062 | 9658 | 595  |
| 730 | 863.3229 | 3823 | 4418 | 5013 | 5608 | 595  |
| 731 | 9174     | 9768 | 0362 | 0956 | 1550 | 594  |
| 732 | 864.5111 | 5704 | 6297 | 6890 | 7483 | 593  |
| 733 | 865.1040 | 1632 | 2225 | 2817 | 3409 | 592  |
| 734 | 6961     | 7552 | 8144 | 8735 | 9327 | 591  |
| 735 | 866.2873 | 3464 | 4055 | 4646 | 5236 | 590  |
| 736 | 8778     | 9368 | 9958 | 0548 | 1138 | 590  |
| 737 | 867.4675 | 5264 | 5853 | 6442 | 7031 | 589  |
| 738 | 868.0564 | 1152 | 1740 | 2329 | 2917 | 588  |

| N°  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 701 | 846.0277 | 0896 | 1515 | 2134 | 2752 | 619 |
| 702 | 6463     | 7081 | 7700 | 8318 | 8935 | 618 |
| 703 | 847.2641 | 3258 | 3876 | 4493 | 5110 | 618 |
| 704 | 88.10    | 9426 | 0043 | 0659 | 1275 | 617 |
| 705 | 848.4970 | 5586 | 6201 | 6817 | 7432 | 616 |
| 706 | 849.1122 | 1736 | 2351 | 2965 | 3580 | 615 |
| 707 | 7264     | 7878 | 8492 | 9106 | 9719 | 614 |
| 708 | 850.3399 | 4011 | 4624 | 5237 | 5850 | 613 |
| 709 | 9524     | 0136 | 0748 | 1360 | 1972 | 612 |
| 710 | 851.5641 | 6252 | 6863 | 7474 | 8085 | 611 |
| 711 | 852.1749 | 2359 | 2970 | 3580 | 4190 | 611 |
| 712 | 7849     | 8458 | 9068 | 9677 | 0286 | 610 |
| 713 | 853.3940 | 4548 | 5157 | 5765 | 6374 | 609 |
| 714 | 854.0022 | 0630 | 1238 | 1845 | 2453 | 608 |
| 715 | 6096     | 6703 | 7310 | 7917 | 8524 | 607 |
| 716 | 855.2162 | 2768 | 3374 | 3980 | 4586 | 606 |
| 717 | 8219     | 8824 | 9429 | 0035 | 0640 | 605 |
| 718 | 856.4268 | 4872 | 5476 | 6081 | 6685 | 604 |
| 719 | 857.0308 | 0912 | 1515 | 2118 | 2722 | 604 |
| 720 | 6340     | 6943 | 7545 | 8148 | 8750 | 603 |
| 721 | 858.2363 | 2965 | 3567 | 4169 | 4770 | 602 |
| 722 | 8379     | 8980 | 9581 | 0181 | 0782 | 601 |
| 723 | 859.4385 | 4986 | 5586 | 6186 | 6786 | 600 |
| 724 | 860.0384 | 0983 | 1583 | 2182 | 2781 | 600 |
| 725 | 6374     | 6973 | 7571 | 8170 | 8768 | 599 |
| 726 | 861.2356 | 2954 | 3552 | 4149 | 4747 | 598 |
| 727 | 8330     | 8927 | 9524 | 0121 | 0717 | 597 |
| 728 | 862.4296 | 4892 | 5488 | 6084 | 6680 | 596 |
| 729 | 863.0253 | 0848 | 1443 | 2039 | 2634 | 595 |
| 730 | 6202     | 6797 | 7391 | 7985 | 8580 | 595 |
| 731 | 864.2143 | 2737 | 3331 | 3924 | 4517 | 594 |
| 732 | 8076     | 8669 | 9262 | 9855 | 0447 | 593 |
| 733 | 865.4001 | 4593 | 5185 | 5777 | 6369 | 592 |
| 734 | 9918     | 0509 | 1100 | 1691 | 2282 | 592 |
| 735 | 866.5827 | 6417 | 7008 | 7598 | 8188 | 590 |
| 736 | 867.1728 | 2317 | 2907 | 3496 | 4086 | 590 |
| 737 | 7620     | 8209 | 8798 | 9387 | 9975 | 589 |
| 738 | 868.3505 | 4093 | 4681 | 5269 | 5857 | 588 |

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| N°  | °        | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 739 | 868.6444 | 7032 | 7620 | 8207 | 8794 | 587 |
| 740 | 869.2317 | 2904 | 3491 | 4077 | 4664 | 586 |
| 741 | 8182     | 8768 | 9354 | 9940 | 0526 | 586 |
| 742 | 870.4039 | 4624 | 5210 | 5795 | 6380 | 585 |
| 743 | 9888     | 0473 | 1057 | 1641 | 2226 | 584 |
| 744 | 871.5729 | 6313 | 6897 | 7480 | 8064 | 583 |
| 745 | 872.1563 | 2146 | 2728 | 3311 | 3894 | 582 |
| 746 | 7388     | 7970 | 8552 | 9134 | 9716 | 582 |
| 747 | 873.3206 | 3787 | 4369 | 4950 | 5531 | 581 |
| 748 | 9016     | 9597 | 0177 | 0757 | 1338 | 580 |
| 749 | 874.4818 | 5398 | 5978 | 6557 | 7137 | 579 |
| 750 | 875.0613 | 1192 | 1771 | 2349 | 2928 | 579 |
| 751 | 6399     | 6978 | 7556 | 8134 | 8712 | 578 |
| 752 | 876.2178 | 2756 | 3333 | 3911 | 4488 | 577 |
| 753 | 7950     | 8526 | 9103 | 9680 | 0256 | 576 |
| 754 | 877.3713 | 4289 | 4865 | 5441 | 6017 | 576 |
| 755 | 9470     | 0045 | 0620 | 1195 | 1770 | 575 |
| 756 | 878.5218 | 5792 | 6367 | 6941 | 7515 | 574 |
| 757 | 879.0959 | 1532 | 2106 | 2680 | 3253 | 573 |
| 758 | 6692     | 7265 | 7838 | 8411 | 8983 | 572 |
| 759 | 880.2418 | 2990 | 3562 | 4134 | 4706 | 572 |
| 760 | 8136     | 8707 | 9279 | 9850 | 0421 | 571 |
| 761 | 881.3847 | 4417 | 4988 | 5558 | 6129 | 570 |
| 762 | 9550     | 0120 | 0689 | 1259 | 1829 | 569 |
| 763 | 882.5245 | 5815 | 6384 | 6953 | 7522 | 569 |
| 764 | 883.0934 | 1502 | 2070 | 2639 | 3207 | 568 |
| 765 | 6614     | 7182 | 7750 | 8317 | 8885 | 567 |
| 766 | 884.2288 | 2855 | 3421 | 3988 | 4555 | 567 |
| 767 | 7954     | 8520 | 9086 | 9652 | 0218 | 566 |
| 768 | 885.3612 | 4178 | 4743 | 5308 | 5874 | 565 |
| 769 | 9263     | 9828 | 0393 | 0957 | 1522 | 564 |
| 770 | 886.4907 | 5471 | 6035 | 6599 | 7163 | 564 |
| 771 | 887.0544 | 1107 | 1670 | 2233 | 2796 | 563 |
| 772 | 6173     | 6736 | 7298 | 7860 | 8423 | 562 |
| 773 | 888.1795 | 2357 | 2918 | 3480 | 4042 | 561 |
| 774 | 7410     | 7971 | 8532 | 9093 | 9653 | 561 |
| 775 | 889.3017 | 3577 | 4138 | 4694 | 5258 | 560 |
| 776 | 8617     | 9177 | 9736 | 0296 | 0855 | 559 |

| N°  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 739 | 868.9382 | 9969 | 0556 | 1143 | 1730 | 587 |
| 740 | 869.5251 | 5837 | 6423 | 7010 | 7596 | 586 |
| 741 | 870.1112 | 1697 | 2283 | 2868 | 3454 | 586 |
| 742 | 6965     | 7549 | 8134 | 8719 | 9304 | 585 |
| 743 | 871.2810 | 3394 | 3978 | 4562 | 5146 | 584 |
| 744 | 8647     | 9230 | 9814 | 0397 | 0980 | 583 |
| 745 | 872.4476 | 5059 | 5641 | 6224 | 6806 | 582 |
| 746 | 873.0298 | 0880 | 1462 | 2043 | 2625 | 582 |
| 747 | 6112     | 6693 | 7274 | 7855 | 8435 | 581 |
| 748 | 874.1918 | 2498 | 3078 | 3658 | 4238 | 580 |
| 749 | 7716     | 8296 | 8875 | 9454 | 0034 | 579 |
| 750 | 875.3507 | 4086 | 4664 | 5243 | 5821 | 579 |
| 751 | 9290     | 9868 | 0446 | 1023 | 1601 | 578 |
| 752 | 876.5065 | 5642 | 6219 | 6796 | 7373 | 577 |
| 753 | 877.0833 | 1409 | 1985 | 2561 | 3137 | 576 |
| 754 | 6592     | 7168 | 7743 | 8319 | 8894 | 576 |
| 755 | 878.2345 | 2919 | 3494 | 4069 | 4643 | 575 |
| 756 | 8089     | 8663 | 9237 | 9811 | 0385 | 574 |
| 757 | 879.3826 | 4400 | 4973 | 5546 | 6119 | 573 |
| 758 | 9556     | 0128 | 0701 | 1273 | 1846 | 572 |
| 759 | 880.5278 | 5850 | 6421 | 6993 | 7564 | 572 |
| 760 | 881.0992 | 1563 | 2134 | 2705 | 3276 | 571 |
| 761 | 6699     | 7269 | 7840 | 8412 | 8980 | 570 |
| 762 | 882.2398 | 2968 | 3537 | 4107 | 4676 | 569 |
| 763 | 8090     | 8659 | 9228 | 9797 | 0365 | 569 |
| 764 | 883.3775 | 4343 | 4911 | 5479 | 6047 | 568 |
| 765 | 9452     | 0019 | 0586 | 1154 | 1721 | 567 |
| 766 | 884.5122 | 5688 | 6255 | 6821 | 7387 | 567 |
| 767 | 885.0784 | 1350 | 1915 | 2481 | 3047 | 566 |
| 768 | 6439     | 7004 | 7569 | 8134 | 8699 | 565 |
| 769 | 886.2086 | 2651 | 3215 | 3779 | 4343 | 564 |
| 770 | 7726     | 8290 | 8854 | 9417 | 9980 | 564 |
| 771 | 887.3359 | 3922 | 4485 | 5048 | 5610 | 563 |
| 772 | 8985     | 9547 | 0109 | 0671 | 1233 | 562 |
| 773 | 888.4603 | 5165 | 5726 | 6287 | 6848 | 561 |
| 774 | 889.0214 | 0775 | 1336 | 1896 | 2457 | 561 |
| 775 | 5818     | 6378 | 6938 | 7498 | 8058 | 560 |
| 776 | 890.1415 | 1974 | 2533 | 3092 | 3651 | 559 |

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| 739 | 868.6444 | 7032 | 7620 | 8207 | 8794 | 587 |
| 740 | 869.2317 | 2904 | 3491 | 4077 | 4664 | 586 |
| 741 | 8182     | 8768 | 9354 | 9940 | 0526 | 586 |
| 742 | 870.4039 | 4624 | 5210 | 5795 | 6380 | 585 |
| 743 | 9888     | 0473 | 1057 | 1641 | 2226 | 584 |
| 744 | 871.5729 | 6313 | 6897 | 7480 | 8064 | 583 |
| 745 | 872.1563 | 2146 | 2728 | 3311 | 3894 | 582 |
| 746 | 7388     | 7970 | 8552 | 9134 | 9716 | 582 |
| 747 | 873.3206 | 3787 | 4369 | 4950 | 5531 | 581 |
| 748 | 9016     | 9597 | 0177 | 0757 | 1338 | 580 |
| 749 | 874.4818 | 5398 | 5978 | 6557 | 7137 | 579 |
| 750 | 875.0613 | 1192 | 1771 | 2349 | 2928 | 579 |
| 751 | 6399     | 6978 | 7556 | 8134 | 8712 | 578 |
| 752 | 876.2178 | 2756 | 3333 | 3911 | 4488 | 577 |
| 753 | 7950     | 8526 | 9103 | 9680 | 0256 | 576 |
| 754 | 877.3713 | 4289 | 4865 | 5441 | 6017 | 576 |
| 755 | 9470     | 0045 | 0620 | 1195 | 1770 | 575 |
| 756 | 878.5218 | 5792 | 6367 | 6941 | 7515 | 574 |
| 757 | 879.0959 | 1532 | 2106 | 2680 | 3253 | 573 |
| 758 | 6692     | 7265 | 7838 | 8411 | 8983 | 572 |
| 759 | 880.2418 | 2990 | 3562 | 4134 | 4706 | 572 |
| 760 | 8136     | 8707 | 9279 | 9850 | 0421 | 571 |
| 761 | 881.3847 | 4417 | 4988 | 5558 | 6129 | 570 |
| 762 | 9550     | 0120 | 0689 | 1259 | 1829 | 569 |
| 763 | 882.5245 | 5815 | 6384 | 6953 | 7522 | 569 |
| 764 | 883.0934 | 1502 | 2070 | 2639 | 3207 | 568 |
| 765 | 6614     | 7182 | 7750 | 8317 | 8885 | 567 |
| 766 | 884.2288 | 2855 | 3421 | 3988 | 4555 | 567 |
| 767 | 7954     | 8520 | 9086 | 9652 | 0218 | 566 |
| 768 | 885.3612 | 4178 | 4743 | 5308 | 5874 | 565 |
| 769 | 9263     | 9828 | 0393 | 0957 | 1522 | 564 |
| 770 | 886.4907 | 5471 | 6035 | 6599 | 7163 | 564 |
| 771 | 887.0544 | 1107 | 1670 | 2233 | 2796 | 563 |
| 772 | 6173     | 6736 | 7298 | 7860 | 8423 | 562 |
| 773 | 888.1795 | 2357 | 2918 | 3480 | 4042 | 561 |
| 774 | 7410     | 7971 | 8532 | 9093 | 9653 | 561 |
| 775 | 889.3017 | 3577 | 4138 | 4694 | 5258 | 560 |
| 776 | 8617     | 9177 | 9736 | 0296 | 0855 | 559 |

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| 777 | 890.7004 | 7563 | 8121 | 8679 | 9238 | 559 |
| 778 | 891 2586 | 3144 | 3702 | 4259 | 4817 | 558 |
| 779 | 8161     | 8718 | 9275 | 9832 | 0389 | 557 |
| 780 | 892.3729 | 4285 | 4842 | 5398 | 5954 | 556 |
| 781 | 9290     | 9846 | 0401 | 0957 | 1512 | 556 |
| 782 | 893.4843 | 5398 | 5953 | 6508 | 7061 | 555 |
| 783 | 894.0390 | 0944 | 1498 | 2053 | 2607 | 554 |
| 784 | 5929     | 6483 | 7037 | 7590 | 8143 | 554 |
| 785 | 895.1462 | 2015 | 2568 | 3120 | 3673 | 553 |
| 786 | 6987     | 7539 | 8092 | 8644 | 9195 | 552 |
| 787 | 896.2506 | 3057 | 3608 | 4160 | 4711 | 552 |
| 788 | 8017     | 8568 | 9118 | 9669 | 0220 | 551 |
| 789 | 897.3521 | 4071 | 4621 | 5171 | 5721 | 550 |
| 790 | 9019     | 9568 | 0117 | 0667 | 1216 | 549 |
| 791 | 898.4509 | 5058 | 5606 | 6155 | 6703 | 549 |
| 792 | 9993     | 0541 | 1089 | 1636 | 2184 | 548 |
| 793 | 899.5469 | 6017 | 6564 | 7111 | 7658 | 547 |
| 794 | 900.0939 | 1486 | 2032 | 2579 | 3125 | 547 |
| 795 | 6402     | 6948 | 7494 | 8039 | 8585 | 546 |
| 796 | 901.1858 | 2403 | 2948 | 3493 | 4038 | 545 |
| 797 | 7307     | 7851 | 8396 | 8940 | 9485 | 544 |
| 798 | 902.2749 | 3293 | 3837 | 4381 | 4924 | 544 |
| 799 | 8185     | 8728 | 9271 | 9814 | 0357 | 543 |
| 800 | 903.3613 | 4156 | 4698 | 5241 | 5783 | 543 |
| 801 | 9035     | 9577 | 0119 | 0661 | 1202 | 542 |
| 802 | 904.4450 | 4992 | 5533 | 6074 | 6615 | 541 |
| 803 | 9859     | 0399 | 0940 | 1480 | 2020 | 540 |
| 804 | 905.5260 | 5800 | 6340 | 6880 | 7419 | 540 |
| 805 | 906.0655 | 1195 | 1734 | 2273 | 2812 | 539 |
| 806 | 6044     | 6582 | 7121 | 7659 | 8197 | 539 |
| 807 | 907.1425 | 1963 | 2501 | 3038 | 3576 | 538 |
| 808 | 6800     | 7337 | 7874 | 8411 | 8948 | 537 |
| 809 | 908.2169 | 2705 | 3241 | 3778 | 4314 | 536 |
| 810 | 7530     | 8066 | 8602 | 9137 | 9673 | 536 |
| 811 | 909.2885 | 3420 | 3955 | 4490 | 5025 | 535 |
| 812 | 8234     | 8768 | 9303 | 9837 | 0371 | 534 |
| 813 | 910.3576 | 4109 | 4643 | 5177 | 5710 | 534 |
| 814 | 8911     | 9444 | 9977 | 0510 | 1043 | 533 |

| N°  | 0        | 1    | 2    | 3    | 4    | Dif |
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| 815 | 911.1576 | 2109 | 2642 | 3174 | 3707 | 533 |
| 816 | 6902     | 7434 | 7966 | 8498 | 9030 | 532 |
| 817 | 912.2221 | 2752 | 3284 | 3815 | 4346 | 531 |
| 818 | 7533     | 8064 | 8595 | 9126 | 9656 | 530 |
| 819 | 913.2839 | 3369 | 3899 | 4430 | 4960 | 530 |
| 820 | 8139     | 8668 | 9198 | 9727 | 0257 | 529 |
| 821 | 914.3432 | 3961 | 4489 | 5018 | 5547 | 529 |
| 822 | 8718     | 9246 | 9775 | 0303 | 0831 | 528 |
| 823 | 915.3998 | 4526 | 5054 | 5581 | 6109 | 527 |
| 824 | 9272     | 9799 | 0326 | 0853 | 1380 | 527 |
| 825 | 916.4539 | 5066 | 5592 | 6118 | 6645 | 526 |
| 826 | 9800     | 0326 | 0852 | 1378 | 1903 | 526 |
| 827 | 917.5055 | 5580 | 6105 | 6630 | 7155 | 525 |
| 828 | 918.0303 | 0828 | 1352 | 1877 | 2401 | 524 |
| 829 | 5545     | 6069 | 6593 | 7117 | 7640 | 524 |
| 830 | 919.0781 | 1304 | 1827 | 2350 | 2873 | 523 |
| 831 | 6010     | 6533 | 7055 | 7578 | 8100 | 522 |
| 832 | 920.1233 | 1755 | 2277 | 2799 | 3321 | 522 |
| 833 | 6450     | 6971 | 7493 | 8014 | 8535 | 521 |
| 834 | 921.1661 | 2181 | 2702 | 3222 | 3743 | 521 |
| 835 | 6865     | 7385 | 7905 | 8425 | 8945 | 520 |
| 836 | 922.2063 | 2582 | 3102 | 3621 | 4140 | 519 |
| 837 | 7255     | 7773 | 8292 | 8811 | 9330 | 518 |
| 838 | 923.2440 | 2958 | 3477 | 3995 | 4513 | 518 |
| 839 | 7620     | 8137 | 8655 | 9172 | 9690 | 517 |
| 840 | 924.2793 | 3310 | 3827 | 4344 | 4860 | 517 |
| 841 | 7960     | 8476 | 8993 | 9509 | 0025 | 516 |
| 842 | 925.3121 | 3637 | 4152 | 4668 | 5184 | 516 |
| 843 | 8276     | 8791 | 9306 | 9821 | 0336 | 515 |
| 844 | 926.3424 | 3939 | 4453 | 4968 | 5482 | 514 |
| 845 | 8567     | 9081 | 9595 | 0109 | 0622 | 514 |
| 846 | 927.3704 | 4217 | 4730 | 5243 | 5757 | 513 |
| 847 | 8834     | 9347 | 9859 | 0372 | 0885 | 512 |
| 848 | 928.3959 | 4471 | 4983 | 5495 | 6007 | 512 |
| 849 | 9077     | 9588 | 0100 | 0611 | 1123 | 511 |
| 850 | 929.4189 | 4700 | 5211 | 5722 | 6233 | 511 |
| 851 | 9296     | 9806 | 0316 | 0826 | 1336 | 510 |
| 852 | 930.4396 | 4906 | 5415 | 5925 | 6434 | 509 |



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| 816 | 9562     | 0094 | 0626 | 1157 | 1689 | 532 |
| 817 | 912.4878 | 5409 | 5940 | 6471 | 7002 | 531 |
| 818 | 913.0189 | 0717 | 1248 | 1778 | 2309 | 530 |
| 819 | 5490     | 6019 | 6549 | 7079 | 7609 | 530 |
| 820 | 914.0786 | 1315 | 1844 | 2373 | 2903 | 529 |
| 821 | 6076     | 6604 | 7133 | 7661 | 8190 | 529 |
| 822 | 915.1359 | 1887 | 2415 | 2943 | 3471 | 528 |
| 823 | 6636     | 7163 | 7691 | 8218 | 8745 | 527 |
| 824 | 916.1907 | 2433 | 2960 | 3487 | 4013 | 527 |
| 825 | 7171     | 7697 | 8223 | 8749 | 9275 | 526 |
| 826 | 917.2429 | 2954 | 3479 | 4005 | 4530 | 526 |
| 827 | 7680     | 8205 | 8730 | 9254 | 9779 | 525 |
| 828 | 918.2925 | 3449 | 3973 | 4497 | 5021 | 524 |
| 829 | 8164     | 8687 | 9211 | 9734 | 0258 | 524 |
| 830 | 919.3396 | 3919 | 4442 | 4965 | 5488 | 523 |
| 831 | 8623     | 9145 | 9667 | 0189 | 0711 | 522 |
| 832 | 920.3842 | 4364 | 4886 | 5407 | 5929 | 522 |
| 833 | 9056     | 9577 | 0098 | 0619 | 1140 | 521 |
| 834 | 921.4263 | 4784 | 5304 | 5824 | 6345 | 521 |
| 835 | 9465     | 9984 | 0504 | 1024 | 1543 | 520 |
| 836 | 922.4659 | 5179 | 5698 | 6217 | 6736 | 519 |
| 837 | 9848     | 0367 | 0885 | 1404 | 1922 | 518 |
| 838 | 923.5031 | 5549 | 6066 | 6584 | 7102 | 518 |
| 839 | 924.0207 | 0724 | 1242 | 1759 | 2276 | 517 |
| 840 | 5377     | 5894 | 6410 | 6927 | 7444 | 517 |
| 841 | 925.0541 | 1057 | 1573 | 2089 | 2605 | 516 |
| 842 | 5699     | 6215 | 6730 | 7245 | 7761 | 516 |
| 843 | 926.0851 | 1366 | 1880 | 2395 | 2910 | 515 |
| 844 | 5997     | 6511 | 7025 | 7539 | 8035 | 514 |
| 845 | 927.1136 | 1650 | 2163 | 2677 | 3190 | 514 |
| 846 | 6270     | 6783 | 7296 | 7808 | 8321 | 513 |
| 847 | 928.1397 | 1909 | 2422 | 2934 | 3446 | 512 |
| 848 | 6518     | 7030 | 7542 | 8054 | 8565 | 512 |
| 849 | 929.1634 | 2145 | 2656 | 3167 | 3678 | 511 |
| 850 | 6743     | 7254 | 7764 | 8275 | 8785 | 511 |
| 851 | 930.1847 | 2357 | 2866 | 3376 | 3886 | 510 |
| 852 | 6944     | 7453 | 7963 | 8472 | 8981 | 509 |

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| 854 | 931.4579 | 5087 | 5596 | 6104 | 6612 | 508 |
| 855 | 9661     | 0169 | 0677 | 1185 | 1692 | 508 |
| 856 | 932.4738 | 5245 | 5752 | 6259 | 6767 | 507 |
| 857 | 9808     | 0315 | 0822 | 1328 | 1835 | 506 |
| 858 | 933.4873 | 5379 | 5885 | 6391 | 6897 | 106 |
| 859 | 9932     | 0437 | 0943 | 1448 | 1953 | 505 |
| 860 | 934.4985 | 5489 | 5994 | 6499 | 7004 | 505 |
| 861 | 935.0032 | 0536 | 1040 | 1544 | 2049 | 504 |
| 862 | 5073     | 5576 | 6080 | 6584 | 7087 | 503 |
| 863 | 936.0108 | 0611 | 1114 | 1617 | 2120 | 503 |
| 864 | 5137     | 5640 | 6143 | 6645 | 7148 | 502 |
| 865 | 937.0161 | 0663 | 1165 | 1667 | 2169 | 502 |
| 866 | 5179     | 5680 | 6182 | 6683 | 7184 | 501 |
| 867 | 938.0191 | 0692 | 1193 | 1693 | 2194 | 501 |
| 868 | 5197     | 5698 | 6198 | 6698 | 7198 | 500 |
| 869 | 939.0198 | 0697 | 1197 | 1697 | 2197 | 499 |
| 870 | 5193     | 5694 | 6191 | 6690 | 7189 | 499 |
| 871 | 940.0182 | 0680 | 1179 | 1677 | 2176 | 498 |
| 872 | 5165     | 5663 | 6161 | 6659 | 7157 | 498 |
| 873 | 941.0142 | 0640 | 1137 | 1635 | 2132 | 497 |
| 874 | 5114     | 5611 | 6108 | 6605 | 7101 | 497 |
| 875 | 942.0081 | 0577 | 1073 | 1569 | 2065 | 496 |
| 876 | 5041     | 5537 | 6032 | 6528 | 7024 | 496 |
| 877 | 9996     | 0491 | 0986 | 1481 | 1976 | 495 |
| 878 | 943.4945 | 5440 | 5934 | 6429 | 6923 | 494 |
| 879 | 9889     | 0383 | 0877 | 1371 | 1865 | 494 |
| 880 | 944.4827 | 5320 | 5814 | 6307 | 6800 | 493 |
| 881 | 9759     | 0252 | 0745 | 1238 | 1730 | 493 |
| 882 | 945.4686 | 5178 | 5671 | 6163 | 6655 | 492 |
| 883 | 9607     | 0099 | 0591 | 1082 | 1574 | 492 |
| 884 | 946.4523 | 5014 | 5505 | 5996 | 6487 | 491 |
| 885 | 9433     | 9923 | 0414 | 0905 | 1395 | 490 |
| 886 | 947.4337 | 4827 | 5317 | 5807 | 6297 | 490 |
| 887 | 9236     | 9726 | 0215 | 0705 | 1194 | 489 |
| 888 | 948.4130 | 4619 | 5108 | 5597 | 6085 | 489 |
| 889 | 9018     | 9506 | 9995 | 0483 | 0971 | 488 |
| 890 | 949.3900 | 4388 | 4876 | 5364 | 5852 | 488 |

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| 854 | 7121     | 7029 | 8137 | 8645 | 9153 | 508 |
| 855 | 932.2200 | 2708 | 3215 | 3723 | 4230 | 508 |
| 856 | 7274     | 7781 | 8288 | 8795 | 9301 | 507 |
| 857 | 933.2311 | 2848 | 3354 | 3860 | 4367 | 506 |
| 858 | 7403     | 7909 | 8415 | 8920 | 9426 | 506 |
| 859 | 934.2409 | 2964 | 3469 | 3974 | 4479 | 505 |
| 860 | 7509     | 8013 | 8518 | 9023 | 9527 | 505 |
| 861 | 935.2553 | 3057 | 3561 | 4065 | 4569 | 504 |
| 862 | 7591     | 8095 | 8598 | 9101 | 9605 | 503 |
| 863 | 936.2623 | 3120 | 3629 | 4132 | 4635 | 503 |
| 864 | 7650     | 8152 | 8655 | 9157 | 9659 | 502 |
| 865 | 937.2671 | 3172 | 3674 | 4176 | 4677 | 502 |
| 866 | 7686     | 8187 | 8688 | 9189 | 9690 | 501 |
| 867 | 938.2695 | 3195 | 3696 | 4196 | 4697 | 501 |
| 868 | 7698     | 8198 | 8698 | 9198 | 9698 | 500 |
| 869 | 939.2696 | 3195 | 3695 | 4194 | 4693 | 499 |
| 870 | 7688     | 8187 | 8685 | 9184 | 9683 | 499 |
| 871 | 940.2674 | 3172 | 3670 | 4169 | 4667 | 498 |
| 872 | 7654     | 8152 | 8650 | 9147 | 9645 | 498 |
| 873 | 941.2629 | 3126 | 3623 | 4120 | 4617 | 497 |
| 874 | 7598     | 8095 | 8591 | 9088 | 9584 | 497 |
| 875 | 942.2562 | 3053 | 3553 | 4049 | 4545 | 496 |
| 876 | 7519     | 8015 | 8510 | 9005 | 9501 | 496 |
| 877 | 943.2471 | 2966 | 3461 | 3956 | 4450 | 495 |
| 878 | 7418     | 7912 | 8406 | 8900 | 9395 | 494 |
| 879 | 944.2358 | 2852 | 3346 | 3840 | 4333 | 494 |
| 880 | 7294     | 7787 | 8280 | 8773 | 9266 | 493 |
| 881 | 945.2223 | 2716 | 3208 | 3701 | 4193 | 493 |
| 882 | 7147     | 7639 | 8131 | 8623 | 9115 | 492 |
| 883 | 946.2066 | 2557 | 3049 | 3540 | 4031 | 492 |
| 884 | 6978     | 7469 | 7960 | 8451 | 8942 | 491 |
| 885 | 947.1886 | 2376 | 2866 | 3357 | 3847 | 490 |
| 886 | 6787     | 7277 | 7767 | 8257 | 8747 | 490 |
| 887 | 948.1684 | 2173 | 2662 | 3151 | 3641 | 489 |
| 888 | 6574     | 7063 | 7552 | 8040 | 8529 | 489 |
| 889 | 949.1460 | 1948 | 2436 | 2924 | 3412 | 488 |
| 890 | 6339     | 6827 | 7315 | 7802 | 8290 | 488 |

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| 891 | 949.8777 | 9264 | 9752 | 0239 | 0726 | 487 |
| 892 | 950.3649 | 4135 | 4622 | 5109 | 5596 | 487 |
| 893 | 8515     | 9001 | 9487 | 9973 | 0459 | 486 |
| 894 | 951.3375 | 3861 | 4347 | 4832 | 5318 | 486 |
| 895 | 8230     | 8716 | 9201 | 9686 | 0171 | 485 |
| 896 | 952.3080 | 3565 | 4049 | 4534 | 5018 | 485 |
| 897 | 7924     | 8409 | 8893 | 9377 | 9861 | 484 |
| 898 | 953.2763 | 3247 | 3731 | 4214 | 4697 | 484 |
| 899 | 7597     | 8080 | 8563 | 9046 | 9529 | 483 |
| 900 | 954.2425 | 2908 | 3390 | 3873 | 4355 | 482 |
| 901 | 7248     | 7730 | 8212 | 8694 | 9176 | 482 |
| 902 | 955.2065 | 2547 | 3028 | 3510 | 3991 | 481 |
| 903 | 6878     | 7358 | 7839 | 8320 | 8801 | 481 |
| 904 | 956.1684 | 2165 | 2645 | 3125 | 3606 | 480 |
| 905 | 6486     | 6966 | 7445 | 7925 | 8405 | 480 |
| 906 | 957.1282 | 1761 | 2241 | 2720 | 3199 | 479 |
| 907 | 6073     | 6552 | 7030 | 7509 | 7988 | 479 |
| 908 | 958.0858 | 1337 | 1815 | 2293 | 2771 | 478 |
| 909 | 5639     | 6117 | 6594 | 7072 | 7549 | 478 |
| 910 | 959.0414 | 0891 | 1368 | 1845 | 2322 | 477 |
| 911 | 5184     | 5660 | 6137 | 6614 | 7090 | 477 |
| 912 | 9948     | 0425 | 0901 | 1377 | 1853 | 476 |
| 913 | 960.4708 | 5183 | 5659 | 6135 | 6610 | 476 |
| 914 | 9462     | 9937 | 0412 | 0887 | 1362 | 475 |
| 915 | 961.4211 | 4686 | 5160 | 5635 | 6109 | 474 |
| 916 | 8955     | 9429 | 9903 | 0377 | 0851 | 474 |
| 917 | 962.3693 | 4167 | 4640 | 5114 | 5587 | 473 |
| 918 | 8427     | 8900 | 9373 | 9846 | 0319 | 473 |
| 919 | 963.3155 | 3628 | 4100 | 4573 | 5045 | 472 |
| 920 | 7878     | 8350 | 8822 | 9294 | 9766 | 472 |
| 921 | 964.2596 | 3068 | 3539 | 4011 | 4482 | 471 |
| 922 | 7309     | 7780 | 8251 | 8722 | 9193 | 471 |
| 923 | 965.2017 | 2488 | 2958 | 3428 | 3899 | 470 |
| 924 | 6720     | 7190 | 7660 | 8130 | 8599 | 470 |
| 925 | 966.7417 | 1887 | 2356 | 2826 | 3295 | 469 |
| 926 | 6110     | 6579 | 7048 | 7517 | 7985 | 469 |
| 927 | 967.0797 | 1266 | 1734 | 2203 | 2671 | 468 |
| 928 | 5480     | 5948 | 6416 | 6884 | 7351 | 468 |

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| 891 | 950.1213 | 1701 | 2188 | 2675 | 3162 | 487 |
| 892 | 6082     | 6569 | 7055 | 7542 | 8028 | 487 |
| 893 | 951.0946 | 1432 | 1918 | 2404 | 2889 | 486 |
| 894 | 5803     | 6289 | 6774 | 7260 | 7745 | 486 |
| 895 | 952.0656 | 1141 | 1626 | 2111 | 2595 | 485 |
| 896 | 5503     | 5987 | 6472 | 6956 | 7440 | 485 |
| 897 | 953.0345 | 0828 | 1312 | 1796 | 2280 | 484 |
| 898 | 5181     | 5664 | 6147 | 6631 | 7114 | 484 |
| 899 | 954.0012 | 0494 | 0977 | 1460 | 1943 | 483 |
| 900 | 4837     | 5319 | 5802 | 6284 | 6766 | 482 |
| 901 | 9657     | 0139 | 0621 | 1102 | 1584 | 482 |
| 902 | 655.4472 | 4953 | 5434 | 5916 | 6397 | 481 |
| 903 | 9282     | 9762 | 0243 | 0723 | 1204 | 481 |
| 904 | 956.4086 | 4566 | 5046 | 5526 | 6006 | 480 |
| 905 | 8885     | 9364 | 9844 | 0323 | 0803 | 480 |
| 906 | 957.3678 | 4157 | 4636 | 5115 | 5594 | 479 |
| 907 | 8466     | 8945 | 9423 | 9902 | 0380 | 479 |
| 908 | 958.3249 | 3727 | 4205 | 4683 | 5161 | 478 |
| 909 | 8027     | 8505 | 8982 | 9459 | 9937 | 478 |
| 910 | 859.2800 | 3276 | 3753 | 4230 | 4707 | 477 |
| 911 | 7567     | 8043 | 8520 | 8996 | 9472 | 477 |
| 912 | 960.2329 | 2805 | 3281 | 3756 | 4232 | 476 |
| 913 | 7086     | 7561 | 8036 | 8512 | 8987 | 476 |
| 914 | 961.1837 | 2312 | 2787 | 3262 | 3736 | 475 |
| 915 | 6583     | 7058 | 7532 | 8006 | 8481 | 474 |
| 916 | 962.1325 | 1799 | 2272 | 2746 | 3220 | 474 |
| 917 | 6061     | 6534 | 7007 | 7481 | 7954 | 473 |
| 918 | 963.0792 | 1264 | 1737 | 2210 | 2683 | 473 |
| 919 | 5517     | 5990 | 6462 | 6934 | 7406 | 472 |
| 920 | 964.0238 | 0710 | 1181 | 1653 | 2125 | 472 |
| 921 | 4953     | 5425 | 5896 | 6367 | 6838 | 471 |
| 922 | 9664     | 0135 | 0605 | 1076 | 1546 | 471 |
| 923 | 965.4369 | 4839 | 5309 | 5780 | 6250 | 470 |
| 924 | 9069     | 9539 | 0009 | 0478 | 0948 | 470 |
| 925 | 966.3764 | 4233 | 4703 | 5172 | 5641 | 469 |
| 926 | 8454     | 8923 | 9392 | 9860 | 0329 | 469 |
| 927 | 967.3139 | 3607 | 4076 | 4544 | 5012 | 468 |
| 928 | 7819     | 8287 | 8754 | 9222 | 9690 | 468 |

| N°  | °        | 1    | 2    | 3    | 4    | Diff |
|-----|----------|------|------|------|------|------|
| 929 | 968.0157 | 0625 | 1092 | 1559 | 2027 | 467  |
| 930 | 4829     | 5296 | 5763 | 6230 | 6697 | 467  |
| 931 | 9497     | 9961 | 0430 | 0896 | 1362 | 466  |
| 932 | 969.4159 | 4625 | 5091 | 5557 | 6023 | 466  |
| 933 | 8816     | 9282 | 9747 | 0213 | 0678 | 465  |
| 934 | 970.3469 | 3934 | 4399 | 4863 | 5328 | 465  |
| 935 | 8116     | 8581 | 9045 | 9509 | 9974 | 464  |
| 936 | 971.2758 | 3222 | 3686 | 4150 | 4614 | 464  |
| 937 | 7396     | 7859 | 8323 | 8786 | 9249 | 463  |
| 938 | 972.2028 | 2491 | 2954 | 3417 | 3880 | 463  |
| 939 | 6656     | 7118 | 7581 | 8043 | 8506 | 462  |
| 940 | 973.1279 | 1741 | 2202 | 2664 | 3120 | 462  |
| 941 | 5896     | 6358 | 6819 | 7281 | 7742 | 461  |
| 942 | 974.0509 | 0970 | 1431 | 1892 | 2353 | 461  |
| 943 | 5117     | 5577 | 6038 | 6498 | 6959 | 460  |
| 944 | 9720     | 0180 | 0640 | 1100 | 1560 | 460  |
| 945 | 975.4318 | 4778 | 5237 | 5697 | 6156 | 459  |
| 946 | 8911     | 9370 | 9829 | 0288 | 0747 | 459  |
| 947 | 976.3500 | 3958 | 4417 | 4875 | 5334 | 458  |
| 948 | 8083     | 8541 | 9000 | 9458 | 9915 | 458  |
| 949 | 977.2662 | 3120 | 3577 | 4035 | 4492 | 457  |
| 950 | 7236     | 7693 | 8150 | 8607 | 9064 | 457  |
| 951 | 978.1805 | 2261 | 2718 | 3175 | 3631 | 457  |
| 952 | 6369     | 6826 | 7282 | 7738 | 8194 | 456  |
| 953 | 979.0929 | 1385 | 1840 | 2296 | 2751 | 456  |
| 954 | 5414     | 5939 | 6394 | 6849 | 7304 | 455  |
| 955 | 980.0034 | 0488 | 0943 | 1398 | 1852 | 455  |
| 956 | 4579     | 5033 | 5487 | 5942 | 6396 | 454  |
| 957 | 9119     | 9573 | 0027 | 0481 | 0934 | 454  |
| 958 | 981.3655 | 4108 | 4502 | 5015 | 5468 | 453  |
| 959 | 8186     | 8639 | 9092 | 9544 | 9997 | 453  |
| 960 | 982.2712 | 3165 | 3617 | 4069 | 4522 | 452  |
| 961 | 7234     | 7686 | 8138 | 8589 | 9041 | 452  |
| 962 | 983.1751 | 2202 | 2654 | 3105 | 3556 | 451  |
| 963 | 6263     | 6714 | 7165 | 7616 | 8066 | 451  |
| 964 | 984.0770 | 1221 | 1671 | 2122 | 2572 | 450  |
| 965 | 5273     | 5723 | 6173 | 6623 | 7073 | 450  |
| 966 | 9771     | 0221 | 0670 | 1120 | 1569 | 449  |

| Nº  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 929 | 968.2494 | 2961 | 3428 | 3895 | 4362 | 467 |
| 930 | 7164     | 7630 | 8097 | 8564 | 9030 | 467 |
| 931 | 969.1829 | 2295 | 2751 | 3227 | 3693 | 466 |
| 932 | 6488     | 6954 | 7420 | 7885 | 8351 | 466 |
| 933 | 970.1143 | 1608 | 2074 | 2539 | 3004 | 465 |
| 934 | 5793     | 6258 | 6722 | 7187 | 7652 | 465 |
| 935 | 971.0438 | 0902 | 1366 | 1830 | 2294 | 464 |
| 936 | 5078     | 5542 | 6005 | 6469 | 6932 | 464 |
| 937 | 9713     | 0176 | 0639 | 1102 | 1565 | 463 |
| 938 | 972.4343 | 4805 | 5268 | 5731 | 6193 | 463 |
| 939 | 8968     | 9430 | 9892 | 0354 | 0816 | 462 |
| 940 | 973.3588 | 4950 | 4511 | 4973 | 5435 | 462 |
| 941 | 8203     | 8664 | 9126 | 9587 | 0048 | 461 |
| 942 | 974.2814 | 3274 | 3735 | 4196 | 4656 | 461 |
| 943 | 7419     | 7879 | 8340 | 8800 | 9260 | 460 |
| 944 | 975.2020 | 2479 | 2939 | 3399 | 3858 | 460 |
| 945 | 6615     | 7075 | 7534 | 7993 | 8452 | 459 |
| 946 | 976.1206 | 1695 | 2124 | 2582 | 3041 | 459 |
| 947 | 5792     | 6251 | 6709 | 7167 | 7625 | 458 |
| 948 | 977.0373 | 0831 | 1289 | 1747 | 2204 | 458 |
| 949 | 4950     | 5407 | 5864 | 6322 | 6779 | 457 |
| 950 | 9521     | 9978 | 0435 | 0892 | 1348 | 457 |
| 951 | 978.4088 | 4544 | 5001 | 5457 | 5913 | 457 |
| 952 | 8650     | 9106 | 9562 | 0017 | 0473 | 456 |
| 953 | 979.3205 | 3662 | 4118 | 4573 | 5028 | 456 |
| 954 | 7759     | 8214 | 8669 | 9124 | 9579 | 455 |
| 955 | 980.2307 | 2761 | 3216 | 3670 | 4125 | 455 |
| 956 | 6850     | 7304 | 7758 | 8212 | 8666 | 454 |
| 957 | 981.1388 | 1841 | 2295 | 2748 | 3202 | 454 |
| 958 | 5921     | 6374 | 6827 | 7280 | 7733 | 453 |
| 959 | 982.0450 | 0902 | 1355 | 1807 | 2260 | 453 |
| 960 | 4974     | 5426 | 5878 | 6330 | 6782 | 452 |
| 961 | 983.9493 | 9945 | 0396 | 0848 | 1299 | 452 |
| 962 | 4007     | 4459 | 4910 | 5361 | 5812 | 451 |
| 963 | 8517     | 8968 | 9419 | 9869 | 0320 | 451 |
| 964 | 984.3022 | 3473 | 3923 | 4373 | 4823 | 450 |
| 965 | 7523     | 7973 | 8422 | 8872 | 9322 | 450 |
| 966 | 985.2019 | 2468 | 2917 | 3366 | 3816 | 449 |

| N°  |          | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 967 | 985.4265 | 4714 | 5163 | 5612 | 6061 | 449 |
| 968 | 8754     | 9202 | 9651 | 6099 | 0548 | 449 |
| 969 | 986.3238 | 3686 | 4134 | 4582 | 5030 | 448 |
| 970 | 7717     | 8165 | 8613 | 9060 | 9508 | 448 |
| 971 | 987.2192 | 2640 | 3087 | 3534 | 3981 | 447 |
| 972 | 6663     | 7109 | 7556 | 8003 | 8450 | 447 |
| 973 | 988.1128 | 1575 | 2021 | 2467 | 2913 | 446 |
| 974 | 5590     | 6035 | 6481 | 6927 | 7373 | 446 |
| 975 | 989.0046 | 0492 | 0937 | 1382 | 1828 | 445 |
| 976 | 4498     | 4943 | 5388 | 5833 | 6278 | 445 |
| 977 | 8946     | 9390 | 9835 | 0279 | 0723 | 444 |
| 978 | 990.3389 | 3833 | 4277 | 4721 | 5164 | 444 |
| 979 | 7827     | 8271 | 8714 | 9158 | 9601 | 443 |
| 980 | 991.2261 | 2704 | 3147 | 3590 | 4033 | 443 |
| 981 | 6690     | 7133 | 7575 | 8018 | 8461 | 442 |
| 982 | 992.1115 | 1557 | 1999 | 2441 | 2884 | 442 |
| 983 | 5535     | 5977 | 6419 | 6860 | 7302 | 442 |
| 984 | 9951     | 0392 | 0834 | 1275 | 1716 | 441 |
| 985 | 993.4362 | 4803 | 5244 | 5685 | 6126 | 441 |
| 986 | 8769     | 9209 | 9650 | 0090 | 0531 | 440 |
| 987 | 994.3171 | 3611 | 4051 | 4491 | 4931 | 440 |
| 988 | 7569     | 8009 | 8448 | 8888 | 9327 | 440 |
| 989 | 995.1963 | 2402 | 2841 | 3280 | 3719 | 439 |
| 990 | 6352     | 6791 | 7229 | 7668 | 8106 | 439 |
| 991 | 996.0736 | 1175 | 1613 | 2051 | 2489 | 438 |
| 992 | 5117     | 5554 | 5992 | 6430 | 6867 | 438 |
| 993 | 9492     | 9930 | 0367 | 0804 | 1241 | 437 |
| 994 | 997.3864 | 4301 | 4737 | 5174 | 5611 | 437 |
| 995 | 8231     | 8667 | 9104 | 9540 | 9976 | 436 |
| 996 | 998.2593 | 3029 | 3465 | 3901 | 4337 | 436 |
| 997 | 6951     | 7387 | 7823 | 8258 | 8694 | 435 |
| 998 | 999.1305 | 1740 | 2176 | 2611 | 3046 | 435 |
| 999 | 5655     | 6089 | 6524 | 6959 | 7393 | 434 |



| Nº  | 5        | 6    | 7    | 8    | 9    | Dif |
|-----|----------|------|------|------|------|-----|
| 967 | 985.6510 | 6959 | 7407 | 7856 | 8305 | 449 |
| 968 | 986.0996 | 1445 | 1893 | 2341 | 2790 | 449 |
| 969 | 5478     | 5926 | 6374 | 6822 | 7270 | 448 |
| 970 | 9955     | 0403 | 0850 | 1298 | 1745 | 448 |
| 971 | 987.4428 | 4875 | 5322 | 5769 | 6216 | 447 |
| 972 | 8896     | 9343 | 9789 | 0236 | 0682 | 447 |
| 973 | 988.3360 | 3806 | 4252 | 4698 | 5144 | 446 |
| 974 | 7818     | 8264 | 8710 | 9155 | 9601 | 446 |
| 975 | 989.2273 | 2718 | 3163 | 3608 | 4053 | 445 |
| 976 | 6722     | 7167 | 7612 | 8057 | 8501 | 445 |
| 977 | 990.1168 | 1612 | 2056 | 2500 | 2944 | 444 |
| 978 | 5608     | 6052 | 6496 | 6940 | 7383 | 444 |
| 979 | 991.0044 | 0488 | 0931 | 1374 | 1818 | 443 |
| 980 | 4476     | 4919 | 5362 | 5805 | 6247 | 443 |
| 981 | 8903     | 9345 | 9788 | 0230 | 0673 | 442 |
| 982 | 992.3326 | 3768 | 4210 | 4651 | 5093 | 442 |
| 983 | 7744     | 8185 | 8627 | 9068 | 9510 | 442 |
| 984 | 993.2157 | 2598 | 3039 | 3480 | 3921 | 441 |
| 985 | 6566     | 7007 | 7447 | 7888 | 8329 | 441 |
| 986 | 994.0971 | 1411 | 1851 | 2291 | 2731 | 440 |
| 987 | 5371     | 5811 | 6250 | 6690 | 7130 | 440 |
| 988 | 9767     | 0206 | 0645 | 1085 | 1524 | 440 |
| 989 | 995.4158 | 4597 | 5036 | 5474 | 5913 | 439 |
| 990 | 8545     | 8983 | 9422 | 9860 | 0298 | 439 |
| 991 | 996.2927 | 3365 | 3803 | 4241 | 4679 | 438 |
| 992 | 7305     | 7743 | 8180 | 8618 | 9055 | 438 |
| 993 | 997.1679 | 2116 | 2553 | 2990 | 3427 | 437 |
| 994 | 6048     | 6484 | 6921 | 7358 | 7794 | 437 |
| 995 | 998.0413 | 0849 | 1285 | 1721 | 2157 | 436 |
| 996 | 4773     | 5209 | 5645 | 6080 | 6516 | 436 |
| 997 | 9129     | 9564 | 0000 | 0435 | 0870 | 435 |
| 998 | 999.3481 | 3916 | 4350 | 4785 | 5220 | 435 |
| 999 | 7828     | 8262 | 8697 | 9131 | 9566 | 434 |

| N°  | ●        | 1    | 2    | 3    | 4    | Dif |
|-----|----------|------|------|------|------|-----|
| 967 | 985.4265 | 4714 | 5163 | 5612 | 6061 | 449 |
| 968 | 8754     | 9202 | 9651 | 6099 | 0548 | 449 |
| 969 | 986.3238 | 3686 | 4134 | 4582 | 5030 | 448 |
| 970 | 7717     | 8165 | 8613 | 9060 | 9508 | 448 |
| 971 | 987.2192 | 2640 | 3087 | 3534 | 3981 | 447 |
| 972 | 6663     | 7109 | 7556 | 8003 | 8450 | 447 |
| 973 | 988.1128 | 1575 | 2021 | 2467 | 2913 | 446 |
| 974 | 5590     | 6035 | 6481 | 6927 | 7373 | 446 |
| 975 | 989.0046 | 0492 | 0937 | 1382 | 1828 | 445 |
| 976 | 4498     | 4943 | 5388 | 5833 | 6278 | 445 |
| 977 | 8946     | 9390 | 9835 | 0279 | 0723 | 444 |
| 978 | 990.3389 | 3833 | 4277 | 4721 | 5164 | 444 |
| 979 | 7827     | 8271 | 8714 | 9158 | 9601 | 443 |
| 980 | 991.2261 | 2704 | 3147 | 3590 | 4033 | 443 |
| 981 | 6690     | 7133 | 7575 | 8018 | 8461 | 442 |
| 982 | 992.1115 | 1557 | 1999 | 2441 | 2884 | 442 |
| 983 | 5535     | 5977 | 6419 | 6860 | 7302 | 442 |
| 984 | 9951     | 0392 | 0834 | 1275 | 1716 | 441 |
| 985 | 993.4362 | 4803 | 5244 | 5685 | 6126 | 441 |
| 986 | 8769     | 9209 | 9650 | 0090 | 0531 | 440 |
| 987 | 994.3171 | 3611 | 4051 | 4491 | 4931 | 440 |
| 988 | 7569     | 8009 | 8448 | 8888 | 9327 | 440 |
| 989 | 995.1963 | 2402 | 2841 | 3280 | 3719 | 439 |
| 990 | 6352     | 6791 | 7229 | 7668 | 8106 | 439 |
| 991 | 996.0736 | 1175 | 1613 | 2051 | 2489 | 438 |
| 992 | 5117     | 5554 | 5992 | 6430 | 6867 | 438 |
| 993 | 9492     | 9930 | 0367 | 0804 | 1241 | 437 |
| 994 | 997.3864 | 4301 | 4737 | 5174 | 5611 | 437 |
| 995 | 8231     | 8667 | 9104 | 9540 | 9976 | 436 |
| 996 | 998.2593 | 3029 | 3465 | 3901 | 4337 | 436 |
| 997 | 6951     | 7387 | 7823 | 8258 | 8694 | 435 |
| 998 | 999.1305 | 1740 | 2176 | 2611 | 3046 | 435 |
| 999 | 5655     | 6089 | 6524 | 6959 | 7393 | 434 |

# SINES AND TANGENTS.

0 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 7,9408419 | 9,9999835 | 7,9408584 | 12,0591416 | 30   |
| 31   | 7,9550819 | 9,9999823 | 7,9550996 | 12,0449004 | 29   |
| 32   | 7,9688698 | 9,9999812 | 7,9688886 | 12,0311114 | 28   |
| 33   | 7,9822334 | 9,9999800 | 7,9822534 | 12,0177466 | 27   |
| 34   | 7,9951980 | 9,9999788 | 7,9952192 | 12,0047808 | 26   |
| 35   | 8,0077867 | 9,9999775 | 8,0078092 | 11,9921908 | 25   |
| 36   | 8,0200207 | 9,9999762 | 8,0208445 | 11,9799555 | 24   |
| 37   | 8,0319195 | 9,9999748 | 8,0319446 | 11,9680554 | 23   |
| 38   | 8,0435009 | 9,9999735 | 8,0435274 | 11,9564726 | 22   |
| 39   | 8,0547814 | 9,9999721 | 8,0548094 | 11,9451906 | 21   |
| 40   | 8,0657763 | 9,9999706 | 8,0658057 | 11,9341943 | 20   |
| 41   | 8,0764997 | 9,9999691 | 8,0765306 | 11,9234694 | 19   |
| 42   | 8,0869646 | 9,9999676 | 8,0869970 | 11,9130030 | 18   |
| 43   | 8,0971822 | 9,9999660 | 8,0972172 | 11,9027828 | 17   |
| 44   | 8,1071669 | 9,9999644 | 8,1072025 | 11,8927975 | 16   |
| 45   | 8,1169262 | 9,9999628 | 8,1169634 | 11,8830366 | 15   |
| 46   | 8,1264710 | 9,9999611 | 8,1265099 | 11,8734901 | 14   |
| 47   | 8,1358104 | 9,9999594 | 8,1358510 | 11,8641490 | 13   |
| 48   | 8,1449532 | 9,9999577 | 8,1449956 | 11,8550044 | 12   |
| 49   | 8,1539075 | 9,9999559 | 8,1539516 | 11,8460484 | 11   |
| 50   | 8,1626808 | 9,9999541 | 8,1627267 | 11,8372733 | 10   |
| 51   | 8,1712804 | 9,9999522 | 8,1713282 | 11,8286718 | 9    |
| 52   | 8,1797129 | 9,9999503 | 8,1797626 | 11,8202374 | 8    |
| 53   | 8,1879848 | 9,9999484 | 8,1880354 | 11,8119636 | 7    |
| 54   | 8,1961020 | 9,9999464 | 8,1961556 | 11,8038444 | 6    |
| 55   | 8,2040703 | 9,9999444 | 8,2041259 | 11,7958741 | 5    |
| 56   | 8,2118949 | 9,9999424 | 8,2119526 | 11,7880474 | 4    |
| 57   | 8,2195811 | 9,9999403 | 8,2196408 | 11,7803592 | 3    |
| 58   | 8,2271335 | 9,9999382 | 8,2271953 | 11,7728047 | 2    |
| 59   | 8,2345568 | 9,9999360 | 8,2346208 | 11,7653792 | 1    |
| 60   | 8,2418553 | 9,9999338 | 8,2419215 | 11,7580785 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

89 Degrees.

# 1 Degree.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 8,2418553 | 9,9999338 | 8,2419215 | 11,7580785 | 60   |
| 1    | 8,2490332 | 9,9999316 | 8,2491015 | 11,7508985 | 59   |
| 2    | 8,2560943 | 9,9999294 | 8,2561649 | 11,7438351 | 58   |
| 3    | 8,2630424 | 9,9999271 | 8,2631153 | 11,7368847 | 57   |
| 4    | 8,2698810 | 9,9999247 | 8,2699563 | 11,7300437 | 56   |
| 5    | 8,2766156 | 9,9999224 | 8,2766912 | 11,7233088 | 55   |
| 6    | 8,2832434 | 9,9999200 | 8,2833234 | 11,7166766 | 54   |
| 7    | 8,2897734 | 9,9999175 | 8,2898559 | 11,7101441 | 53   |
| 8    | 8,2962067 | 9,9999150 | 8,2962917 | 11,7037083 | 52   |
| 9    | 8,3025460 | 9,9999125 | 8,3026335 | 11,6973665 | 51   |
| 10   | 8,3087941 | 9,9999100 | 8,3088842 | 11,6911158 | 50   |
| 11   | 8,3149536 | 9,9999074 | 8,3150462 | 11,6849538 | 49   |
| 12   | 8,3210269 | 9,9999047 | 8,3211221 | 11,6788779 | 48   |
| 13   | 8,3270163 | 9,9999021 | 8,3271143 | 11,6728857 | 47   |
| 14   | 8,3329243 | 9,9998994 | 8,3330249 | 11,6669751 | 46   |
| 15   | 8,3387529 | 9,9998966 | 8,3388563 | 11,6611437 | 45   |
| 16   | 8,3445043 | 9,9998939 | 8,3446105 | 11,6553895 | 44   |
| 17   | 8,3501805 | 9,9998911 | 8,3502895 | 11,6497105 | 43   |
| 18   | 8,3557835 | 9,9998882 | 8,3558953 | 11,6441047 | 42   |
| 19   | 8,3613150 | 9,9998853 | 8,3614297 | 11,6385703 | 41   |
| 20   | 8,3667769 | 9,9998824 | 8,3668945 | 11,6331055 | 40   |
| 21   | 8,3721710 | 9,9998794 | 8,3722915 | 11,6277085 | 39   |
| 22   | 8,3774988 | 9,9998764 | 8,3776223 | 11,6223777 | 38   |
| 23   | 8,3827620 | 9,9998734 | 8,3828886 | 11,6171114 | 37   |
| 24   | 8,3879622 | 9,9998703 | 8,3880918 | 11,6119082 | 36   |
| 25   | 8,3931008 | 9,9998672 | 8,3932330 | 11,6067664 | 35   |
| 26   | 8,3981793 | 9,9998641 | 8,3983152 | 11,6016848 | 34   |
| 27   | 8,4031990 | 9,9998609 | 8,4033381 | 11,5966619 | 33   |
| 28   | 8,4081614 | 9,9998577 | 8,4083037 | 11,5916963 | 32   |
| 29   | 8,4130676 | 9,9998544 | 8,4132132 | 11,5867868 | 31   |
| 30   | 8,4179190 | 9,9998512 | 8,4180679 | 11,5819321 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tang.      | Min. |

# 88 Degrees.

# 1 Degree.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 8,4179190 | 9,9998512 | 8,4180679 | 11,5819321 | 30   |
| 31   | 8,4227168 | 9,9998478 | 8,4228690 | 11,5771310 | 29   |
| 32   | 8,4274621 | 9,9998445 | 8,4276176 | 11,5723824 | 28   |
| 33   | 8,4321561 | 9,9998411 | 8,4323150 | 11,5676850 | 27   |
| 34   | 8,4367999 | 9,9998367 | 8,4369622 | 11,5630378 | 26   |
| 35   | 8,4413944 | 9,9998342 | 8,4415603 | 11,5584397 | 25   |
| 36   | 8,4459409 | 9,9998306 | 8,4461103 | 11,5538897 | 24   |
| 37   | 8,4504402 | 9,9998271 | 8,4506131 | 11,5493869 | 23   |
| 38   | 8,4548934 | 9,9998235 | 8,4550699 | 11,5449301 | 22   |
| 39   | 8,4593013 | 9,9998199 | 8,4594814 | 11,5405186 | 21   |
| 40   | 8,4636649 | 9,9998162 | 8,4638486 | 11,5361514 | 20   |
| 41   | 8,4679850 | 9,9998125 | 8,4681725 | 11,5318275 | 19   |
| 42   | 8,4722626 | 9,9998088 | 8,4724538 | 11,5275462 | 18   |
| 43   | 8,4764984 | 9,9998050 | 8,4766933 | 11,5233067 | 17   |
| 44   | 8,4806932 | 9,9998012 | 8,4808920 | 11,5191080 | 16   |
| 45   | 8,4848479 | 9,9997974 | 8,4850505 | 11,5149495 | 15   |
| 46   | 8,4889632 | 9,9997935 | 8,4891696 | 11,5108304 | 14   |
| 47   | 8,4930398 | 9,9997896 | 8,4932502 | 11,5067498 | 13   |
| 48   | 8,4970784 | 9,9997856 | 8,4972928 | 11,5027072 | 12   |
| 49   | 8,5010798 | 9,9997817 | 8,5012982 | 11,4987018 | 11   |
| 50   | 8,5050447 | 9,9997776 | 8,5052671 | 11,4947329 | 10   |
| 51   | 8,5089736 | 9,9997736 | 8,5092001 | 11,4907999 | 9    |
| 52   | 8,5128673 | 9,9997695 | 8,5130978 | 11,4869022 | 8    |
| 53   | 8,5167264 | 9,9997653 | 8,5169610 | 11,4830390 | 7    |
| 54   | 8,5205514 | 9,9997612 | 8,5207902 | 11,4792098 | 6    |
| 55   | 8,5243430 | 9,9997570 | 8,5245860 | 11,4754140 | 5    |
| 56   | 8,5281017 | 9,9997527 | 8,5283490 | 11,4716510 | 4    |
| 57   | 8,5318281 | 9,9997484 | 8,5320797 | 11,4679203 | 3    |
| 58   | 8,5355228 | 9,9997441 | 8,5357787 | 11,4642213 | 2    |
| 59   | 8,5391863 | 9,9997398 | 8,5394466 | 11,4605534 | 1    |
| 60   | 8,5428192 | 9,9997354 | 8,5430838 | 11,4569162 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 88 Degrees.

| 2 Degrees.  |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 8,5428192 | 9,9997354 | 8,5430838 | 11,4569162 | 60   |
| 1           | 8,5464218 | 9,9997309 | 8,5466909 | 11,4533091 | 59   |
| 2           | 8,5499948 | 9,9997265 | 8,5502683 | 11,4497317 | 58   |
| 3           | 8,5535386 | 9,9997220 | 8,5538166 | 11,4461834 | 57   |
| 4           | 8,5570536 | 9,9997174 | 8,5573362 | 11,4426638 | 56   |
| 5           | 8,5605404 | 9,9997128 | 8,5608276 | 11,4391724 | 55   |
| 6           | 8,5639994 | 9,9997082 | 8,5642912 | 11,4357088 | 54   |
| 7           | 8,5674310 | 9,9997036 | 8,5677275 | 11,4322725 | 53   |
| 8           | 8,5708357 | 9,9996989 | 8,5711368 | 11,4288632 | 52   |
| 9           | 8,5742139 | 9,9996942 | 8,5745197 | 11,4254803 | 51   |
| 10          | 8,5775660 | 9,9996894 | 8,5778766 | 11,4221234 | 50   |
| 11          | 8,5808923 | 9,9996846 | 8,5812077 | 11,4187923 | 49   |
| 12          | 8,5841933 | 9,9996798 | 8,5845136 | 11,4154864 | 48   |
| 13          | 8,5874694 | 9,9996749 | 8,5877945 | 11,4122055 | 47   |
| 14          | 8,5907209 | 9,9996700 | 8,5910509 | 11,4089491 | 46   |
| 15          | 8,5939481 | 9,9996650 | 8,5942832 | 11,4057168 | 45   |
| 16          | 8,5971517 | 9,9996601 | 8,5974917 | 11,4025083 | 44   |
| 17          | 8,6003317 | 9,9996550 | 8,6006767 | 11,3993233 | 43   |
| 18          | 8,6034886 | 9,9996500 | 8,603886  | 11,3961614 | 42   |
| 19          | 8,6066220 | 9,9996449 | 8,6069777 | 11,3930223 | 41   |
| 20          | 8,609734  | 9,9996398 | 8,6100943 | 11,3899057 | 40   |
| 21          | 8,612823  | 9,9996346 | 8,6131889 | 11,3868111 | 39   |
| 22          | 8,6158910 | 9,9996294 | 8,6162616 | 11,3837384 | 38   |
| 23          | 8,6189369 | 9,9996242 | 8,6193127 | 11,3806873 | 37   |
| 24          | 8,6219616 | 9,9996189 | 8,6223427 | 11,3776573 | 36   |
| 25          | 8,6249653 | 9,9996136 | 8,6253518 | 11,3746482 | 35   |
| 26          | 8,6279484 | 9,9996082 | 8,6283402 | 11,3716598 | 34   |
| 27          | 8,6309111 | 9,9996028 | 8,6313083 | 11,3686917 | 33   |
| 28          | 8,6338537 | 9,9995974 | 8,6342563 | 11,3657437 | 32   |
| 29          | 8,6367764 | 9,9995919 | 8,6371845 | 11,3628155 | 31   |
| 30          | 8,6396796 | 9,9995865 | 8,6400931 | 11,3599069 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 87 Degrees. |           |           |           |            |      |

# 2 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 8,6396795 | 9,9995865 | 8,6400934 | 11,3599069 | 30   |
| 31   | 8,6425634 | 9,9995809 | 8,6429825 | 11,3570175 | 29   |
| 32   | 8,6454282 | 9,9995753 | 8,6458528 | 11,3541472 | 28   |
| 33   | 8,6482742 | 9,9995697 | 8,6487044 | 11,3512956 | 27   |
| 34   | 8,6511016 | 9,9995641 | 8,6515375 | 11,3484625 | 26   |
| 35   | 8,6539107 | 9,9995584 | 8,6543522 | 11,3456478 | 25   |
| 36   | 8,6567017 | 9,9995527 | 8,6571490 | 11,3428510 | 24   |
| 37   | 8,6594748 | 9,9995469 | 8,6599279 | 11,3400721 | 23   |
| 38   | 8,6622303 | 9,9995411 | 8,6626891 | 11,3373109 | 22   |
| 39   | 8,6649684 | 9,9995353 | 8,6654331 | 11,3345669 | 21   |
| 40   | 8,6676893 | 9,9995295 | 8,6681598 | 11,3318402 | 20   |
| 41   | 8,6703932 | 9,9995236 | 8,6708697 | 11,3291303 | 19   |
| 42   | 8,6730804 | 9,9995176 | 8,6735628 | 11,3264372 | 18   |
| 43   | 8,6757510 | 9,9995116 | 8,6762393 | 11,3237607 | 17   |
| 44   | 8,6784052 | 9,9995056 | 8,6788996 | 11,3211004 | 16   |
| 45   | 8,6810433 | 9,9994996 | 8,6815437 | 11,3184563 | 15   |
| 46   | 8,6836654 | 9,9994935 | 8,6841719 | 11,3158281 | 14   |
| 47   | 8,6862718 | 9,9994874 | 8,6867844 | 11,3132156 | 13   |
| 48   | 8,6888625 | 9,9994812 | 8,6893813 | 11,3106187 | 12   |
| 49   | 8,6914379 | 9,9994750 | 8,6919629 | 11,3080371 | 11   |
| 50   | 8,6939980 | 9,9994688 | 8,6945292 | 11,3054708 | 10   |
| 51   | 8,6965431 | 9,9994625 | 8,6970806 | 11,3029194 | 9    |
| 52   | 8,6990734 | 9,9994562 | 8,6996172 | 11,3003828 | 8    |
| 53   | 8,7015889 | 9,9994498 | 8,7021390 | 11,2978610 | 7    |
| 54   | 8,7040899 | 9,9994435 | 8,7046465 | 11,2953535 | 6    |
| 55   | 8,7065766 | 9,9994370 | 8,7071395 | 11,2928605 | 5    |
| 56   | 8,7090490 | 9,9994306 | 8,7096135 | 11,2903815 | 4    |
| 57   | 8,7115075 | 9,9994241 | 8,7120834 | 11,2879166 | 3    |
| 58   | 8,7139520 | 9,9994176 | 8,7145345 | 11,2854655 | 2    |
| 59   | 8,7163829 | 9,9994110 | 8,7169719 | 11,2830281 | 1    |
| 60   | 8,7188002 | 9,9994044 | 8,7193958 | 11,2806042 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 87 Degrees.

| 3 Degrees.  |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 8,7188002 | 9,9994044 | 8,7193958 | 11,2806042 | 60   |
| 1           | 8,7212040 | 9,9993978 | 8,7218063 | 11,2781937 | 59   |
| 2           | 8,7235946 | 9,9993911 | 8,7242035 | 11,2757965 | 58   |
| 3           | 8,7259721 | 9,9993844 | 8,7265877 | 11,2734123 | 57   |
| 4           | 8,7283366 | 9,9993776 | 8,7289589 | 11,2710411 | 56   |
| 5           | 8,7306882 | 9,9993708 | 8,7313174 | 11,2686826 | 55   |
| 6           | 8,7330272 | 9,9993640 | 8,7336631 | 11,2663369 | 54   |
| 7           | 8,7353535 | 9,9993572 | 8,7359964 | 11,2640036 | 53   |
| 8           | 8,7376675 | 9,9993503 | 8,7383172 | 11,2616828 | 52   |
| 9           | 8,7399691 | 9,9993433 | 8,7406258 | 11,2593742 | 51   |
| 10          | 8,7422586 | 9,9993364 | 8,7429222 | 11,2570778 | 50   |
| 11          | 8,7445360 | 9,9993293 | 8,7452067 | 11,2547933 | 49   |
| 12          | 8,7468015 | 9,9993223 | 8,7474792 | 11,2525208 | 48   |
| 13          | 8,7490553 | 9,9993152 | 8,7497400 | 11,2502600 | 47   |
| 14          | 8,7512973 | 9,9993081 | 8,7519892 | 11,2480108 | 46   |
| 15          | 8,7535278 | 9,9993009 | 8,7542269 | 11,2457731 | 45   |
| 16          | 8,7557469 | 9,9992938 | 8,7564531 | 11,2435469 | 44   |
| 17          | 8,7579546 | 9,9992865 | 8,7586681 | 11,2413319 | 43   |
| 18          | 8,7601512 | 9,9992793 | 8,7608719 | 11,2391281 | 42   |
| 19          | 8,7623366 | 9,9992720 | 8,7630647 | 11,2369353 | 41   |
| 20          | 8,7645111 | 9,9992646 | 8,7652465 | 11,2347535 | 40   |
| 21          | 8,7666747 | 9,9992572 | 8,7674175 | 11,2325825 | 39   |
| 22          | 8,7688275 | 9,9992498 | 8,7695777 | 11,2304223 | 38   |
| 23          | 8,7709697 | 9,9992424 | 8,7717274 | 11,2282726 | 37   |
| 24          | 8,7731014 | 9,9992349 | 8,7738665 | 11,2261335 | 36   |
| 25          | 8,7752226 | 9,9992274 | 8,7759952 | 11,2240048 | 35   |
| 26          | 8,7773334 | 9,9992198 | 8,7781136 | 11,2218864 | 34   |
| 27          | 8,7794340 | 9,9992122 | 8,7802218 | 11,2197782 | 33   |
| 28          | 8,7815244 | 9,9992046 | 8,7823199 | 11,2176801 | 32   |
| 29          | 8,7836048 | 9,9991969 | 8,7844079 | 11,2155921 | 31   |
| 30          | 8,7856753 | 9,9991892 | 8,7864861 | 11,2135139 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 86 Degrees. |           |           |           |            |      |



# 3 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 8,7856753 | 9,9991892 | 8,7864861 | 11,2135139 | 30   |
| 31   | 8,7877359 | 9,9991815 | 8,7885544 | 11,2114456 | 29   |
| 32   | 8,7897867 | 9,9991737 | 8,7906130 | 11,2093870 | 28   |
| 33   | 8,7918278 | 9,9991659 | 8,7926620 | 11,2073380 | 27   |
| 34   | 8,7938594 | 9,9991580 | 8,7947014 | 11,2052986 | 26   |
| 35   | 8,7958814 | 9,9991501 | 8,7967313 | 11,2032687 | 25   |
| 36   | 8,7978941 | 9,9991422 | 8,7987519 | 11,2012481 | 24   |
| 37   | 8,7998974 | 9,9991342 | 8,8007632 | 11,1992368 | 23   |
| 38   | 8,8018915 | 9,9991262 | 8,8027653 | 11,1972347 | 22   |
| 39   | 8,8038764 | 9,9991182 | 8,8047583 | 11,1952417 | 21   |
| 40   | 8,8058523 | 9,9991101 | 8,8067422 | 11,1932578 | 20   |
| 41   | 8,8078192 | 9,9991020 | 8,8087172 | 11,1912828 | 19   |
| 42   | 8,8097772 | 9,9990938 | 8,8106834 | 11,1893166 | 18   |
| 43   | 8,8117264 | 9,9990856 | 8,8126407 | 11,1873593 | 17   |
| 44   | 8,8136668 | 9,9990774 | 8,8145894 | 11,1854106 | 16   |
| 45   | 8,8155985 | 9,9990691 | 8,8165294 | 11,1834706 | 15   |
| 46   | 8,8175217 | 9,9990608 | 8,8184608 | 11,1815392 | 14   |
| 47   | 8,8194363 | 9,9990525 | 8,8203838 | 11,1796162 | 13   |
| 48   | 8,8213425 | 9,9990441 | 8,8222984 | 11,1777016 | 12   |
| 49   | 8,8232404 | 9,9990357 | 8,8242046 | 11,1757954 | 11   |
| 50   | 8,8251299 | 9,9990273 | 8,8261026 | 11,1738974 | 10   |
| 51   | 8,8270112 | 9,9990188 | 8,8279924 | 11,1720076 | 9    |
| 52   | 8,8288844 | 9,9990103 | 8,8298741 | 11,1701259 | 8    |
| 53   | 8,8307495 | 9,9990017 | 8,8317478 | 11,1682522 | 7    |
| 54   | 8,8326066 | 9,9989931 | 8,8336134 | 11,1663866 | 6    |
| 55   | 8,8344557 | 9,9989845 | 8,8354712 | 11,1645288 | 5    |
| 56   | 8,8362969 | 9,9989758 | 8,8373211 | 11,1626789 | 4    |
| 57   | 8,8381304 | 9,9989671 | 8,8391633 | 11,1608367 | 3    |
| 58   | 8,8399561 | 9,9989584 | 8,8409977 | 11,1590023 | 2    |
| 59   | 8,8417741 | 9,9989496 | 8,8428245 | 11,1571755 | 1    |
| 60   | 8,8435845 | 9,9989408 | 8,8446437 | 11,1553563 | 0    |
|      | Sine Com. | Sine      | Tan. Com. | Tangent.   | Min. |

# 86 Degrees.

| 4 Degrees.  |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 8,8435845 | 9,9989408 | 8,8446437 | 11,1553563 | 60   |
| 1           | 8,8453874 | 9,9989319 | 8,8464554 | 11,1535446 | 59   |
| 2           | 8,8471827 | 9,9989230 | 8,8482597 | 11,1517403 | 58   |
| 3           | 8,8489707 | 9,9989141 | 8,8500566 | 11,1499434 | 57   |
| 4           | 8,8507512 | 9,9989052 | 8,8518461 | 11,1481539 | 56   |
| 5           | 8,8525245 | 9,9988962 | 8,8536283 | 11,1463717 | 55   |
| 6           | 8,8542905 | 9,9988871 | 8,8554034 | 11,1445966 | 54   |
| 7           | 8,8560493 | 9,9988780 | 8,8571713 | 11,1428287 | 53   |
| 8           | 8,8578010 | 9,9988689 | 8,8589321 | 11,1410679 | 52   |
| 9           | 8,8595457 | 9,9988598 | 8,8606859 | 11,1393141 | 51   |
| 10          | 8,8612833 | 9,9988506 | 8,8624327 | 11,1375673 | 50   |
| 11          | 8,8630139 | 9,9988414 | 8,8641725 | 11,1358275 | 49   |
| 12          | 8,8647376 | 9,9988321 | 8,8659055 | 11,1340945 | 48   |
| 13          | 8,8664545 | 9,9988228 | 8,8676317 | 11,1323683 | 47   |
| 14          | 8,8681646 | 9,9988135 | 8,8693511 | 11,1306489 | 46   |
| 15          | 8,8698680 | 9,9988041 | 8,8710638 | 11,1289362 | 45   |
| 16          | 8,8715646 | 9,9987947 | 8,8727699 | 11,1272301 | 44   |
| 17          | 8,8732546 | 9,9987853 | 8,8744694 | 11,1255306 | 43   |
| 18          | 8,8749381 | 9,9987758 | 8,8761623 | 11,1238377 | 42   |
| 19          | 8,8766150 | 9,9987663 | 8,8778487 | 11,1221513 | 41   |
| 20          | 8,8782854 | 9,9987567 | 8,8795286 | 11,1204714 | 40   |
| 21          | 8,8799493 | 9,9987471 | 8,8812022 | 11,1187978 | 39   |
| 22          | 8,8816069 | 9,9987375 | 8,8828694 | 11,1171306 | 38   |
| 23          | 8,8832581 | 9,9987278 | 8,8845303 | 11,1154697 | 37   |
| 24          | 8,8849031 | 9,9987181 | 8,8861850 | 11,1138150 | 36   |
| 25          | 8,8865418 | 9,9987084 | 8,8878334 | 11,1121666 | 35   |
| 26          | 8,8881743 | 9,9986986 | 8,8894757 | 11,1105243 | 34   |
| 27          | 8,8898007 | 9,9986888 | 8,8911119 | 11,1088881 | 33   |
| 28          | 8,8914209 | 9,9986790 | 8,8927420 | 11,1072580 | 32   |
| 29          | 8,8930351 | 9,9986691 | 8,8943660 | 11,1056340 | 31   |
| 30          | 8,8946433 | 9,9986591 | 8,8959842 | 11,1040158 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 85 Degrees. |           |           |           |            |      |

# 4 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 8,8946433 | 9,9986591 | 8,8959842 | 11,1040158 | 30   |
| 31   | 8,8962455 | 9,9986492 | 8,8975963 | 11,1024037 | 29   |
| 32   | 8,8978418 | 9,9986392 | 8,8992026 | 11,1007974 | 28   |
| 33   | 8,8994322 | 9,9986292 | 8,9008030 | 11,0991970 | 27   |
| 34   | 8,9010168 | 9,9986191 | 8,9023977 | 11,0976023 | 26   |
| 35   | 8,9025955 | 9,9986090 | 8,9039866 | 11,0960134 | 25   |
| 36   | 8,9041685 | 9,9985988 | 8,9055697 | 11,0944303 | 24   |
| 37   | 8,9057358 | 9,9985886 | 8,9071472 | 11,0928528 | 23   |
| 38   | 8,9072975 | 9,9985784 | 8,9087190 | 11,0912810 | 22   |
| 39   | 8,9088535 | 9,9985682 | 8,9102853 | 11,0897147 | 21   |
| 40   | 8,9104039 | 9,9985579 | 8,9118460 | 11,0881540 | 20   |
| 41   | 8,9119487 | 9,9985475 | 8,9134012 | 11,0865988 | 19   |
| 42   | 8,9134881 | 9,9985372 | 8,9149509 | 11,0850491 | 18   |
| 43   | 8,9150219 | 9,9985268 | 8,9164952 | 11,0835048 | 17   |
| 44   | 8,9165504 | 9,9985163 | 8,9180340 | 11,0819660 | 16   |
| 45   | 8,9180734 | 9,9985058 | 8,9195675 | 11,0804325 | 15   |
| 46   | 8,9195911 | 9,9984953 | 8,9210957 | 11,0789043 | 14   |
| 47   | 8,9211034 | 9,9984848 | 8,9226186 | 11,0773814 | 13   |
| 48   | 8,9226105 | 9,9984742 | 8,9241363 | 11,0758637 | 12   |
| 49   | 8,9241123 | 9,9984636 | 8,9256487 | 11,0743513 | 11   |
| 50   | 8,9256089 | 9,9984529 | 8,9271560 | 11,0728440 | 10   |
| 51   | 8,9271003 | 9,9984422 | 8,9286581 | 11,0713419 | 9    |
| 52   | 8,9285866 | 9,9984315 | 8,9301552 | 11,0698448 | 8    |
| 53   | 8,9300678 | 9,9984207 | 8,9316471 | 11,0683529 | 7    |
| 54   | 8,9315439 | 9,9984099 | 8,9331340 | 11,0668660 | 6    |
| 55   | 8,9330150 | 9,9983990 | 8,9346160 | 11,0653840 | 5    |
| 56   | 8,9344811 | 9,9983881 | 8,9360929 | 11,0639071 | 4    |
| 57   | 8,9359422 | 9,9983772 | 8,9375650 | 11,0624350 | 3    |
| 58   | 8,9373983 | 9,9983663 | 8,9390321 | 11,0609679 | 2    |
| 59   | 8,9388496 | 9,9983553 | 8,9404944 | 11,0595056 | 1    |
| 60   | 8,9402960 | 9,9983442 | 8,9419518 | 11,0580482 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 85 Degrees.

# 5 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 8,9402960 | 9,9983442 | 8,9419518 | 11,0580482 | 60   |
| 1    | 8,9417376 | 9,9983332 | 8,9434044 | 11,0565956 | 59   |
| 2    | 8,9431743 | 9,9983220 | 8,9448523 | 11,0551477 | 58   |
| 3    | 8,9446063 | 9,9983109 | 8,9462954 | 11,0537046 | 57   |
| 4    | 8,9460335 | 9,9982997 | 8,9477338 | 11,0522662 | 56   |
| 5    | 8,9474561 | 9,9982885 | 8,9491676 | 11,0508324 | 55   |
| 6    | 8,9488739 | 9,9982772 | 8,9505967 | 11,0494033 | 54   |
| 7    | 8,9502871 | 9,9982660 | 8,9520211 | 11,0479789 | 53   |
| 8    | 8,9516957 | 9,9982546 | 8,9534410 | 11,0465590 | 52   |
| 9    | 8,9530996 | 9,9982433 | 8,9548564 | 11,0451436 | 51   |
| 10   | 8,9544991 | 9,9982318 | 8,9562672 | 11,0437328 | 50   |
| 11   | 8,9558940 | 9,9982204 | 8,9576735 | 11,0423265 | 49   |
| 12   | 8,9572843 | 9,9982089 | 8,9590754 | 11,0409246 | 48   |
| 13   | 8,9586703 | 9,9981974 | 8,9604728 | 11,0395272 | 47   |
| 14   | 8,9600517 | 9,9981859 | 8,9618659 | 11,0381341 | 46   |
| 15   | 8,9614288 | 9,9981743 | 8,9632545 | 11,0367455 | 45   |
| 16   | 8,9628014 | 9,9981629 | 8,9646388 | 11,0353612 | 44   |
| 17   | 8,9641697 | 9,9981510 | 8,9660188 | 11,0339812 | 43   |
| 18   | 8,9655337 | 9,9981393 | 8,9673944 | 11,0326056 | 42   |
| 19   | 8,9668934 | 9,9981275 | 8,9687658 | 11,0312342 | 41   |
| 20   | 8,9682487 | 9,9981158 | 8,9701330 | 11,0298670 | 40   |
| 21   | 8,9695999 | 9,9981040 | 8,9714959 | 11,0285041 | 39   |
| 22   | 8,9709468 | 9,9980921 | 8,9728547 | 11,0271453 | 38   |
| 23   | 8,9722895 | 9,9980802 | 8,9742092 | 11,0257908 | 37   |
| 24   | 8,9736280 | 9,9980683 | 8,9755597 | 11,0244403 | 36   |
| 25   | 8,9749624 | 9,9980563 | 8,9769060 | 11,0230940 | 35   |
| 26   | 8,9762926 | 9,9980443 | 8,9782483 | 11,0217517 | 34   |
| 27   | 8,9776188 | 9,9980323 | 8,9795865 | 11,0204135 | 33   |
| 28   | 8,9789408 | 9,9980202 | 8,9809206 | 11,0190794 | 32   |
| 29   | 8,9802589 | 9,9980081 | 8,9822507 | 11,0177493 | 31   |
| 30   | 8,9815729 | 9,9979960 | 8,9835769 | 11,0164231 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 84 Degrees.

# 5 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 8,9815729 | 9,9979960 | 8,9835769 | 11,0164231 | 30   |
| 31   | 8,9828829 | 9,9979838 | 8,9848991 | 11,0151009 | 29   |
| 32   | 8,9841889 | 9,9979716 | 8,9862173 | 11,0137827 | 28   |
| 33   | 8,9854910 | 9,9979593 | 8,9875317 | 11,0124683 | 27   |
| 34   | 8,9867891 | 9,9979470 | 8,9888421 | 11,0111579 | 26   |
| 35   | 8,9880834 | 9,9979347 | 8,9901487 | 11,0098513 | 25   |
| 36   | 8,9893737 | 9,9979223 | 8,9914514 | 11,0085486 | 24   |
| 37   | 8,9906602 | 9,9979099 | 8,9927503 | 11,0072497 | 23   |
| 38   | 8,9919429 | 9,9978975 | 8,9940454 | 11,0059546 | 22   |
| 39   | 8,9932217 | 9,9978850 | 8,9953367 | 11,0046633 | 21   |
| 40   | 8,9944968 | 9,9978725 | 8,9966243 | 11,0033757 | 20   |
| 41   | 8,9957681 | 9,9978599 | 8,9979081 | 11,0020919 | 19   |
| 42   | 8,9970356 | 9,9978473 | 8,9991883 | 11,0008117 | 18   |
| 43   | 8,9982994 | 9,9978347 | 9,0004647 | 10,9995353 | 17   |
| 44   | 8,9995595 | 9,9978220 | 9,0017375 | 10,9982625 | 16   |
| 45   | 9,0008160 | 9,9978093 | 9,0030066 | 10,9969934 | 15   |
| 46   | 9,0020687 | 9,9977966 | 9,0042721 | 10,9957279 | 14   |
| 47   | 9,0033179 | 9,9977838 | 9,0055340 | 10,9944660 | 13   |
| 48   | 9,0045634 | 9,9977710 | 9,0067924 | 10,9932076 | 12   |
| 49   | 9,0058053 | 9,9977582 | 9,0080471 | 10,9919529 | 11   |
| 50   | 9,0070436 | 9,9977453 | 9,0092984 | 10,9907016 | 10   |
| 51   | 9,0082784 | 9,9977323 | 9,0105461 | 10,9894539 | 9    |
| 52   | 9,0095096 | 9,9977194 | 9,0117903 | 10,9882097 | 8    |
| 53   | 9,0107374 | 9,9977064 | 9,0130310 | 10,9869690 | 7    |
| 54   | 9,0119616 | 9,9976933 | 9,0142682 | 10,9857318 | 6    |
| 55   | 9,0131823 | 9,9976803 | 9,0155021 | 10,9844979 | 5    |
| 56   | 9,0143996 | 9,9976672 | 9,0167325 | 10,9832675 | 4    |
| 57   | 9,0156135 | 9,9976540 | 9,0179594 | 10,9820406 | 3    |
| 58   | 9,0168239 | 9,9976408 | 9,0191831 | 10,9808169 | 2    |
| 59   | 9,0180309 | 9,9976276 | 9,0204033 | 10,9795967 | 1    |
| 60   | 9,0192346 | 9,9976143 | 9,0216202 | 10,9783798 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 84 Degrees.

# 6 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,0192346 | 9,9976143 | 9,0216202 | 10,9783798 | 60   |
| 1    | 9,0204348 | 9,9976011 | 9,0228338 | 10,9771662 | 59   |
| 2    | 9,0216318 | 9,9975877 | 9,0240441 | 10,9759559 | 58   |
| 3    | 9,0228254 | 9,9975743 | 9,0252510 | 10,9747490 | 57   |
| 4    | 9,0240157 | 9,9975609 | 9,0264548 | 10,9735432 | 56   |
| 5    | 9,0252027 | 9,9975475 | 9,0276552 | 10,9723448 | 55   |
| 6    | 9,0263865 | 9,9975340 | 9,0288524 | 10,9711476 | 54   |
| 7    | 9,0275669 | 9,9975205 | 9,0300464 | 10,9699536 | 53   |
| 8    | 9,0287442 | 9,9975069 | 9,0312373 | 10,9687627 | 52   |
| 9    | 9,0299182 | 9,9974933 | 9,0324249 | 10,9675751 | 51   |
| 10   | 9,0310890 | 9,9974797 | 9,0336093 | 10,9663907 | 50   |
| 11   | 9,0322567 | 9,9974660 | 9,0347906 | 10,9652094 | 49   |
| 12   | 9,0334212 | 9,9974523 | 9,0359688 | 10,9640312 | 48   |
| 13   | 9,0345825 | 9,9974386 | 9,0371439 | 10,9628561 | 47   |
| 14   | 9,0357407 | 9,9974248 | 9,0383159 | 10,9616841 | 46   |
| 15   | 9,0368958 | 9,9974110 | 9,0394848 | 10,9605152 | 45   |
| 16   | 9,0380477 | 9,9973971 | 9,0406506 | 10,9593494 | 44   |
| 17   | 9,0391966 | 9,9973833 | 9,0418134 | 10,9581866 | 43   |
| 18   | 9,0403424 | 9,9973693 | 9,0429731 | 10,9570269 | 42   |
| 19   | 9,0414852 | 9,9973554 | 9,0441299 | 10,9558701 | 41   |
| 20   | 9,0426249 | 9,9973414 | 9,0452836 | 10,9547164 | 40   |
| 21   | 9,0437617 | 9,9973273 | 9,0464343 | 10,9535657 | 39   |
| 22   | 9,0448954 | 9,9973132 | 9,0475821 | 10,9524179 | 38   |
| 23   | 9,0460261 | 9,9972991 | 9,0487270 | 10,9512730 | 37   |
| 24   | 9,0471538 | 9,9972850 | 9,0498689 | 10,9501311 | 36   |
| 25   | 9,0482786 | 9,9972708 | 9,0510078 | 10,9489922 | 35   |
| 26   | 9,0494005 | 9,9972566 | 9,0521439 | 10,9478561 | 34   |
| 27   | 9,0505194 | 9,9972423 | 9,0532771 | 10,9467229 | 33   |
| 28   | 9,0516354 | 9,9972280 | 9,0544074 | 10,9455926 | 32   |
| 29   | 9,0527485 | 9,9972137 | 9,0555349 | 10,9444651 | 31   |
| 30   | 9,0538588 | 9,9971993 | 9,0566595 | 10,9433405 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tange n .  | Min. |

# 83 Degrees.

# 6 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,9538588 | 9,9971993 | 9,0566595 | 10,9433405 | 30   |
| 31   | 9,0549061 | 9,9971849 | 9,0577813 | 10,9422187 | 29   |
| 32   | 9,0560706 | 9,9971704 | 9,0589002 | 10,9410998 | 28   |
| 33   | 9,0571723 | 9,9971559 | 9,0600164 | 10,9399836 | 27   |
| 34   | 9,0582711 | 9,9971414 | 9,0611297 | 10,9388703 | 26   |
| 35   | 9,0593672 | 9,9971268 | 9,0622403 | 10,9377597 | 25   |
| 36   | 9,0604604 | 9,9971122 | 9,0633482 | 10,9366518 | 24   |
| 37   | 9,0615509 | 9,9970976 | 9,0644533 | 10,9355467 | 23   |
| 38   | 9,0626386 | 9,9970829 | 9,0655556 | 10,9344444 | 22   |
| 39   | 9,0637235 | 9,9970682 | 9,0666553 | 10,9333447 | 21   |
| 40   | 9,0648057 | 9,9970535 | 9,0677522 | 10,9322478 | 20   |
| 41   | 9,0658852 | 9,9970387 | 9,0688465 | 10,9311535 | 19   |
| 42   | 9,0669619 | 9,9970239 | 9,0699381 | 10,9300619 | 18   |
| 43   | 9,0680360 | 9,9970090 | 9,0710270 | 10,9289730 | 17   |
| 44   | 9,0691074 | 9,9969941 | 9,0721133 | 10,9278867 | 16   |
| 45   | 9,0701761 | 9,9969792 | 9,0731969 | 10,9268031 | 15   |
| 46   | 9,0712421 | 9,9969642 | 9,0742779 | 10,9257221 | 14   |
| 47   | 9,0723055 | 9,9969492 | 9,0753563 | 10,9246437 | 13   |
| 48   | 9,0733663 | 9,9969342 | 9,0764321 | 10,9235679 | 12   |
| 49   | 9,0744244 | 9,9969191 | 9,0775053 | 10,9224947 | 11   |
| 50   | 9,0754799 | 9,9969040 | 9,0785760 | 10,9214240 | 10   |
| 51   | 9,0765329 | 9,9968888 | 9,0796441 | 10,9203559 | 9    |
| 52   | 9,0775832 | 9,9968736 | 9,0807096 | 10,9192904 | 8    |
| 53   | 9,0786310 | 9,9968584 | 9,0817726 | 10,9182274 | 7    |
| 54   | 9,0796762 | 9,9968431 | 9,0828331 | 10,9171669 | 6    |
| 55   | 9,0807189 | 9,9968278 | 9,0838911 | 10,9161089 | 5    |
| 56   | 9,0817590 | 9,9968125 | 9,0849466 | 10,9150534 | 4    |
| 57   | 9,0827966 | 9,9967971 | 9,0859996 | 10,9140004 | 3    |
| 58   | 9,0838317 | 9,9967817 | 9,0870501 | 10,9129499 | 2    |
| 59   | 9,0848643 | 9,9967662 | 9,0880981 | 10,9119019 | 1    |
| 60   | 9,0858945 | 9,9967507 | 9,0891438 | 10,9108562 | 0    |
|      | Sine Com  | Sine.     | Tan. Com. | Tangent.   | Min. |

# 83 Degrees.

| 7 Degrees.  |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 9,0858945 | 9,9967507 | 9,0891438 | 10,9108562 | 60   |
| 1           | 9,0869221 | 9,9967352 | 9,0901869 | 10,9098131 | 59   |
| 2           | 9,0879473 | 9,9967196 | 9,0912277 | 10,9087723 | 58   |
| 3           | 9,0889700 | 9,9967040 | 9,0922660 | 10,9077340 | 57   |
| 4           | 9,0899903 | 9,9966884 | 9,0933020 | 10,9066980 | 56   |
| 5           | 9,0910082 | 9,9966727 | 9,0943355 | 10,9056645 | 55   |
| 6           | 9,0920237 | 9,9966570 | 9,0953667 | 10,9046333 | 54   |
| 7           | 9,0930367 | 9,9966412 | 9,0963955 | 10,9036045 | 53   |
| 8           | 9,0940474 | 9,9966254 | 9,0974219 | 10,9025781 | 52   |
| 9           | 9,0950556 | 9,9966096 | 9,0984460 | 10,9015540 | 51   |
| 10          | 9,0960615 | 9,9965937 | 9,0994678 | 10,9005322 | 50   |
| 11          | 9,0970651 | 9,9965778 | 9,1004872 | 10,8995128 | 49   |
| 12          | 9,0980662 | 9,9965619 | 9,1015044 | 10,8984956 | 48   |
| 13          | 9,0990651 | 9,9965459 | 9,1025192 | 10,8974808 | 47   |
| 14          | 9,1000616 | 9,9965299 | 9,1035317 | 10,8964683 | 46   |
| 15          | 9,1010558 | 9,9965138 | 9,1045420 | 10,8954580 | 45   |
| 16          | 9,1020477 | 9,9964977 | 9,1055500 | 10,8944500 | 44   |
| 17          | 9,1030373 | 9,9964816 | 9,1065557 | 10,8934443 | 43   |
| 18          | 9,1040246 | 9,9964655 | 9,1075591 | 10,8924409 | 42   |
| 19          | 9,1050096 | 9,9964493 | 9,1085604 | 10,8914396 | 41   |
| 20          | 9,1059924 | 9,9964330 | 9,1095594 | 10,8904406 | 40   |
| 21          | 9,1069729 | 9,9964167 | 9,1105562 | 10,8894438 | 39   |
| 22          | 9,1079512 | 9,9964004 | 9,1115508 | 10,8884492 | 38   |
| 23          | 9,1089272 | 9,9963841 | 9,1125431 | 10,8874569 | 37   |
| 24          | 9,1099010 | 9,9963677 | 9,1135333 | 10,8864667 | 36   |
| 25          | 9,1108726 | 9,9963513 | 9,1145213 | 10,8854787 | 35   |
| 26          | 9,1118420 | 9,9963348 | 9,1155072 | 10,8844928 | 34   |
| 27          | 9,1128092 | 9,9963183 | 9,1164909 | 10,8835091 | 33   |
| 28          | 9,1137742 | 9,9963018 | 9,1174724 | 10,8825276 | 32   |
| 29          | 9,1147370 | 9,9962852 | 9,1184518 | 10,8815482 | 31   |
| 30          | 9,1156977 | 9,9962686 | 9,1194291 | 10,8805709 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 82 Degrees. |           |           |           |            |      |



# 7 Degrees

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,1156977 | 9,9962686 | 9,1194291 | 10,8805709 | 30   |
| 31   | 9,1166562 | 9,9962519 | 9,1204043 | 10,8795957 | 29   |
| 32   | 9,1176125 | 9,9962352 | 9,1213773 | 10,8786227 | 28   |
| 33   | 9,1185667 | 9,9962185 | 9,1223482 | 10,8776518 | 27   |
| 34   | 9,1195188 | 9,9962017 | 9,1233171 | 10,8766829 | 26   |
| 35   | 9,1204688 | 9,9961849 | 9,1242839 | 10,8757161 | 25   |
| 36   | 9,1214167 | 9,9961681 | 9,1252486 | 10,8747514 | 24   |
| 37   | 9,1223624 | 9,9961512 | 9,1262112 | 10,8737888 | 23   |
| 38   | 9,1233061 | 9,9961343 | 9,1271718 | 10,8728282 | 22   |
| 39   | 9,1242477 | 9,9961174 | 9,1281303 | 10,8718697 | 21   |
| 40   | 9,1251872 | 9,9961004 | 9,1290868 | 10,8709132 | 20   |
| 41   | 9,1261246 | 9,9960834 | 9,1300413 | 10,8699587 | 19   |
| 42   | 9,1270600 | 9,9960663 | 9,1309937 | 10,8690063 | 18   |
| 43   | 9,1279934 | 9,9960492 | 9,1319442 | 10,8680558 | 17   |
| 44   | 9,1289247 | 9,9960321 | 9,1328926 | 10,8671074 | 16   |
| 45   | 9,1298539 | 9,9960149 | 9,1338391 | 10,8661609 | 15   |
| 46   | 9,1307812 | 9,9959977 | 9,1347835 | 10,8652165 | 14   |
| 47   | 9,1317064 | 9,9959804 | 9,1357260 | 10,8642740 | 13   |
| 48   | 9,1326297 | 9,9959631 | 9,1366665 | 10,8633335 | 12   |
| 49   | 9,1335509 | 9,9959458 | 9,1376051 | 10,8623949 | 11   |
| 50   | 9,1344702 | 9,9959284 | 9,1385417 | 10,8614583 | 10   |
| 51   | 9,1353875 | 9,9959111 | 9,1394764 | 10,8605236 | 9    |
| 52   | 9,1363028 | 9,9958936 | 9,1404092 | 10,8595908 | 8    |
| 53   | 9,1372161 | 9,9958761 | 9,1413400 | 10,8586600 | 7    |
| 54   | 9,1381275 | 9,9958586 | 9,1422689 | 10,8577311 | 6    |
| 55   | 9,1390370 | 9,9958411 | 9,1431959 | 10,8568041 | 5    |
| 56   | 9,1399445 | 9,9958235 | 9,1441210 | 10,8558790 | 4    |
| 57   | 9,1408501 | 9,9958059 | 9,1450442 | 10,8549558 | 3    |
| 58   | 9,1417537 | 9,9957882 | 9,1459655 | 10,8540345 | 2    |
| 59   | 9,1426555 | 9,9957705 | 9,1468849 | 10,8531151 | 1    |
| 60   | 9,1435553 | 9,9957528 | 9,1478025 | 10,8521975 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

| 8 Degrees.  |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 9,1435553 | 9,9957528 | 9,1478025 | 10,8521975 | 60   |
| 1           | 9,1444532 | 9,9957350 | 9,1487182 | 10,8512818 | 59   |
| 2           | 9,1453493 | 9,9957172 | 9,1496321 | 10,8503679 | 58   |
| 3           | 9,1462435 | 9,9956993 | 9,1505441 | 10,8494559 | 57   |
| 4           | 9,1471358 | 9,9956815 | 9,1514543 | 10,8485457 | 56   |
| 5           | 9,1480262 | 9,9956635 | 9,1523627 | 10,8476373 | 55   |
| 6           | 9,1489148 | 9,9956456 | 9,1532692 | 10,8467308 | 54   |
| 7           | 9,1498015 | 9,9956276 | 9,1541739 | 10,8458261 | 53   |
| 8           | 9,1506864 | 9,9956095 | 9,1550769 | 10,8449231 | 52   |
| 9           | 9,1515694 | 9,9955915 | 9,1559780 | 10,8440220 | 51   |
| 10          | 9,1524507 | 9,9955734 | 9,1568773 | 10,8431227 | 50   |
| 11          | 9,1533301 | 9,9955552 | 9,1577748 | 10,8422252 | 49   |
| 12          | 9,1542076 | 9,9955370 | 9,1586706 | 10,8413294 | 48   |
| 13          | 9,1550834 | 9,9955188 | 9,1595646 | 10,8404354 | 47   |
| 14          | 9,1559574 | 9,9955005 | 9,1604569 | 10,8395431 | 46   |
| 15          | 9,1568296 | 9,9954822 | 9,1613473 | 10,8386527 | 45   |
| 16          | 9,1577000 | 9,9954639 | 9,1622361 | 10,8377639 | 44   |
| 17          | 9,1585686 | 9,9954455 | 9,1631231 | 10,8368769 | 43   |
| 18          | 9,1594354 | 9,9954271 | 9,1640083 | 10,8359917 | 42   |
| 19          | 9,1603005 | 9,9954087 | 9,1648919 | 10,8351081 | 41   |
| 20          | 9,1611639 | 9,9953902 | 9,1657737 | 10,8342263 | 40   |
| 21          | 9,1620254 | 9,9953717 | 9,1666538 | 10,8333462 | 39   |
| 22          | 9,1628853 | 9,9953531 | 9,1675322 | 10,8324678 | 38   |
| 23          | 9,1637434 | 9,9953345 | 9,1684089 | 10,8315911 | 37   |
| 24          | 9,1645998 | 9,9953159 | 9,1692839 | 10,8307161 | 36   |
| 25          | 9,1654544 | 9,9952972 | 9,1701572 | 10,8298428 | 35   |
| 26          | 9,1663074 | 9,9952785 | 9,1710289 | 10,8289711 | 34   |
| 27          | 9,1671586 | 9,9952597 | 9,1718989 | 10,8281011 | 33   |
| 28          | 9,1680081 | 9,9952409 | 9,1727672 | 10,8272328 | 32   |
| 29          | 9,1688559 | 9,9952221 | 9,1736338 | 10,8263662 | 31   |
| 30          | 9,1697021 | 9,9952033 | 9,1744988 | 10,8255012 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tang.      | Min. |
| 81 Degrees. |           |           |           |            |      |

# 8 Degrees.

|    | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|----|-----------|-----------|-----------|------------|------|
| 30 | 9,1697621 | 9,9952033 | 9,1744988 | 10,8255072 | 30   |
| 31 | 9,1705465 | 9,9951844 | 9,1753622 | 10,8246378 | 29   |
| 32 | 9,1713893 | 9,9951654 | 9,1762239 | 10,8237761 | 28   |
| 33 | 9,1722305 | 9,9951464 | 9,1770840 | 10,8229160 | 27   |
| 34 | 9,1730699 | 9,9951274 | 9,1779425 | 10,8220575 | 26   |
| 35 | 9,1739077 | 9,9951084 | 9,1788005 | 10,8212007 | 25   |
| 36 | 9,1747439 | 9,9950893 | 9,1796579 | 10,8203454 | 24   |
| 37 | 9,1755784 | 9,9950702 | 9,1805148 | 10,8194918 | 23   |
| 38 | 9,1764112 | 9,9950516 | 9,1813712 | 10,8186398 | 22   |
| 39 | 9,1772425 | 9,9950328 | 9,1822270 | 10,8177894 | 21   |
| 40 | 9,1780721 | 9,9950126 | 9,1830825 | 10,8169405 | 20   |
| 41 | 9,1789001 | 9,9949933 | 9,1839378 | 10,8160932 | 19   |
| 42 | 9,1797265 | 9,9949740 | 9,1847925 | 10,8152475 | 18   |
| 43 | 9,1805512 | 9,9949546 | 9,1856466 | 10,8144034 | 17   |
| 44 | 9,1813744 | 9,9949352 | 9,1865002 | 10,8135608 | 16   |
| 45 | 9,1821960 | 9,9949158 | 9,1873532 | 10,8127198 | 15   |
| 46 | 9,1830160 | 9,9948964 | 9,1882056 | 10,8118804 | 14   |
| 47 | 9,1838344 | 9,9948769 | 9,1890575 | 10,8110425 | 13   |
| 48 | 9,1846512 | 9,9948573 | 9,1899089 | 10,8102061 | 12   |
| 49 | 9,1854665 | 9,9948377 | 9,1907597 | 10,8093713 | 11   |
| 50 | 9,1862802 | 9,9948181 | 9,1916101 | 10,8085379 | 10   |
| 51 | 9,1870923 | 9,9947985 | 9,1924601 | 10,8077061 | 9    |
| 52 | 9,1879029 | 9,9947788 | 9,1933124 | 10,8068759 | 8    |
| 53 | 9,1887120 | 9,9947591 | 9,1941641 | 10,8060471 | 7    |
| 54 | 9,1895195 | 9,9947393 | 9,1950152 | 10,8052198 | 6    |
| 55 | 9,1903254 | 9,9947195 | 9,1958659 | 10,8043941 | 5    |
| 56 | 9,1911299 | 9,9946997 | 9,1967162 | 10,8035698 | 4    |
| 57 | 9,1919328 | 9,9946798 | 9,1975660 | 10,8027470 | 3    |
| 58 | 9,1927342 | 9,9946599 | 9,1984153 | 10,8019257 | 2    |
| 59 | 9,1935341 | 9,9946399 | 9,1992641 | 10,8011059 | 1    |
| 60 | 9,1943324 | 9,9946199 | 9,1997125 | 10,8002875 | 0    |
|    | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 81 Degrees.

9 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,1943324 | 9,9946199 | 9,1997125 | 10,8002875 | 60   |
| 1    | 9,1951293 | 9,9945999 | 9,2005294 | 10,7994706 | 59   |
| 2    | 9,1959247 | 9,9945798 | 9,2013449 | 10,7986551 | 58   |
| 3    | 9,1967186 | 9,9945597 | 9,2021588 | 10,7978412 | 57   |
| 4    | 9,1975110 | 9,9945396 | 9,2029714 | 10,7970286 | 56   |
| 5    | 9,1983019 | 9,9945194 | 9,2037825 | 10,7962175 | 55   |
| 6    | 9,1990913 | 9,9944992 | 9,2045922 | 10,7954078 | 54   |
| 7    | 9,1998793 | 9,9944789 | 9,2054004 | 10,7945996 | 53   |
| 8    | 9,2006658 | 9,9944587 | 9,2062072 | 10,7937928 | 52   |
| 9    | 9,2014509 | 9,9944383 | 9,2070126 | 10,7929874 | 51   |
| 10   | 9,2022345 | 9,9944180 | 9,2078165 | 10,7921835 | 50   |
| 11   | 9,2030167 | 9,9943975 | 9,2086191 | 10,7913809 | 49   |
| 12   | 9,2037974 | 9,9943771 | 9,2094203 | 10,7905797 | 48   |
| 13   | 9,2045766 | 9,9943566 | 9,2102200 | 10,7897800 | 47   |
| 14   | 9,2053545 | 9,9943361 | 9,2110184 | 10,7889816 | 46   |
| 15   | 9,2061309 | 9,9943156 | 9,2118153 | 10,7881847 | 45   |
| 16   | 9,2069059 | 9,9942950 | 9,2126109 | 10,7873891 | 44   |
| 17   | 9,2076795 | 9,9942743 | 9,2134051 | 10,7865949 | 43   |
| 18   | 9,2084516 | 9,9942537 | 9,2141980 | 10,7858020 | 42   |
| 19   | 9,2092224 | 9,9942330 | 9,2149894 | 10,7850106 | 41   |
| 20   | 9,2099917 | 9,9942122 | 9,2157795 | 10,7842205 | 40   |
| 21   | 9,2107597 | 9,9941914 | 9,2165683 | 10,7834317 | 39   |
| 22   | 9,2115263 | 9,9941706 | 9,2173556 | 10,7826444 | 38   |
| 23   | 9,2122914 | 9,9941498 | 9,2181417 | 10,7818583 | 37   |
| 24   | 9,2130552 | 9,9941289 | 9,2189264 | 10,7810736 | 36   |
| 25   | 9,2138176 | 9,9941079 | 9,2197097 | 10,7802903 | 35   |
| 26   | 9,2145787 | 9,9940870 | 9,2204917 | 10,7795083 | 34   |
| 27   | 9,2153384 | 9,9940659 | 9,2212724 | 10,7787276 | 33   |
| 28   | 9,2160967 | 9,9940449 | 9,2220518 | 10,7779482 | 32   |
| 29   | 9,2168536 | 9,9940238 | 9,2228298 | 10,7771702 | 31   |
| 30   | 9,2176092 | 9,9940027 | 9,2236065 | 10,7763935 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

80 Degrees.

# 9 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,2176092 | 9,9940027 | 9,2236065 | 10,7763935 | 30   |
| 31   | 9,2183635 | 9,9939815 | 9,2243819 | 10,7756181 | 29   |
| 32   | 9,2191164 | 9,9939603 | 9,2251561 | 10,7748439 | 28   |
| 33   | 9,2198680 | 9,9939391 | 9,2259289 | 10,7740711 | 27   |
| 34   | 9,2206182 | 9,9939178 | 9,2267004 | 10,7732996 | 26   |
| 35   | 9,2213671 | 9,9938965 | 9,2274706 | 10,7725294 | 25   |
| 36   | 9,2221147 | 9,9938752 | 9,2282395 | 10,7717605 | 24   |
| 37   | 9,2228609 | 9,9938538 | 9,2290071 | 10,7709929 | 23   |
| 38   | 9,2236059 | 9,9938324 | 9,2297735 | 10,7702265 | 22   |
| 39   | 9,2243495 | 9,9938109 | 9,2305386 | 10,7694614 | 21   |
| 40   | 9,2250918 | 9,9937894 | 9,2313024 | 10,7686976 | 20   |
| 41   | 9,2258328 | 9,9937679 | 9,2320650 | 10,7679350 | 19   |
| 42   | 9,2265725 | 9,9937463 | 9,2328262 | 10,7671738 | 18   |
| 43   | 9,2273110 | 9,9937247 | 9,2335863 | 10,7664137 | 17   |
| 44   | 9,2280481 | 9,9937030 | 9,2343451 | 10,7656549 | 16   |
| 45   | 9,2287839 | 9,9936813 | 9,2351026 | 10,7648974 | 15   |
| 46   | 9,2295185 | 9,9936596 | 9,2358589 | 10,7641411 | 14   |
| 47   | 9,2302518 | 9,9936378 | 9,2366139 | 10,7633861 | 13   |
| 48   | 9,2309838 | 9,9936160 | 9,2373678 | 10,7626322 | 12   |
| 49   | 9,2317145 | 9,9935942 | 9,2381203 | 10,7618797 | 11   |
| 50   | 9,2324440 | 9,9935723 | 9,2388717 | 10,7611283 | 10   |
| 51   | 9,2331722 | 9,9935504 | 9,2396218 | 10,7603782 | 9    |
| 52   | 9,2338992 | 9,9935285 | 9,2403708 | 10,7596292 | 8    |
| 53   | 9,2346249 | 9,9935065 | 9,2411185 | 10,7588815 | 7    |
| 54   | 9,2353494 | 9,9934844 | 9,2418650 | 10,7581350 | 6    |
| 55   | 9,2360726 | 9,9934624 | 9,2426103 | 10,7573897 | 5    |
| 56   | 9,2367946 | 9,9934403 | 9,2433543 | 10,7566457 | 4    |
| 57   | 9,2375153 | 9,9934181 | 9,2440972 | 10,7559028 | 3    |
| 58   | 9,2382349 | 9,9933959 | 9,2448389 | 10,7551611 | 2    |
| 59   | 9,2389532 | 9,9933737 | 9,2455794 | 10,7544206 | 1    |
| 60   | 9,2396702 | 9,9933515 | 9,2463188 | 10,7536812 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

80 Degrees.

10 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,2396702 | 9,9933515 | 9,2463188 | 10,7536812 | 60   |
| 1    | 9,2403861 | 9,9933292 | 9,2470569 | 10,7529431 | 59   |
| 2    | 9,2411007 | 9,9933068 | 9,2477939 | 10,7522061 | 58   |
| 3    | 9,2418141 | 9,9932845 | 9,2485297 | 10,7514703 | 57   |
| 4    | 9,2425264 | 9,9932621 | 9,2492643 | 10,7507357 | 56   |
| 5    | 9,2432374 | 9,9932396 | 9,2499978 | 10,7500022 | 55   |
| 6    | 9,2439472 | 9,9932171 | 9,2507301 | 10,7492699 | 54   |
| 7    | 9,2446558 | 9,9931946 | 9,2514612 | 10,7485388 | 53   |
| 8    | 9,2453632 | 9,9931720 | 9,2521912 | 10,7478088 | 52   |
| 9    | 9,2460695 | 9,9931494 | 9,2529200 | 10,7470800 | 51   |
| 10   | 9,2467746 | 9,9931268 | 9,2536477 | 10,7463523 | 50   |
| 11   | 9,2474784 | 9,9931041 | 9,2543743 | 10,7456257 | 49   |
| 12   | 9,2481811 | 9,9930814 | 9,2550997 | 10,7449003 | 48   |
| 13   | 9,2488827 | 9,9930587 | 9,2558240 | 10,7441760 | 47   |
| 14   | 9,2495830 | 9,9930359 | 9,2565472 | 10,7434528 | 46   |
| 15   | 9,2502822 | 9,9930131 | 9,2572692 | 10,7427308 | 45   |
| 16   | 9,2509803 | 9,9929902 | 9,2579901 | 10,7420099 | 44   |
| 17   | 9,2516772 | 9,9929673 | 9,2587099 | 10,7412901 | 43   |
| 18   | 9,2523729 | 9,9929444 | 9,2594285 | 10,7405715 | 42   |
| 19   | 9,2530675 | 9,9929214 | 9,2601461 | 10,7398539 | 41   |
| 20   | 9,2537609 | 9,9928984 | 9,2608625 | 10,7391375 | 40   |
| 21   | 9,2544532 | 9,9928753 | 9,2615779 | 10,7384221 | 39   |
| 22   | 9,2551444 | 9,9928522 | 9,2622921 | 10,7377079 | 38   |
| 23   | 9,2558344 | 9,9928291 | 9,2630053 | 10,7369947 | 37   |
| 24   | 9,2565233 | 9,9928059 | 9,2637173 | 10,7362827 | 36   |
| 25   | 9,2572110 | 9,9927827 | 9,2644283 | 10,7355717 | 35   |
| 26   | 9,2578977 | 9,9927595 | 9,2651382 | 10,7348618 | 34   |
| 27   | 9,2585832 | 9,9927362 | 9,2658470 | 10,7341530 | 33   |
| 28   | 9,2592676 | 9,9927129 | 9,2665547 | 10,7334453 | 32   |
| 29   | 9,2599509 | 9,9926895 | 9,2672613 | 10,7327387 | 31   |
| 30   | 9,2606330 | 9,9926661 | 9,2679669 | 10,7320331 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

79 Degrees.

10 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,2606330 | 9,9926661 | 9,2679669 | 10,7320331 | 30   |
| 31   | 9,2613141 | 9,9926427 | 9,2686714 | 10,7313286 | 29   |
| 32   | 9,2619941 | 9,9926192 | 9,2693749 | 10,7306251 | 28   |
| 33   | 9,2626729 | 9,9925957 | 9,2700772 | 10,7299228 | 27   |
| 34   | 9,2633507 | 9,9925722 | 9,2707786 | 10,7292214 | 26   |
| 35   | 9,2640274 | 9,9925486 | 9,2714788 | 10,7285212 | 25   |
| 36   | 9,2647030 | 9,9925250 | 9,2721780 | 10,7278220 | 24   |
| 37   | 9,2653775 | 9,9925013 | 9,2728762 | 10,7271238 | 23   |
| 38   | 9,2660509 | 9,9924776 | 9,2735733 | 10,7264267 | 22   |
| 39   | 9,2667232 | 9,9924539 | 9,2742694 | 10,7257306 | 21   |
| 40   | 9,2673945 | 9,9924301 | 9,2749644 | 10,7250356 | 20   |
| 41   | 9,2680647 | 9,9924063 | 9,2756584 | 10,7243416 | 19   |
| 42   | 9,2687338 | 9,9923824 | 9,2763514 | 10,7236486 | 18   |
| 43   | 9,2694019 | 9,9923585 | 9,2770434 | 10,7229566 | 17   |
| 44   | 9,2700689 | 9,9923346 | 9,2777343 | 10,7222657 | 16   |
| 45   | 9,2707348 | 9,9923106 | 9,2784242 | 10,7215758 | 15   |
| 46   | 9,2713997 | 9,9922866 | 9,2791131 | 10,7208869 | 14   |
| 47   | 9,2720635 | 9,9922626 | 9,2798009 | 10,7201991 | 13   |
| 48   | 9,2727263 | 9,9922385 | 9,2804878 | 10,7195122 | 12   |
| 49   | 9,2733880 | 9,9922144 | 9,2811736 | 10,7188264 | 11   |
| 50   | 9,2740487 | 9,9921902 | 9,2818585 | 10,7181415 | 10   |
| 51   | 9,2747083 | 9,9921660 | 9,2825423 | 10,7174577 | 9    |
| 52   | 9,2753669 | 9,9921418 | 9,2832251 | 10,7167749 | 8    |
| 53   | 9,2760245 | 9,9921175 | 9,2839070 | 10,7160930 | 7    |
| 54   | 9,2766811 | 9,9920932 | 9,2845878 | 10,7154122 | 6    |
| 55   | 9,2773366 | 9,9920689 | 9,2852677 | 10,7147323 | 5    |
| 56   | 9,2779911 | 9,9920445 | 9,2859466 | 10,7140534 | 4    |
| 57   | 9,2786445 | 9,9920201 | 9,2866245 | 10,7133755 | 3    |
| 58   | 9,2792970 | 9,9919956 | 9,2873014 | 10,7126986 | 2    |
| 59   | 9,2799484 | 9,9919711 | 9,2879773 | 10,7120227 | 1    |
| 60   | 9,2805988 | 9,9919466 | 9,2886523 | 10,7113477 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

79 Degrees.

# 11 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,2805988 | 9,9919466 | 9,2886523 | 10,7113477 | 60   |
| 1    | 9,2812483 | 9,9919220 | 9,2893263 | 10,7106737 | 59   |
| 2    | 9,2818967 | 9,9918974 | 9,2899993 | 10,7100007 | 58   |
| 3    | 9,2825441 | 9,9918727 | 9,2906713 | 10,7093287 | 57   |
| 4    | 9,2831905 | 9,9918480 | 9,2913424 | 10,7086576 | 56   |
| 5    | 9,2838359 | 9,9918233 | 9,2920126 | 10,7079874 | 55   |
| 6    | 9,2844803 | 9,9917986 | 9,2926817 | 10,7073183 | 54   |
| 7    | 9,2851237 | 9,9917737 | 9,2933500 | 10,7066500 | 53   |
| 8    | 9,2857661 | 9,9917489 | 9,2940172 | 10,7059828 | 52   |
| 9    | 9,2864076 | 9,9917240 | 9,2946836 | 10,7053164 | 51   |
| 10   | 9,2870480 | 9,9916991 | 9,2953489 | 10,7046511 | 50   |
| 11   | 9,2876875 | 9,9916741 | 9,2960134 | 10,7039866 | 49   |
| 12   | 9,2883260 | 9,9916492 | 9,2966769 | 10,7033231 | 48   |
| 13   | 9,2889636 | 9,9916241 | 9,2973395 | 10,7026605 | 47   |
| 14   | 9,2896001 | 9,9915990 | 9,2980011 | 10,7019989 | 46   |
| 15   | 9,2902357 | 9,9915739 | 9,2986618 | 10,7013382 | 45   |
| 16   | 9,2908704 | 9,9915488 | 9,2993216 | 10,7006784 | 44   |
| 17   | 9,2915040 | 9,9915236 | 9,2999804 | 10,7000196 | 43   |
| 18   | 9,2921367 | 9,9914984 | 9,3006383 | 10,6993617 | 42   |
| 19   | 9,2927685 | 9,9914731 | 9,3012954 | 10,6987046 | 41   |
| 20   | 9,2933993 | 9,9914478 | 9,3019514 | 10,6980486 | 40   |
| 21   | 9,2940291 | 9,9914225 | 9,3026066 | 10,6973934 | 39   |
| 22   | 9,2946580 | 9,9913971 | 9,3032609 | 10,6967391 | 38   |
| 23   | 9,2952859 | 9,9913717 | 9,3039143 | 10,6960857 | 37   |
| 24   | 9,2959129 | 9,9913462 | 9,3045667 | 10,6954333 | 36   |
| 25   | 9,2965390 | 9,9913207 | 9,3052183 | 10,6947817 | 35   |
| 26   | 9,2971641 | 9,9912952 | 9,3058689 | 10,6941311 | 34   |
| 27   | 9,2977883 | 9,9912696 | 9,3065187 | 10,6934813 | 33   |
| 28   | 9,2984116 | 9,9912440 | 9,3071675 | 10,6928325 | 32   |
| 29   | 9,2990339 | 9,9912184 | 9,3078155 | 10,6921845 | 31   |
| 30   | 9,2996553 | 9,9911927 | 9,3084626 | 10,6915374 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 78 Degrees.



# 77 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,2996553 | 9,9911927 | 9,3084626 | 10,6915374 | 30   |
| 31   | 9,3002758 | 9,9911670 | 9,3091088 | 10,6908912 | 29   |
| 32   | 9,3008953 | 9,9911412 | 9,3097541 | 10,6902459 | 28   |
| 33   | 9,3015140 | 9,9911154 | 9,3103985 | 10,6896015 | 27   |
| 34   | 9,3021317 | 9,9910896 | 9,3110421 | 10,6889579 | 26   |
| 35   | 9,3027485 | 9,9910637 | 9,3116848 | 10,6883152 | 25   |
| 36   | 9,3033644 | 9,9910378 | 9,3123266 | 10,6876734 | 24   |
| 37   | 9,3039794 | 9,9910119 | 9,3129675 | 10,6870325 | 23   |
| 38   | 9,3045934 | 9,9909859 | 9,3136076 | 10,6863924 | 22   |
| 39   | 9,3052066 | 9,9909598 | 9,3142468 | 10,6857532 | 21   |
| 40   | 9,3058189 | 9,9909338 | 9,3148851 | 10,6851149 | 20   |
| 41   | 9,3064303 | 9,9909077 | 9,3155226 | 10,6844774 | 19   |
| 42   | 9,3070407 | 9,9908815 | 9,3161592 | 10,6838408 | 18   |
| 43   | 9,3076503 | 9,9908553 | 9,3167950 | 10,6832050 | 17   |
| 44   | 9,3082590 | 9,9908291 | 9,3174299 | 10,6825701 | 16   |
| 45   | 9,3088668 | 9,9908029 | 9,3180640 | 10,6819360 | 15   |
| 46   | 9,3094737 | 9,9907766 | 9,3186972 | 10,6813028 | 14   |
| 47   | 9,3100798 | 9,9907502 | 9,3193295 | 10,6806705 | 13   |
| 48   | 9,3106849 | 9,9907239 | 9,3199611 | 10,6800389 | 12   |
| 49   | 9,3112892 | 9,9906974 | 9,3205918 | 10,6794082 | 11   |
| 50   | 9,3118926 | 9,9906710 | 9,3212216 | 10,6787784 | 10   |
| 51   | 9,3124951 | 9,9906445 | 9,3218506 | 10,6781494 | 9    |
| 52   | 9,3130968 | 9,9906180 | 9,3224788 | 10,6775212 | 8    |
| 53   | 9,3136976 | 9,9905914 | 9,3231061 | 10,6768939 | 7    |
| 54   | 9,3142975 | 9,9905648 | 9,3237327 | 10,6762673 | 6    |
| 55   | 9,3148965 | 9,9905382 | 9,3243584 | 10,6756416 | 5    |
| 56   | 9,3154947 | 9,9905115 | 9,3249832 | 10,6750168 | 4    |
| 57   | 9,3160921 | 9,9904848 | 9,3256073 | 10,6743927 | 3    |
| 58   | 9,3166885 | 9,9904580 | 9,3262305 | 10,6737695 | 2    |
| 59   | 9,3172841 | 9,9904312 | 9,3268629 | 10,6731471 | 1    |
| 60   | 9,3178789 | 9,9904044 | 9,3274745 | 10,6725255 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 78 Degrees.

# 12 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,3178789 | 9,9904044 | 9,3274745 | 10,6725255 | 60   |
| 1    | 9,3184728 | 9,9903775 | 9,3280953 | 10,6719047 | 59   |
| 2    | 9,3190659 | 9,9903506 | 9,3287153 | 10,6712847 | 58   |
| 3    | 9,3196581 | 9,9903237 | 9,3293345 | 10,6706655 | 57   |
| 4    | 9,3202495 | 9,9902967 | 9,3299528 | 10,6700472 | 56   |
| 5    | 9,3208400 | 9,9902697 | 9,3305704 | 10,6694296 | 55   |
| 6    | 9,3214297 | 9,9902426 | 9,3311872 | 10,6688128 | 54   |
| 7    | 9,3220186 | 9,9902155 | 9,3318031 | 10,6681969 | 53   |
| 8    | 9,3226066 | 9,9901883 | 9,3324183 | 10,6675817 | 52   |
| 9    | 9,3231938 | 9,9901612 | 9,3330327 | 10,6669673 | 51   |
| 10   | 9,3237802 | 9,9901339 | 9,3336463 | 10,6663537 | 50   |
| 11   | 9,3243657 | 9,9901067 | 9,3342591 | 10,6657409 | 49   |
| 12   | 9,3249505 | 9,9900794 | 9,3348711 | 10,6651289 | 48   |
| 13   | 9,3255344 | 9,9900521 | 9,3354823 | 10,6645177 | 47   |
| 14   | 9,3261174 | 9,9900247 | 9,3360927 | 10,6639073 | 46   |
| 15   | 9,3266997 | 9,9899973 | 9,3367024 | 10,6632976 | 45   |
| 16   | 9,3272811 | 9,9899698 | 9,3373113 | 10,6626887 | 44   |
| 17   | 9,3278617 | 9,9899423 | 9,3379194 | 10,6620806 | 43   |
| 18   | 9,3284416 | 9,9899148 | 9,3385267 | 10,6614733 | 42   |
| 19   | 9,3290206 | 9,9898873 | 9,3391333 | 10,6608667 | 41   |
| 20   | 9,3295988 | 9,9898597 | 9,3397391 | 10,6602609 | 40   |
| 21   | 9,3301761 | 9,9898320 | 9,3403441 | 10,6596559 | 39   |
| 22   | 9,3307527 | 9,9898043 | 9,3409484 | 10,6590516 | 38   |
| 23   | 9,3313285 | 9,9897766 | 9,3415519 | 10,6584481 | 37   |
| 24   | 9,3319035 | 9,9897489 | 9,3421546 | 10,6578454 | 36   |
| 25   | 9,3324777 | 9,9897211 | 9,3427566 | 10,6572434 | 35   |
| 26   | 9,3330511 | 9,9896932 | 9,3433578 | 10,6566422 | 34   |
| 27   | 9,3336237 | 9,9896654 | 9,3439583 | 10,6560417 | 33   |
| 28   | 9,3341955 | 9,9896374 | 9,3445580 | 10,6554420 | 32   |
| 29   | 9,3347665 | 9,9896095 | 9,3451570 | 10,6548430 | 31   |
| 30   | 9,3353368 | 9,9895815 | 9,3457552 | 10,6542448 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

## 77 Degrees.

# 12 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9.3353368 | 9.9895815 | 9.3457552 | 10.6542448 | 30   |
| 31   | 9.3359062 | 9.9895535 | 9.3463527 | 10.6536473 | 29   |
| 32   | 9.3364749 | 9.9895254 | 9.3469494 | 10.6530506 | 28   |
| 33   | 9.3370428 | 9.9894973 | 9.3475454 | 10.6524546 | 27   |
| 34   | 9.3376099 | 9.9894692 | 9.3481407 | 10.6518593 | 26   |
| 35   | 9.3381762 | 9.9894410 | 9.3487352 | 10.6512648 | 25   |
| 36   | 9.3387418 | 9.9894128 | 9.3493290 | 10.6506710 | 24   |
| 37   | 9.3393056 | 9.9893845 | 9.3499220 | 10.6500780 | 23   |
| 38   | 9.3398706 | 9.9893562 | 9.3504143 | 10.6494857 | 22   |
| 39   | 9.3404338 | 9.9893279 | 9.3511059 | 10.6488941 | 21   |
| 40   | 9.3409963 | 9.9892995 | 9.3516968 | 10.6483032 | 20   |
| 41   | 9.3415580 | 9.9892711 | 9.3522869 | 10.6477131 | 19   |
| 42   | 9.3421190 | 9.9892427 | 9.3528763 | 10.6471237 | 18   |
| 43   | 9.3426792 | 9.9892142 | 9.3534650 | 10.6465350 | 17   |
| 44   | 9.3432386 | 9.9891856 | 9.3540530 | 10.6459470 | 16   |
| 45   | 9.3437973 | 9.9891571 | 9.3546402 | 10.6453598 | 15   |
| 46   | 9.3443552 | 9.9891285 | 9.3552267 | 10.6447733 | 14   |
| 47   | 9.3449124 | 9.9890998 | 9.3558126 | 10.6441874 | 13   |
| 48   | 9.3454688 | 9.9890711 | 9.3563977 | 10.6436023 | 12   |
| 49   | 9.3460245 | 9.9890424 | 9.3569821 | 10.6430179 | 11   |
| 50   | 9.3465794 | 9.9890137 | 9.3575658 | 10.6424342 | 10   |
| 51   | 9.3471336 | 9.9889849 | 9.3581487 | 10.6418513 | 9    |
| 52   | 9.3476870 | 9.9889560 | 9.3587310 | 10.6412690 | 8    |
| 53   | 9.3482397 | 9.9889271 | 9.3593126 | 10.6406874 | 7    |
| 54   | 9.3487917 | 9.9888982 | 9.3598935 | 10.6401065 | 6    |
| 55   | 9.3493429 | 9.9888693 | 9.3604736 | 10.6395264 | 5    |
| 56   | 9.3498934 | 9.9888403 | 9.3610531 | 10.6389469 | 4    |
| 57   | 9.3504432 | 9.9888113 | 9.3616319 | 10.6383681 | 3    |
| 58   | 9.3509922 | 9.9887822 | 9.3622100 | 10.6377900 | 2    |
| 59   | 9.3515405 | 9.9887531 | 9.3627874 | 10.6372126 | 1    |
| 60   | 9.3520880 | 9.9887239 | 9.3633641 | 10.6366359 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 77 Degrees.

L

# 13 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,3520880 | 9,9887239 | 9,3633641 | 10,6366359 | 60   |
| 1    | 9,3526349 | 9,9886947 | 9,3639401 | 10,6360599 | 59   |
| 2    | 9,3531810 | 9,9886655 | 9,3645155 | 10,6354845 | 58   |
| 3    | 9,3537264 | 9,9886363 | 9,3650901 | 10,6349099 | 57   |
| 4    | 9,3542710 | 9,9886070 | 9,3656641 | 10,6343359 | 56   |
| 5    | 9,3548150 | 9,9885776 | 9,3662374 | 10,6337626 | 55   |
| 6    | 9,3553582 | 9,9885482 | 9,3668100 | 10,6331900 | 54   |
| 7    | 9,3559007 | 9,9885188 | 9,3673819 | 10,6326181 | 53   |
| 8    | 9,3564426 | 9,9884894 | 9,3679532 | 10,6320468 | 52   |
| 9    | 9,3569836 | 9,9884599 | 9,3685238 | 10,6314762 | 51   |
| 10   | 9,3575240 | 9,9884303 | 9,3690937 | 10,6309063 | 50   |
| 11   | 9,3580637 | 9,9884008 | 9,3696629 | 10,6303371 | 49   |
| 12   | 9,3586027 | 9,9883712 | 9,3702315 | 10,6297685 | 48   |
| 13   | 9,3591409 | 9,9883415 | 9,3707994 | 10,6292006 | 47   |
| 14   | 9,3596785 | 9,9883118 | 9,3713667 | 10,6286333 | 46   |
| 15   | 9,3602154 | 9,9882821 | 9,3719333 | 10,6280667 | 45   |
| 16   | 9,3607515 | 9,9882523 | 9,3724992 | 10,6275008 | 44   |
| 17   | 9,3612870 | 9,9882225 | 9,3730645 | 10,6269355 | 43   |
| 18   | 9,3618217 | 9,9881927 | 9,3736291 | 10,6263709 | 42   |
| 19   | 9,3623558 | 9,9881628 | 9,3741930 | 10,6258070 | 41   |
| 20   | 9,3628892 | 9,9881329 | 9,3747563 | 10,6252437 | 40   |
| 21   | 9,3634219 | 9,9881029 | 9,3753190 | 10,6246810 | 39   |
| 22   | 9,3639539 | 9,9880729 | 9,3758810 | 10,6241190 | 38   |
| 23   | 9,3644852 | 9,9880429 | 9,3764423 | 10,6235577 | 37   |
| 24   | 9,3650158 | 9,9880128 | 9,3770030 | 10,6229970 | 36   |
| 25   | 9,3655458 | 9,9879827 | 9,3775631 | 10,6224369 | 35   |
| 26   | 9,3660750 | 9,9879525 | 9,3781225 | 10,6218775 | 34   |
| 27   | 9,3666036 | 9,9879223 | 9,3786813 | 10,6213187 | 33   |
| 28   | 9,3671315 | 9,9878921 | 9,3792394 | 10,6207606 | 32   |
| 29   | 9,3676587 | 9,9878618 | 9,3797969 | 10,6202031 | 31   |
| 30   | 9,3681853 | 9,9878315 | 9,3803537 | 10,6196463 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

76 Degrees

# 73 Degrees.

| Min.      | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|-----------|-----------|-----------|-----------|------------|------|
| 30        | 9,3681853 | 9,9878315 | 9,3803537 | 10,6196463 | 30   |
| 31        | 9,3687111 | 9,9878012 | 9,3809100 | 10,6190900 | 29   |
| 32        | 9,3692363 | 9,9877708 | 9,3814655 | 10,6185345 | 28   |
| 33        | 9,3697608 | 9,9877404 | 9,3820205 | 10,6179795 | 27   |
| 34        | 9,3702847 | 9,9877099 | 9,3825748 | 10,6174252 | 26   |
| 35        | 9,3708079 | 9,9876794 | 9,3831285 | 10,6168715 | 25   |
| 36        | 9,3713304 | 9,9876488 | 9,3836816 | 10,6163184 | 24   |
| 37        | 9,3718523 | 9,9876183 | 9,3842340 | 10,6157660 | 23   |
| 38        | 9,3723735 | 9,9875876 | 9,3847858 | 10,6152142 | 22   |
| 39        | 9,3728940 | 9,9875570 | 9,3853370 | 10,6146630 | 21   |
| 40        | 9,3734139 | 9,9875263 | 9,3858876 | 10,6141124 | 20   |
| 41        | 9,3739331 | 9,9874955 | 9,3864376 | 10,6135624 | 19   |
| 42        | 9,3744517 | 9,9874648 | 9,3869869 | 10,6130131 | 18   |
| 43        | 9,3749696 | 9,9874339 | 9,3875356 | 10,6124644 | 17   |
| 44        | 9,3754868 | 9,9874031 | 9,3880837 | 10,6119163 | 16   |
| 45        | 9,3760034 | 9,9873722 | 9,3886312 | 10,6113688 | 15   |
| 46        | 9,3765194 | 9,9873413 | 9,3891781 | 10,6108219 | 14   |
| 47        | 9,3770347 | 9,9873103 | 9,3897244 | 10,6102756 | 13   |
| 48        | 9,3775493 | 9,9872793 | 9,3902700 | 10,6097300 | 12   |
| 49        | 9,3780633 | 9,9872482 | 9,3908151 | 10,6091849 | 11   |
| 50        | 9,3785767 | 9,9872171 | 9,3913595 | 10,6086405 | 10   |
| 51        | 9,3790894 | 9,9871860 | 9,3919034 | 10,6080966 | 9    |
| 52        | 9,3796015 | 9,9871549 | 9,3924466 | 10,6075534 | 8    |
| 53        | 9,3801129 | 9,9871236 | 9,3929893 | 10,6070107 | 7    |
| 54        | 9,3806237 | 9,9870924 | 9,3935313 | 10,6064687 | 6    |
| 55        | 9,3811339 | 9,9870611 | 9,3940727 | 10,6059275 | 5    |
| 56        | 9,3816434 | 9,9870298 | 9,3946136 | 10,6053864 | 4    |
| 57        | 9,3821523 | 9,9869984 | 9,3951538 | 10,6048462 | 3    |
| 58        | 9,3826605 | 9,9869670 | 9,3956935 | 10,6043065 | 2    |
| 59        | 9,3831682 | 9,9869356 | 9,3962326 | 10,6037674 | 1    |
| 60        | 9,3836752 | 9,9869041 | 9,3967711 | 10,6032289 | 0    |
| Sine Com. | Sine.     | Tan. Com. | Tangent.  |            | Min. |

# 76 Degrees.

# 14 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,3836752 | 9,9869041 | 9,3967711 | 10,6032289 | 60   |
| 1    | 9,3841815 | 9,9868726 | 9,3973089 | 10,6026911 | 59   |
| 2    | 9,3846873 | 9,9868410 | 9,3978463 | 10,6021537 | 58   |
| 3    | 9,3851924 | 9,9868094 | 9,3983830 | 10,6016170 | 57   |
| 4    | 9,3856969 | 9,9867778 | 9,3989191 | 10,6010809 | 56   |
| 5    | 9,3862008 | 9,9867461 | 9,3994547 | 10,6005453 | 55   |
| 6    | 9,3867040 | 9,9867144 | 9,3999895 | 10,6000104 | 54   |
| 7    | 9,3872067 | 9,9866827 | 9,4005240 | 10,5994760 | 53   |
| 8    | 9,3877087 | 9,9866509 | 9,4010578 | 10,5989422 | 52   |
| 9    | 9,3882101 | 9,9866191 | 9,4015910 | 10,5984090 | 51   |
| 10   | 9,3887109 | 9,9865872 | 9,4021237 | 10,5978763 | 50   |
| 11   | 9,3892111 | 9,9865553 | 9,4026558 | 10,5973442 | 49   |
| 12   | 9,3897106 | 9,9865233 | 9,4031873 | 10,5968127 | 48   |
| 13   | 9,3902096 | 9,9864913 | 9,4037182 | 10,5962818 | 47   |
| 14   | 9,3907079 | 9,9864593 | 9,4042486 | 10,5957514 | 46   |
| 15   | 9,3912057 | 9,9864273 | 9,4047784 | 10,5952216 | 45   |
| 16   | 9,3917028 | 9,9863952 | 9,4053076 | 10,5946924 | 44   |
| 17   | 9,3921993 | 9,9863630 | 9,4058363 | 10,5941637 | 43   |
| 18   | 9,3926952 | 9,9863308 | 9,4063644 | 10,5936356 | 42   |
| 19   | 9,3931905 | 9,9862986 | 9,4068919 | 10,5931081 | 41   |
| 20   | 9,3936852 | 9,9862663 | 9,4074189 | 10,5925811 | 40   |
| 21   | 9,3941794 | 9,9862340 | 9,4079453 | 10,5920547 | 39   |
| 22   | 9,3946729 | 9,9862017 | 9,4084712 | 10,5915288 | 38   |
| 23   | 9,3951658 | 9,9861693 | 9,4089965 | 10,5910035 | 37   |
| 24   | 9,3956581 | 9,9861369 | 9,4095212 | 10,5904788 | 36   |
| 25   | 9,3961499 | 9,9861045 | 9,4100454 | 10,5899546 | 35   |
| 26   | 9,3966410 | 9,9860720 | 9,4105690 | 10,5894310 | 34   |
| 27   | 9,3971315 | 9,9860394 | 9,4110921 | 10,5889079 | 33   |
| 28   | 9,3976215 | 9,9860069 | 9,4116146 | 10,5883854 | 32   |
| 29   | 9,3981109 | 9,9859742 | 9,4121366 | 10,5878634 | 31   |
| 30   | 9,3985996 | 9,9859416 | 9,4126581 | 10,5873419 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 75 Degrees.

# 14 Degrees

|    | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|----|-----------|-----------|-----------|------------|------|
| 30 | 9,3985996 | 9,9859416 | 9,4126581 | 10,5873410 | 30   |
| 31 | 9,3990878 | 9,9859089 | 9,4131789 | 10,5868211 | 29   |
| 32 | 9,3995754 | 9,9858762 | 9,4136993 | 10,5863007 | 28   |
| 33 | 9,4000625 | 9,9858434 | 9,4142191 | 10,5857809 | 27   |
| 34 | 9,4005489 | 9,9858106 | 9,4147383 | 10,5852617 | 26   |
| 35 | 9,4010348 | 9,9857777 | 9,4152570 | 10,5847430 | 25   |
| 36 | 9,4015201 | 9,9857449 | 9,4157752 | 10,5842248 | 24   |
| 37 | 9,4020048 | 9,9857119 | 9,4162928 | 10,5837072 | 23   |
| 38 | 9,4024889 | 9,9856790 | 9,4168099 | 10,5831901 | 22   |
| 39 | 9,4029724 | 9,9856460 | 9,4173265 | 10,5826735 | 21   |
| 40 | 9,4034554 | 9,9856129 | 9,4178425 | 10,5821575 | 20   |
| 41 | 9,4039378 | 9,9855798 | 9,4183580 | 10,5816420 | 19   |
| 42 | 9,4044196 | 9,9855467 | 9,4188729 | 10,5811271 | 18   |
| 43 | 9,4049009 | 9,9855135 | 9,4193874 | 10,5806126 | 17   |
| 44 | 9,4053816 | 9,9854803 | 9,4199013 | 10,5800987 | 16   |
| 45 | 9,4058617 | 9,9854471 | 9,4204146 | 10,5795854 | 15   |
| 46 | 9,4063413 | 9,9854138 | 9,4209275 | 10,5790725 | 14   |
| 47 | 9,4068203 | 9,9853805 | 9,4214398 | 10,5785602 | 13   |
| 48 | 9,4072987 | 9,9853471 | 9,4219515 | 10,5780485 | 12   |
| 49 | 9,4077766 | 9,9853138 | 9,4224628 | 10,5775372 | 11   |
| 50 | 9,4082539 | 9,9852803 | 9,4229735 | 10,5770265 | 10   |
| 51 | 9,4087306 | 9,9852468 | 9,4234838 | 10,5765162 | 9    |
| 52 | 9,4092068 | 9,9852133 | 9,4239935 | 10,5760065 | 8    |
| 53 | 9,4096824 | 9,9851798 | 9,4245026 | 10,5754974 | 7    |
| 54 | 9,4101575 | 9,9851462 | 9,4250113 | 10,5749887 | 6    |
| 55 | 9,4106320 | 9,9851125 | 9,4255194 | 10,5744806 | 5    |
| 56 | 9,4111059 | 9,9850789 | 9,4260271 | 10,5739729 | 4    |
| 57 | 9,4115793 | 9,9850452 | 9,4265342 | 10,5734658 | 3    |
| 58 | 9,4120522 | 9,9850114 | 9,4270408 | 10,5729592 | 2    |
| 59 | 9,4125245 | 9,9849776 | 9,4275469 | 10,5724531 | 1    |
| 60 | 9,4129962 | 9,9849438 | 9,4280525 | 10,5719475 | 0    |
|    | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

75 Degrees.



# 75 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,4129962 | 9,9849438 | 9,4280525 | 10,5719475 | 60   |
| 1    | 9,4134674 | 9,9849099 | 9,4285575 | 10,5714425 | 59   |
| 2    | 9,4139381 | 9,9848760 | 9,4290621 | 10,5709379 | 58   |
| 3    | 9,4144082 | 9,9848420 | 9,4295661 | 10,5704339 | 57   |
| 4    | 9,4148778 | 9,9848081 | 9,4300697 | 10,5699303 | 56   |
| 5    | 9,4153468 | 9,9847740 | 9,4305727 | 10,5694273 | 55   |
| 6    | 9,4158152 | 9,9847400 | 9,4310753 | 10,5689247 | 54   |
| 7    | 9,4162832 | 9,9847059 | 9,4315773 | 10,5684227 | 53   |
| 8    | 9,4167506 | 9,9846717 | 9,4320789 | 10,5679211 | 52   |
| 9    | 9,4172174 | 9,9846375 | 9,4325799 | 10,5674201 | 51   |
| 10   | 9,4176837 | 9,9846033 | 9,4330804 | 10,5669196 | 50   |
| 11   | 9,4181495 | 9,9845690 | 9,4335805 | 10,5664195 | 49   |
| 12   | 9,4186148 | 9,9845347 | 9,4340800 | 10,5659200 | 48   |
| 13   | 9,4190795 | 9,9845004 | 9,4345791 | 10,5654209 | 47   |
| 14   | 9,4195436 | 9,9844660 | 9,4350776 | 10,5649224 | 46   |
| 15   | 9,4200073 | 9,9844316 | 9,4355757 | 10,5644243 | 45   |
| 16   | 9,4204704 | 9,9843971 | 9,4360733 | 10,5639267 | 44   |
| 17   | 9,4209330 | 9,9843626 | 9,4365704 | 10,5634296 | 43   |
| 18   | 9,4213950 | 9,9843281 | 9,4370670 | 10,5629330 | 42   |
| 19   | 9,4218566 | 9,9842935 | 9,4375631 | 10,5624369 | 41   |
| 20   | 9,4223176 | 9,9842589 | 9,4380587 | 10,5619413 | 40   |
| 21   | 9,4227780 | 9,9842242 | 9,4385538 | 10,5614462 | 39   |
| 22   | 9,4232380 | 9,9841895 | 9,4390485 | 10,5609515 | 38   |
| 23   | 9,4236974 | 9,9841548 | 9,4395426 | 10,5604574 | 37   |
| 24   | 9,4241563 | 9,9841200 | 9,4400363 | 10,5599637 | 36   |
| 25   | 9,4246147 | 9,9840852 | 9,4405295 | 10,5594705 | 35   |
| 26   | 9,4250726 | 9,9840503 | 9,4410222 | 10,5589778 | 34   |
| 27   | 9,4255299 | 9,9840154 | 9,4415145 | 10,5584855 | 33   |
| 28   | 9,4259867 | 9,9839805 | 9,4420062 | 10,5579938 | 32   |
| 29   | 9,4264430 | 9,9839455 | 9,4424975 | 10,5575025 | 31   |
| 30   | 9,4268988 | 9,9839105 | 9,4429883 | 10,5570117 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 74 Degrees.



# 15 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,4268988 | 9, 0105   | 9,4429883 | 10,5570117 | 30   |
| 31   | 9,4273541 | 9,98 155  | 9,4434786 | 10,5565214 | 29   |
| 32   | 9,4278089 | 9,9838404 | 9,4439685 | 10,5560315 | 28   |
| 33   | 9,4282631 | 9,9838052 | 9,4444579 | 10,5555421 | 27   |
| 34   | 9,4287169 | 9,9837701 | 9,4449468 | 10,5550532 | 26   |
| 35   | 9,4291701 | 9,9837348 | 9,4454352 | 10,5545648 | 25   |
| 36   | 9,4296228 | 9,9836996 | 9,4459232 | 10,5540768 | 24   |
| 37   | 9,4300750 | 9,9836643 | 9,4464107 | 10,5535893 | 23   |
| 38   | 9,4305267 | 9,9836290 | 9,4468978 | 10,5531022 | 22   |
| 39   | 9,4309779 | 9,9835936 | 9,4473843 | 10,5526157 | 21   |
| 40   | 9,4314286 | 9,9835582 | 9,4478704 | 10,5521296 | 20   |
| 41   | 9,4318788 | 9,9835227 | 9,4483561 | 10,5516439 | 19   |
| 42   | 9,4323285 | 9,9834872 | 9,4488413 | 10,5511587 | 18   |
| 43   | 9,4327777 | 9,9834517 | 9,4493260 | 10,5506740 | 17   |
| 44   | 9,4332264 | 9,9834161 | 9,4498102 | 10,5501898 | 16   |
| 45   | 9,4336746 | 9,9833805 | 9,4502940 | 10,5497060 | 15   |
| 46   | 9,4341223 | 9,9833449 | 9,4507774 | 10,5492226 | 14   |
| 47   | 9,4345694 | 9,9833092 | 9,4512602 | 10,5487398 | 13   |
| 48   | 9,4350161 | 9,9832735 | 9,4517427 | 10,5482573 | 12   |
| 49   | 9,4354623 | 9,9832377 | 9,4522246 | 10,5477754 | 11   |
| 50   | 9,4359080 | 9,9832019 | 9,4527061 | 10,5472939 | 10   |
| 51   | 9,4363532 | 9,9831661 | 9,4531872 | 10,5468128 | 9    |
| 52   | 9,4367980 | 9,9831302 | 9,4536678 | 10,5463322 | 8    |
| 53   | 9,4372422 | 9,9830942 | 9,4541479 | 10,5458521 | 7    |
| 54   | 9,4376859 | 9,9830583 | 9,4546276 | 10,5453724 | 6    |
| 55   | 9,4381292 | 9,9830223 | 9,4551069 | 10,5448931 | 5    |
| 56   | 9,4385719 | 9,9829862 | 9,4555857 | 10,5444143 | 4    |
| 57   | 9,4390142 | 9,9829501 | 9,4560641 | 10,5439359 | 3    |
| 58   | 9,4394560 | 9,9829140 | 9,4565420 | 10,5434580 | 2    |
| 59   | 9,4398973 | 9,9828778 | 9,4570194 | 10,5429806 | 1    |
| 60   | 9,4403381 | 9,9828416 | 9,4574964 | 10,5425036 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 74 Degrees.

# 16 Degrees

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,4403381 | 9,9828416 | 9,4574964 | 10,5425036 | 60   |
| 1    | 9,4407784 | 9,9828054 | 9,4579730 | 10,5420270 | 59   |
| 2    | 9,4412182 | 9,9827691 | 9,4584491 | 10,5415509 | 58   |
| 3    | 9,4416576 | 9,9827328 | 9,4589248 | 10,5410752 | 57   |
| 4    | 9,4420955 | 9,9826964 | 9,4594001 | 10,5405999 | 56   |
| 5    | 9,4425249 | 9,9826600 | 9,4598749 | 10,5401251 | 55   |
| 6    | 9,4429728 | 9,9826236 | 9,4603492 | 10,5396508 | 54   |
| 7    | 9,4434103 | 9,9825871 | 9,4608232 | 10,5391768 | 53   |
| 8    | 9,4438472 | 9,9825506 | 9,4612957 | 10,5387033 | 52   |
| 9    | 9,4442837 | 9,9825140 | 9,4617697 | 10,5382303 | 51   |
| 10   | 9,4447197 | 9,9824774 | 9,4622423 | 10,5377577 | 50   |
| 11   | 9,4451553 | 9,9824408 | 9,4627145 | 10,5372855 | 49   |
| 12   | 9,4455903 | 9,9824041 | 9,4631863 | 10,5368137 | 48   |
| 13   | 9,4460250 | 9,9823674 | 9,4636576 | 10,5363424 | 47   |
| 14   | 9,4464591 | 9,9823306 | 9,4641285 | 10,5358715 | 46   |
| 15   | 9,4468927 | 9,9822938 | 9,4645990 | 10,5354010 | 45   |
| 16   | 9,4473259 | 9,9822569 | 9,4650690 | 10,5349310 | 44   |
| 17   | 9,4477586 | 9,9822201 | 9,4655386 | 10,5344614 | 43   |
| 18   | 9,4481909 | 9,9821831 | 9,4660078 | 10,5339922 | 42   |
| 19   | 9,4486227 | 9,9821462 | 9,4664765 | 10,5335235 | 41   |
| 20   | 9,4490540 | 9,9821092 | 9,4669448 | 10,5330552 | 40   |
| 21   | 9,4494849 | 9,9820721 | 9,4674127 | 10,5325873 | 39   |
| 22   | 9,4499153 | 9,9820351 | 9,4678802 | 10,5321198 | 38   |
| 23   | 9,4503452 | 9,9819979 | 9,4683473 | 10,5316527 | 37   |
| 24   | 9,4507747 | 9,9819608 | 9,4688139 | 10,5311861 | 36   |
| 25   | 9,4512037 | 9,9819236 | 9,4692801 | 10,5307199 | 35   |
| 26   | 9,4516322 | 9,9818863 | 9,4697459 | 10,5302541 | 34   |
| 27   | 9,4520603 | 9,9818490 | 9,4702112 | 10,5297888 | 33   |
| 28   | 9,4524879 | 9,9818117 | 9,4706762 | 10,5293238 | 32   |
| 29   | 9,4529151 | 9,9817744 | 9,4711407 | 10,5288593 | 31   |
| 30   | 9,4533418 | 9,9817370 | 9,4716048 | 10,5283952 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 73 Degrees

15 ~~beginning~~

M

# 17 Degrees.

| Min. | Sine      | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,4659353 | 9,9805963 | 9,4853390 | 10,5146610 | 60   |
| 1    | 9,4663483 | 9,9805577 | 9,4857907 | 10,5142093 | 59   |
| 2    | 9,4667609 | 9,9805190 | 9,4862419 | 10,5137581 | 58   |
| 3    | 9,4671730 | 9,9804803 | 9,4866928 | 10,5133072 | 57   |
| 4    | 9,4675848 | 9,9804415 | 9,4871433 | 10,5128567 | 56   |
| 5    | 9,4679960 | 9,9804027 | 9,4875933 | 10,5124067 | 55   |
| 6    | 9,4684069 | 9,9803639 | 9,4880430 | 10,5119570 | 54   |
| 7    | 9,4688173 | 9,9803250 | 9,4884924 | 10,5115076 | 53   |
| 8    | 9,4692273 | 9,9802860 | 9,4889413 | 10,5110587 | 52   |
| 9    | 9,4696369 | 9,9802471 | 9,4893898 | 10,5106102 | 51   |
| 10   | 9,4700461 | 9,9802081 | 9,4898380 | 10,5101620 | 50   |
| 11   | 9,4704548 | 9,9801690 | 9,4902858 | 10,5097142 | 49   |
| 12   | 9,4708631 | 9,9801299 | 9,4907332 | 10,5092668 | 48   |
| 13   | 9,4712710 | 9,9800908 | 9,4911802 | 10,5088198 | 47   |
| 14   | 9,4716785 | 9,9800516 | 9,4916269 | 10,5083731 | 46   |
| 15   | 9,4720856 | 9,9800124 | 9,4920731 | 10,5079269 | 45   |
| 16   | 9,4724922 | 9,9799732 | 9,4925190 | 10,5074810 | 44   |
| 17   | 9,4728985 | 9,9799339 | 9,4929646 | 10,5070354 | 43   |
| 18   | 9,4733043 | 9,9798946 | 9,4934097 | 10,5065903 | 42   |
| 19   | 9,4737097 | 9,9798552 | 9,4938545 | 10,5061455 | 41   |
| 20   | 9,4741146 | 9,9798158 | 9,4942988 | 10,5057012 | 40   |
| 21   | 9,4745192 | 9,9797764 | 9,4947429 | 10,5052571 | 39   |
| 22   | 9,4749234 | 9,9797369 | 9,4951865 | 10,5048135 | 38   |
| 23   | 9,4753271 | 9,9796973 | 9,4956298 | 10,5043702 | 37   |
| 24   | 9,4757304 | 9,9796578 | 9,4960727 | 10,5039273 | 36   |
| 25   | 9,4761334 | 9,9796182 | 9,4965152 | 10,5034848 | 35   |
| 26   | 9,4765359 | 9,9795785 | 9,4969574 | 10,5030426 | 34   |
| 27   | 9,4769380 | 9,9795388 | 9,4973991 | 10,5026009 | 33   |
| 28   | 9,4773396 | 9,9794991 | 9,4978406 | 10,5021594 | 32   |
| 29   | 9,4777409 | 9,9794593 | 9,4982816 | 10,5017184 | 31   |
| 30   | 9,4781418 | 9,9794195 | 9,4987223 | 10,5012777 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

72 Degrees

• 17 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,4781418 | 9,9794195 | 9,4987223 | 10,5012777 | 30   |
| 31   | 9,4785423 | 9,9793796 | 9,4991626 | 10,5008374 | 29   |
| 32   | 9,4789423 | 9,9793398 | 9,4996026 | 10,5003974 | 28   |
| 33   | 9,4793420 | 9,9792998 | 9,5000422 | 10,4999578 | 27   |
| 34   | 9,4797412 | 9,9792599 | 9,5004814 | 10,4995186 | 26   |
| 35   | 9,4801401 | 9,9792198 | 9,5009203 | 10,4990797 | 25   |
| 36   | 9,4805385 | 9,9791798 | 9,5013588 | 10,4986412 | 24   |
| 37   | 9,4809366 | 9,9791397 | 9,5017969 | 10,4982031 | 23   |
| 38   | 9,4813342 | 9,9790996 | 9,5022347 | 10,4977653 | 22   |
| 39   | 9,4817315 | 9,9790594 | 9,5026721 | 10,4973279 | 21   |
| 40   | 9,4821283 | 9,9790192 | 9,5031092 | 10,4968908 | 20   |
| 41   | 9,4825248 | 9,9789789 | 9,5035459 | 10,4964541 | 19   |
| 42   | 9,4829208 | 9,9789386 | 9,5039822 | 10,4960178 | 18   |
| 43   | 9,4833165 | 9,9788983 | 9,5044182 | 10,4955818 | 17   |
| 44   | 9,4837117 | 9,9788579 | 9,5048538 | 10,4951462 | 16   |
| 45   | 9,4841066 | 9,9788175 | 9,5052891 | 10,4947109 | 15   |
| 46   | 9,4845010 | 9,9787770 | 9,5057240 | 10,4942760 | 14   |
| 47   | 9,4848951 | 9,9787365 | 9,5061586 | 10,4938414 | 13   |
| 48   | 9,4852888 | 9,9786960 | 9,5065928 | 10,4934072 | 12   |
| 49   | 9,4856820 | 9,9786554 | 9,5070267 | 10,4929733 | 11   |
| 50   | 9,4860749 | 9,9786148 | 9,5074602 | 10,4925398 | 10   |
| 51   | 9,4864674 | 9,9785741 | 9,5078933 | 10,4921067 | 9    |
| 52   | 9,4868595 | 9,9785334 | 9,5083261 | 10,4916739 | 8    |
| 53   | 9,4872512 | 9,9784927 | 9,5087586 | 10,4912414 | 7    |
| 54   | 9,4876426 | 9,9784519 | 9,5091907 | 10,4908093 | 6    |
| 55   | 9,4880335 | 9,9784111 | 9,5096224 | 10,4903776 | 5    |
| 56   | 9,4884240 | 9,9783702 | 9,5100539 | 10,4899461 | 4    |
| 57   | 9,4888142 | 9,9783293 | 9,5104849 | 10,4895151 | 3    |
| 58   | 9,4892040 | 9,9782883 | 9,5109156 | 10,4890844 | 2    |
| 59   | 9,4895934 | 9,9782474 | 9,5113460 | 10,4886540 | 1    |
| 60   | 9,4899824 | 9,9782063 | 9,5117760 | 10,4882240 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

72 Degrees.

18 Degrees

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,4899824 | 9,9782063 | 9,5117760 | 10,4882240 | 60.  |
| 1    | 9,4903710 | 9,9781653 | 9,5122057 | 10,4877943 | 59.  |
| 2    | 9,4907592 | 9,9781241 | 9,5126351 | 10,4873649 | 58.  |
| 3    | 9,4911471 | 9,9780830 | 9,5130641 | 10,4869359 | 57.  |
| 4    | 9,4915345 | 9,9780418 | 9,5134927 | 10,4865073 | 56.  |
| 5    | 9,4919216 | 9,9780006 | 9,5139210 | 10,4860790 | 55.  |
| 6    | 9,4923083 | 9,9779593 | 9,5143490 | 10,4856510 | 54.  |
| 7    | 9,4926946 | 9,9779180 | 9,5147766 | 10,4852234 | 53.  |
| 8    | 9,4930806 | 9,9778766 | 9,5152039 | 10,4847961 | 52.  |
| 9    | 9,4934661 | 9,9778353 | 9,5156309 | 10,4843691 | 51.  |
| 10   | 9,4938513 | 9,9777938 | 9,5160575 | 10,4839425 | 50.  |
| 11   | 9,4942361 | 9,9777523 | 9,5164838 | 10,4835162 | 49.  |
| 12   | 9,4946205 | 9,9777108 | 9,5169097 | 10,4830903 | 48.  |
| 13   | 9,4950046 | 9,9776693 | 9,5173353 | 10,4826647 | 47.  |
| 14   | 9,4953883 | 9,9776277 | 9,5177606 | 10,4822394 | 46.  |
| 15   | 9,4957716 | 9,9775860 | 9,5181855 | 10,4818145 | 45.  |
| 16   | 9,4961545 | 9,9775444 | 9,5186101 | 10,4813899 | 44.  |
| 17   | 9,4965370 | 9,9775026 | 9,5190341 | 10,4809656 | 43.  |
| 18   | 9,4969192 | 9,9774609 | 9,5194583 | 10,4805417 | 42.  |
| 19   | 9,4973010 | 9,9774191 | 9,5198819 | 10,4801181 | 41.  |
| 20   | 9,4976824 | 9,9773772 | 9,5203057 | 10,4796948 | 40.  |
| 21   | 9,4980635 | 9,9773354 | 9,5207282 | 10,4792718 | 39.  |
| 22   | 9,4984442 | 9,9772934 | 9,5211508 | 10,4788492 | 38.  |
| 23   | 9,4988245 | 9,9772515 | 9,5215730 | 10,4784270 | 37.  |
| 24   | 9,4992045 | 9,9772095 | 9,5219950 | 10,4780050 | 36.  |
| 25   | 9,4995840 | 9,9771674 | 9,5224166 | 10,4775834 | 35.  |
| 26   | 9,4999633 | 9,9771253 | 9,5228379 | 10,4771621 | 34.  |
| 27   | 9,5003421 | 9,9770832 | 9,5232589 | 10,4767411 | 33.  |
| 28   | 9,5007206 | 9,9770410 | 9,5236795 | 10,4763205 | 32.  |
| 29   | 9,5010987 | 9,9769988 | 9,5240999 | 10,4759001 | 31.  |
| 30   | 9,5014764 | 9,9769566 | 9,5245199 | 10,4754801 | 30.  |
|      | Sine Com. | Sine.     | Tan. Com  | Tangent.   | Min. |

71 Degrees

# 18 Degrees.

| Min. | Sine.       | Sine. Comp. | Tangent.   | Tan. Comp. |    |
|------|-------------|-------------|------------|------------|----|
| 20   | 9,5014764   | 9,9769566   | 9,5245199  | 10,4754801 | 30 |
| 31   | 9,5018538   | 9,9769143   | 9,5249395  | 10,4750605 | 29 |
| 32   | 9,5022308   | 9,9768720   | 9,5253589  | 10,4746411 | 28 |
| 33   | 9,5026075   | 9,9768296   | 9,5257779  | 10,4742221 | 27 |
| 34   | 9,5029838   | 9,9767872   | 9,5261966  | 10,4738034 | 26 |
| 35   | 9,5033597   | 9,9767447   | 9,5266150  | 10,4733850 | 25 |
| 36   | 9,5037353   | 9,9767022   | 9,5270331  | 10,4729669 | 24 |
| 37   | 9,5041105   | 9,9766597   | 9,5274508  | 10,4725492 | 23 |
| 38   | 9,5044853   | 9,9766171   | 9,5278682  | 10,4721318 | 22 |
| 39   | 9,5048598   | 9,9765745   | 9,5282853  | 10,4717147 | 21 |
| 40   | 9,5052339   | 9,9765318   | 9,5287021  | 10,4712979 | 20 |
| 41   | 9,5056077   | 9,9764891   | 9,5291186  | 10,4708814 | 19 |
| 42   | 9,5059811   | 9,9764464   | 9,5295347  | 10,4704653 | 18 |
| 43   | 9,5063542   | 9,9764036   | 9,5299505  | 10,4700495 | 17 |
| 44   | 9,5067268   | 9,9763608   | 9,5303661  | 10,4696339 | 16 |
| 45   | 9,5070992   | 9,9763179   | 9,5307813  | 10,4692187 | 15 |
| 46   | 9,5074712   | 9,9762750   | 9,5311961  | 10,4688039 | 14 |
| 47   | 9,5078428   | 9,9762321   | 9,5316107  | 10,4683893 | 13 |
| 48   | 9,5082141   | 9,9761891   | 9,5320250  | 10,4679750 | 12 |
| 49   | 9,5085850   | 9,9761461   | 9,5324389  | 10,4675611 | 11 |
| 50   | 9,5089556   | 9,9761030   | 9,5328526  | 10,4671474 | 10 |
| 51   | 9,5093258   | 9,9760599   | 9,5332659  | 10,4667341 | 9  |
| 52   | 9,5096956   | 9,9760167   | 9,5336789  | 10,4663211 | 8  |
| 53   | 9,5100651   | 9,9759736   | 9,5340916  | 10,4659084 | 7  |
| 54   | 9,5104343   | 9,9759303   | 9,5345040  | 10,4654960 | 6  |
| 55   | 9,5108031   | 9,9758870   | 9,5349161  | 10,4650839 | 5  |
| 56   | 9,5111716   | 9,9758437   | 9,5353278  | 10,4646722 | 4  |
| 57   | 9,5115397   | 9,9758004   | 9,5357393  | 10,4642607 | 3  |
| 58   | 9,5119074   | 9,9757570   | 9,5361505  | 10,4638495 | 2  |
| 59   | 9,5122749   | 9,9757135   | 9,5365613  | 10,4634387 | 1  |
| 60   | 9,5126419   | 9,9756701   | 9,5369719  | 10,4630281 | 0  |
|      | Sine. Comp. | Sine.       | Tan. Comp. | Tangent.   |    |

# 71 Degrees.

# 19 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,5126419 | 9,9756701 | 9,5369718 | 10,4630281 | 60   |
| 1    | 9,5130086 | 9,9756265 | 9,5373821 | 10,4626179 | 59   |
| 2    | 9,5133750 | 9,9755830 | 9,5377920 | 10,4622080 | 58   |
| 3    | 9,5137410 | 9,9755394 | 9,5382017 | 10,4617983 | 57   |
| 4    | 9,5141067 | 9,9754957 | 9,5386110 | 10,4613890 | 56   |
| 5    | 9,5144721 | 9,9754521 | 9,5390200 | 10,4609800 | 55   |
| 6    | 9,5148371 | 9,9754083 | 9,5394287 | 10,4605713 | 54   |
| 7    | 9,5152017 | 9,9753646 | 9,5398371 | 10,4601629 | 53   |
| 8    | 9,5155660 | 9,9753208 | 9,5402453 | 10,4597547 | 52   |
| 9    | 9,5159300 | 9,9752769 | 9,5406531 | 10,4593469 | 51   |
| 10   | 9,5162936 | 9,9752330 | 9,5410606 | 10,4589394 | 50   |
| 11   | 9,5166569 | 9,9751891 | 9,5414678 | 10,4585322 | 49   |
| 12   | 9,5170198 | 9,9751451 | 9,5418747 | 10,4581253 | 48   |
| 13   | 9,5173824 | 9,9751011 | 9,5422813 | 10,4577187 | 47   |
| 14   | 9,5177447 | 9,9750570 | 9,5426877 | 10,4573123 | 46   |
| 15   | 9,5181066 | 9,9750129 | 9,5430937 | 10,4569063 | 45   |
| 16   | 9,5184682 | 9,9749688 | 9,5434994 | 10,4565006 | 44   |
| 17   | 9,5188295 | 9,9749246 | 9,5439048 | 10,4560952 | 43   |
| 18   | 9,5191904 | 9,9748804 | 9,5443100 | 10,4556900 | 42   |
| 19   | 9,5195510 | 9,9748361 | 9,5447148 | 10,4552852 | 41   |
| 20   | 9,5199112 | 9,9747918 | 9,5451193 | 10,4548807 | 40   |
| 21   | 9,5202711 | 9,9747475 | 9,5455236 | 10,4544764 | 39   |
| 22   | 9,5206307 | 9,9747031 | 9,5459276 | 10,4540724 | 38   |
| 23   | 9,5209899 | 9,9746587 | 9,5463312 | 10,4536688 | 37   |
| 24   | 9,5213488 | 9,9746142 | 9,5467346 | 10,4532654 | 36   |
| 25   | 9,5217074 | 9,9745697 | 9,5471377 | 10,4528623 | 35   |
| 26   | 9,5220656 | 9,9745252 | 9,5475405 | 10,4524595 | 34   |
| 27   | 9,5224235 | 9,9744806 | 9,5479430 | 10,4520570 | 33   |
| 28   | 9,5227811 | 9,9744359 | 9,5483452 | 10,4516548 | 32   |
| 29   | 9,5231383 | 9,9743913 | 9,5487471 | 10,4512529 | 31   |
| 30   | 9,5234953 | 9,9743466 | 9,5491487 | 10,4508513 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 20 Degrees.



# 19 Degrees.

| Min.      | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|-----------|-----------|-----------|-----------|------------|------|
| 30        | 9,5234953 | 9,9743466 | 9,5491487 | 10,4598513 | 30   |
| 31        | 9,5238518 | 9,9743018 | 9,5495500 | 10,4504500 | 29   |
| 32        | 9,5242081 | 9,9742570 | 9,5499511 | 10,4500489 | 28   |
| 33        | 9,5245640 | 9,9742122 | 9,5503519 | 10,4496481 | 27   |
| 34        | 9,5249196 | 9,9741673 | 9,5507523 | 10,4492477 | 26   |
| 35        | 9,5252749 | 9,9741224 | 9,5511525 | 10,4488475 | 25   |
| 36        | 9,5256298 | 9,9740774 | 9,5515524 | 10,4484476 | 24   |
| 37        | 9,5259844 | 9,9740324 | 9,5519521 | 10,4480479 | 23   |
| 38        | 9,5263387 | 9,9739873 | 9,5523514 | 10,4476486 | 22   |
| 39        | 9,5266927 | 9,9739422 | 9,5527504 | 10,4472496 | 21   |
| 40        | 9,5270463 | 9,9738971 | 9,5531492 | 10,4468508 | 20   |
| 41        | 9,5273997 | 9,9738519 | 9,5535477 | 10,4464523 | 19   |
| 42        | 9,5277526 | 9,9738067 | 9,5539459 | 10,4460541 | 18   |
| 43        | 9,5281053 | 9,9737615 | 9,5543438 | 10,4456562 | 17   |
| 44        | 9,5284577 | 9,9737162 | 9,5547415 | 10,4452585 | 16   |
| 45        | 9,5288097 | 9,9736709 | 9,5551388 | 10,4448612 | 15   |
| 46        | 9,5291614 | 9,9736255 | 9,5555359 | 10,4444641 | 14   |
| 47        | 9,5295128 | 9,9735801 | 9,5559327 | 10,4440673 | 13   |
| 48        | 9,5298638 | 9,9735346 | 9,5563292 | 10,4436708 | 12   |
| 49        | 9,5302146 | 9,9734891 | 9,5567255 | 10,4432745 | 11   |
| 50        | 9,5305650 | 9,9734435 | 9,5571214 | 10,4428786 | 10   |
| 51        | 9,5309151 | 9,9733980 | 9,5575171 | 10,4424829 | 9    |
| 52        | 9,5312649 | 9,9733523 | 9,5579125 | 10,4420875 | 8    |
| 53        | 9,5316143 | 9,9733067 | 9,5583077 | 10,4416923 | 7    |
| 54        | 9,5319635 | 9,9732610 | 9,5587025 | 10,4412975 | 6    |
| 55        | 9,5323123 | 9,9732152 | 9,5590971 | 10,4409029 | 5    |
| 56        | 9,5326608 | 9,9731694 | 9,5594914 | 10,4405086 | 4    |
| 57        | 9,5330090 | 9,9731236 | 9,5598854 | 10,4401146 | 3    |
| 58        | 9,5333569 | 9,9730777 | 9,5602792 | 10,4397208 | 2    |
| 59        | 9,5337044 | 9,9730318 | 9,5606727 | 10,4393273 | 1    |
| 60        | 9,5340517 | 9,9729858 | 9,5610659 | 10,4389341 | 0    |
| Sine Com. | Sine.     | Tan. Com. | Tangent.  |            | Min. |

## 70 Degrees.

| 20 Degrees. |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 9.5340517 | 9.9729858 | 9.5610659 | 10.4289341 | 60   |
| 1           | 9.5343986 | 9.9729398 | 9.5614588 | 10.4385412 | 59   |
| 2           | 9.5347452 | 9.9728938 | 9.5618515 | 10.4381485 | 58   |
| 3           | 9.5350915 | 9.9728477 | 9.5622439 | 10.4377561 | 57   |
| 4           | 9.5354375 | 9.9728016 | 9.5626360 | 10.4373640 | 56   |
| 5           | 9.5357832 | 9.9727554 | 9.5630278 | 10.4369722 | 55   |
| 6           | 9.5361285 | 9.9727092 | 9.5634194 | 10.4365806 | 54   |
| 7           | 9.5364737 | 9.9726629 | 9.5638107 | 10.4361893 | 53   |
| 8           | 9.5368184 | 9.9726166 | 9.5642018 | 10.4357982 | 52   |
| 9           | 9.5371629 | 9.9725703 | 9.5645925 | 10.4354075 | 51   |
| 10          | 9.5375070 | 9.9725239 | 9.5649831 | 10.4350169 | 50   |
| 11          | 9.5378508 | 9.9724775 | 9.5653733 | 10.4346267 | 49   |
| 12          | 9.5381943 | 9.9724310 | 9.5657633 | 10.4342367 | 48   |
| 13          | 9.5385375 | 9.9723845 | 9.5661530 | 10.4338470 | 47   |
| 14          | 9.5388804 | 9.9723380 | 9.5665424 | 10.4334576 | 46   |
| 15          | 9.5392230 | 9.9722914 | 9.5669316 | 10.4330684 | 45   |
| 16          | 9.5395653 | 9.9722448 | 9.5673205 | 10.4326795 | 44   |
| 17          | 9.5399073 | 9.9721981 | 9.5677091 | 10.4322909 | 43   |
| 18          | 9.5402489 | 9.9721514 | 9.5680975 | 10.4319025 | 42   |
| 19          | 9.5405903 | 9.9721047 | 9.5684856 | 10.4315144 | 41   |
| 20          | 9.5409314 | 9.9720579 | 9.5688735 | 10.4311265 | 40   |
| 21          | 9.5412721 | 9.9720110 | 9.5692611 | 10.4307389 | 39   |
| 22          | 9.5416126 | 9.9719642 | 9.5696484 | 10.4303516 | 38   |
| 23          | 9.5419527 | 9.9719172 | 9.5700355 | 10.4299645 | 37   |
| 24          | 9.5422926 | 9.9718703 | 9.5704223 | 10.4295777 | 36   |
| 25          | 9.5426321 | 9.9718233 | 9.5708088 | 10.4291912 | 35   |
| 26          | 9.5429713 | 9.9717762 | 9.5711951 | 10.4288049 | 34   |
| 27          | 9.5433103 | 9.9717291 | 9.5715811 | 10.4284189 | 33   |
| 28          | 9.5436489 | 9.9716820 | 9.5719669 | 10.4280331 | 32   |
| 29          | 9.5439873 | 9.9716348 | 9.5723524 | 10.4276476 | 31   |
| 30          | 9.5443253 | 9.9715876 | 9.5727377 | 10.4272623 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 69 Degrees. |           |           |           |            |      |

# 20 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,5443253 | 9,9715876 | 9,5727377 | 10,4272623 | 30   |
| 31   | 9,5446630 | 9,9715404 | 9,5731227 | 10,4268773 | 29   |
| 32   | 9,5450905 | 9,9714931 | 9,5735074 | 10,4264926 | 28   |
| 33   | 9,5453376 | 9,9714457 | 9,5738919 | 10,4261081 | 27   |
| 34   | 9,5456745 | 9,9713984 | 9,5742761 | 10,4257239 | 26   |
| 35   | 9,5460110 | 9,9713509 | 9,5746601 | 10,4253399 | 25   |
| 36   | 9,5463472 | 9,9713035 | 9,5750438 | 10,4249562 | 24   |
| 37   | 9,5466832 | 9,9712560 | 9,5754272 | 10,4245728 | 23   |
| 38   | 9,5470189 | 9,9712084 | 9,5758104 | 10,4241896 | 22   |
| 39   | 9,5473542 | 9,9711608 | 9,5761934 | 10,4238066 | 21   |
| 40   | 9,5476893 | 9,9711132 | 9,5765761 | 10,4234239 | 20   |
| 41   | 9,5480240 | 9,9710655 | 9,5769585 | 10,4230415 | 19   |
| 42   | 9,5483585 | 9,9710178 | 9,5773407 | 10,4226593 | 18   |
| 43   | 9,5486927 | 9,9709701 | 9,5777226 | 10,4222774 | 17   |
| 44   | 9,5490266 | 9,9709223 | 9,5781043 | 10,4218957 | 16   |
| 45   | 9,5493602 | 9,9708744 | 9,5784858 | 10,4215142 | 15   |
| 46   | 9,5496935 | 9,9708265 | 9,5788669 | 10,4211331 | 14   |
| 47   | 9,5500265 | 9,9707786 | 9,5792479 | 10,4207521 | 13   |
| 48   | 9,5503592 | 9,9707306 | 9,5796286 | 10,4203714 | 12   |
| 49   | 9,5506916 | 9,9706826 | 9,5800090 | 10,4199910 | 11   |
| 50   | 9,5510237 | 9,9706346 | 9,5803892 | 10,4196108 | 10   |
| 51   | 9,5513556 | 9,9705865 | 9,5807691 | 10,4192309 | 9    |
| 52   | 9,5516871 | 9,9705383 | 9,5811488 | 10,4188512 | 8    |
| 53   | 9,5520184 | 9,9704902 | 9,5815282 | 10,4184718 | 7    |
| 54   | 9,5523494 | 9,9704419 | 9,5819074 | 10,4180926 | 6    |
| 55   | 9,5526801 | 9,9703937 | 9,5822864 | 10,4177136 | 5    |
| 56   | 9,5530105 | 9,9703454 | 9,5826651 | 10,4173349 | 4    |
| 57   | 9,5533406 | 9,9702970 | 9,5830435 | 10,4169565 | 3    |
| 58   | 9,5536704 | 9,9702486 | 9,5834217 | 10,4165783 | 2    |
| 59   | 9,5539999 | 9,9702002 | 9,5837997 | 10,4162003 | 1    |
| 60   | 9,5543292 | 9,9701517 | 9,5841774 | 10,4158226 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

69 Degrees.

N

# 21 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,5543292 | 9,9701517 | 9,5841774 | 10,4158226 | 60   |
| 1    | 9,5546581 | 9,9701032 | 9,5845549 | 10,4154451 | 59   |
| 2    | 9,5549868 | 9,9700547 | 9,5849321 | 10,4150679 | 58   |
| 3    | 9,5553152 | 9,9700061 | 9,5853091 | 10,4146909 | 57   |
| 4    | 9,5556433 | 9,9699574 | 9,5856859 | 10,4143141 | 56   |
| 5    | 9,5559711 | 9,9699087 | 9,5860624 | 10,4139376 | 55   |
| 6    | 9,5562987 | 9,9698600 | 9,5864386 | 10,4135614 | 54   |
| 7    | 9,5566259 | 9,9698112 | 9,5868147 | 10,4131853 | 53   |
| 8    | 9,5569529 | 9,9697624 | 9,5871904 | 10,4128096 | 52   |
| 9    | 9,5572796 | 9,9697136 | 9,5875660 | 10,4124340 | 51   |
| 10   | 9,5576060 | 9,9696647 | 9,5879413 | 10,4120587 | 50   |
| 11   | 9,5579321 | 9,9696158 | 9,5883163 | 10,4116837 | 49   |
| 12   | 9,5582579 | 9,9695668 | 9,5886912 | 10,4113088 | 48   |
| 13   | 9,5585835 | 9,9695177 | 9,5890657 | 10,4109343 | 47   |
| 14   | 9,5589088 | 9,9694687 | 9,5894401 | 10,4105599 | 46   |
| 15   | 9,5592338 | 9,9694196 | 9,5898142 | 10,4101858 | 45   |
| 16   | 9,5595585 | 9,9693704 | 9,5901881 | 10,4098119 | 44   |
| 17   | 9,5598829 | 9,9693212 | 9,5905617 | 10,4094383 | 43   |
| 18   | 9,5602071 | 9,9692720 | 9,5909351 | 10,4090649 | 42   |
| 19   | 9,5605310 | 9,9692227 | 9,5913082 | 10,4086918 | 41   |
| 20   | 9,5608546 | 9,9691734 | 9,5916812 | 10,4083188 | 40   |
| 21   | 9,5611779 | 9,9691241 | 9,5920539 | 10,4079461 | 39   |
| 22   | 9,5615010 | 9,9690746 | 9,5924263 | 10,4075737 | 38   |
| 23   | 9,5618237 | 9,9690252 | 9,5927985 | 10,4072015 | 37   |
| 24   | 9,5621462 | 9,9689757 | 9,5931705 | 10,4068295 | 36   |
| 25   | 9,5624685 | 9,9689262 | 9,5935423 | 10,4064577 | 35   |
| 26   | 9,5627904 | 9,9688760 | 9,5939138 | 10,4060862 | 34   |
| 27   | 9,5631121 | 9,9688270 | 9,5942851 | 10,4057149 | 33   |
| 28   | 9,5634335 | 9,9687773 | 9,5946561 | 10,4053439 | 32   |
| 29   | 9,5637546 | 9,9687276 | 9,5950269 | 10,4049731 | 31   |
| 30   | 9,5640754 | 9,9686779 | 9,5953975 | 10,4046025 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

68 Degrees.

## 21 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,5640754 | 9,9686779 | 9,5953975 | 10,4046025 | 30   |
| 31   | 9,5643960 | 9,9686281 | 9,5957679 | 10,4042321 | 29   |
| 32   | 9,5647163 | 9,9685783 | 9,5961380 | 10,4038620 | 28   |
| 33   | 9,5650363 | 9,9685284 | 9,5965079 | 10,4034921 | 27   |
| 34   | 9,5653561 | 9,9684785 | 9,5968776 | 10,4031224 | 26   |
| 35   | 9,5656756 | 9,9684286 | 9,5972470 | 10,4027530 | 25   |
| 36   | 9,5659948 | 9,9683786 | 9,5976162 | 10,4023838 | 24   |
| 37   | 9,5663137 | 9,9683285 | 9,5979852 | 10,4020148 | 23   |
| 38   | 9,5666324 | 9,9682784 | 9,5983540 | 10,4016460 | 22   |
| 39   | 9,5669508 | 9,9682283 | 9,5987225 | 10,4012775 | 21   |
| 40   | 9,5672689 | 9,9681781 | 9,5990908 | 10,4009092 | 20   |
| 41   | 9,5675868 | 9,9681279 | 9,5994588 | 10,4005412 | 19   |
| 42   | 9,5679044 | 9,9680777 | 9,5998267 | 10,4001733 | 18   |
| 43   | 9,5682217 | 9,9680274 | 9,6001943 | 10,3998057 | 17   |
| 44   | 9,5685387 | 9,9679771 | 9,6005617 | 10,3994383 | 16   |
| 45   | 9,5688555 | 9,9679267 | 9,6009289 | 10,3990711 | 15   |
| 46   | 9,5691721 | 9,9678763 | 9,6012958 | 10,3987042 | 14   |
| 47   | 9,5694883 | 9,9678258 | 9,6016625 | 10,3983375 | 13   |
| 48   | 9,5698043 | 9,9677753 | 9,6020290 | 10,3979710 | 12   |
| 49   | 9,5701200 | 9,9677247 | 9,6023953 | 10,3976047 | 11   |
| 50   | 9,5704355 | 9,9676741 | 9,6027613 | 10,3972387 | 10   |
| 51   | 9,5707506 | 9,9676235 | 9,6031271 | 10,3968729 | 9    |
| 52   | 9,5710656 | 9,9675728 | 9,6034927 | 10,3965073 | 8    |
| 53   | 9,5713802 | 9,9675221 | 9,6038581 | 10,3961419 | 7    |
| 54   | 9,5716946 | 9,9674713 | 9,6042233 | 10,3957767 | 6    |
| 55   | 9,5720087 | 9,9674205 | 9,6045882 | 10,3954118 | 5    |
| 56   | 9,5723226 | 9,9673697 | 9,6049529 | 10,3950471 | 4    |
| 57   | 9,5726362 | 9,9673188 | 9,6053174 | 10,3946826 | 3    |
| 58   | 9,5729495 | 9,9672679 | 9,6056817 | 10,3943183 | 2    |
| 59   | 9,5732626 | 9,9672169 | 9,6060457 | 10,3939543 | 1    |
| 60   | 9,5735754 | 9,9671659 | 9,6064096 | 10,3935904 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

## 68 Degrees.

# 32 Degrees

| Min.      | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|-----------|-----------|-----------|-----------|------------|------|
| 0         | 9,5735754 | 9,9671659 | 9,6064096 | 10,3935904 | 60   |
| 1         | 9,5738880 | 9,9671148 | 9,6067732 | 10,3932268 | 59   |
| 2         | 9,5742003 | 9,9670637 | 9,6071366 | 10,3928634 | 58   |
| 3         | 9,5745123 | 9,9670125 | 9,6074997 | 10,3925003 | 57   |
| 4         | 9,5748240 | 9,9669614 | 9,6078627 | 10,3921373 | 56   |
| 5         | 9,5751356 | 9,9669101 | 9,6082254 | 10,3917746 | 55   |
| 6         | 9,5754468 | 9,9668588 | 9,6085880 | 10,3914120 | 54   |
| 7         | 9,5757578 | 9,9668075 | 9,6089503 | 10,3910497 | 53   |
| 8         | 9,5760685 | 9,9667562 | 9,6093124 | 10,3906876 | 52   |
| 9         | 9,5763790 | 9,9667048 | 9,6096742 | 10,3903258 | 51   |
| 10        | 9,5766892 | 9,9666533 | 9,6100359 | 10,3899641 | 50   |
| 11        | 9,5769991 | 9,9666018 | 9,6103973 | 10,3896027 | 49   |
| 12        | 9,5773088 | 9,9665503 | 9,6107586 | 10,3892414 | 48   |
| 13        | 9,5776183 | 9,9664986 | 9,6111196 | 10,3888804 | 47   |
| 14        | 9,5779275 | 9,9664471 | 9,6114804 | 10,3885196 | 46   |
| 15        | 9,5782364 | 9,9663954 | 9,6118409 | 10,3881591 | 45   |
| 16        | 9,5785450 | 9,9663437 | 9,6122013 | 10,3877987 | 44   |
| 17        | 9,5788535 | 9,9662920 | 9,6125615 | 10,3874385 | 43   |
| 18        | 9,5791616 | 9,9662402 | 9,6129214 | 10,3870786 | 42   |
| 19        | 9,5794695 | 9,9661884 | 9,6132812 | 10,3867188 | 41   |
| 20        | 9,5797772 | 9,9661365 | 9,6136407 | 10,3863593 | 40   |
| 21        | 9,5800845 | 9,9660846 | 9,6140000 | 10,3860000 | 39   |
| 22        | 9,5803917 | 9,9660326 | 9,6143591 | 10,3856409 | 38   |
| 23        | 9,5806986 | 9,9659806 | 9,6147180 | 10,3852820 | 37   |
| 24        | 9,5810052 | 9,9659285 | 9,6150766 | 10,3849234 | 36   |
| 25        | 9,5813116 | 9,9658764 | 9,6154351 | 10,3845649 | 35   |
| 26        | 9,5816177 | 9,9658243 | 9,6157934 | 10,3842066 | 34   |
| 27        | 9,5819236 | 9,9657721 | 9,6161514 | 10,3838486 | 33   |
| 28        | 9,5822292 | 9,9657199 | 9,6165093 | 10,3834907 | 32   |
| 29        | 9,5825345 | 9,9656677 | 9,6168669 | 10,3831331 | 31   |
| 30        | 9,5828397 | 9,9656153 | 9,6172243 | 10,3827757 | 30   |
| Sine Com. | Sine.     | Tan. Com. | Tangent.  |            | Min. |

# 67 Degrees

# 22 Degrees

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,5828397 | 9,9656153 | 9,6172243 | 10,3827757 | 30   |
| 31   | 9,5831445 | 9,9655630 | 9,6175815 | 10,3824185 | 29   |
| 32   | 9,5834491 | 9,9655106 | 9,6179385 | 10,3820615 | 28   |
| 33   | 9,5837535 | 9,9654582 | 9,6182953 | 10,3817047 | 27   |
| 34   | 9,5840576 | 9,9654057 | 9,6186519 | 10,3813481 | 26   |
| 35   | 9,5843615 | 9,9653532 | 9,6190083 | 10,3809917 | 25   |
| 36   | 9,5846651 | 9,9653006 | 9,6193645 | 10,3806355 | 24   |
| 37   | 9,5849685 | 9,9652480 | 9,6197205 | 10,3802795 | 23   |
| 38   | 9,5852716 | 9,9651953 | 9,6200762 | 10,3799238 | 22   |
| 39   | 9,5855745 | 9,9651426 | 9,6204318 | 10,3795682 | 21   |
| 40   | 9,5858771 | 9,9650899 | 9,6207872 | 10,3792128 | 20   |
| 41   | 9,5861795 | 9,9650371 | 9,6211423 | 10,3788577 | 19   |
| 42   | 9,5864816 | 9,9649843 | 9,6214973 | 10,3785027 | 18   |
| 43   | 9,5867835 | 9,9649314 | 9,6218520 | 10,3781480 | 17   |
| 44   | 9,5870851 | 9,9648785 | 9,6222066 | 10,3777934 | 16   |
| 45   | 9,5873865 | 9,9648256 | 9,6225609 | 10,3774391 | 15   |
| 46   | 9,5876876 | 9,9647726 | 9,6229150 | 10,3770850 | 14   |
| 47   | 9,5879885 | 9,9647195 | 9,6232690 | 10,3767310 | 13   |
| 48   | 9,5882892 | 9,9646665 | 9,6236227 | 10,3763773 | 12   |
| 49   | 9,5885896 | 9,9646133 | 9,6239763 | 10,3760237 | 11   |
| 50   | 9,5888897 | 9,9645602 | 9,6243296 | 10,3756704 | 10   |
| 51   | 9,5891897 | 9,9645069 | 9,6246827 | 10,3753173 | 9    |
| 52   | 9,5894893 | 9,9644537 | 9,6250356 | 10,3749644 | 8    |
| 53   | 9,5897888 | 9,9644004 | 9,6253884 | 10,3746116 | 7    |
| 54   | 9,5900880 | 9,9643470 | 9,6257409 | 10,3742591 | 6    |
| 55   | 9,5903869 | 9,9642937 | 9,6260932 | 10,3739068 | 5    |
| 56   | 9,5906856 | 9,9642402 | 9,6264454 | 10,3735546 | 4    |
| 57   | 9,5909841 | 9,9641868 | 9,6267973 | 10,3732027 | 3    |
| 58   | 9,5912823 | 9,9641332 | 9,6271491 | 10,3728509 | 2    |
| 59   | 9,5915803 | 9,9640797 | 9,6275006 | 10,3724994 | 1    |
| 60   | 9,5918780 | 9,9640261 | 9,6278519 | 10,3721481 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 27 Degrees



# 23 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,5918780 | 9,9640261 | 9,6278519 | 10,3721481 | 60   |
| 1    | 9,5921755 | 9,9639724 | 9,6282031 | 10,3717969 | 59   |
| 2    | 9,5924728 | 9,9639187 | 9,6285540 | 10,3714460 | 58   |
| 3    | 9,5927698 | 9,9638550 | 9,6289048 | 10,3710952 | 57   |
| 4    | 9,5930666 | 9,9638112 | 9,6292553 | 10,3707447 | 56   |
| 5    | 9,5933631 | 9,9637574 | 9,6296057 | 10,3703943 | 55   |
| 6    | 9,5936594 | 9,9637036 | 9,6299558 | 10,3700442 | 54   |
| 7    | 9,5939555 | 9,9636496 | 9,6303058 | 10,3696942 | 53   |
| 8    | 9,5942513 | 9,9635957 | 9,6306556 | 10,3693444 | 52   |
| 9    | 9,5945469 | 9,9635417 | 9,6310052 | 10,3689948 | 51   |
| 10   | 9,5948422 | 9,9634877 | 9,6313545 | 10,3686455 | 50   |
| 11   | 9,5951373 | 9,9634336 | 9,6317037 | 10,3682963 | 49   |
| 12   | 9,5954322 | 9,9633795 | 9,6320527 | 10,3679473 | 48   |
| 13   | 9,5957268 | 9,9633253 | 9,6324015 | 10,3675985 | 47   |
| 14   | 9,5960212 | 9,9632711 | 9,6327501 | 10,3672499 | 46   |
| 15   | 9,5963154 | 9,9632168 | 9,6330985 | 10,3669015 | 45   |
| 16   | 9,5966093 | 9,9631625 | 9,6334468 | 10,3665532 | 44   |
| 17   | 9,5969030 | 9,9631082 | 9,6337948 | 10,3662052 | 43   |
| 18   | 9,5971965 | 9,9630538 | 9,6341426 | 10,3658574 | 42   |
| 19   | 9,5974897 | 9,9629994 | 9,6344903 | 10,3655097 | 41   |
| 20   | 9,5977827 | 9,9629449 | 9,6348378 | 10,3651622 | 40   |
| 21   | 9,5980754 | 9,9628904 | 9,6351850 | 10,3648150 | 39   |
| 22   | 9,5983679 | 9,9628358 | 9,6355321 | 10,3644679 | 38   |
| 23   | 9,5986602 | 9,9627812 | 9,6358790 | 10,3641210 | 37   |
| 24   | 9,5989523 | 9,9627266 | 9,6362257 | 10,3637743 | 36   |
| 25   | 9,5992441 | 9,9626719 | 9,6365722 | 10,3634278 | 35   |
| 26   | 9,5995357 | 9,9626172 | 9,6369185 | 10,3630815 | 34   |
| 27   | 9,5998270 | 9,9625624 | 9,6372646 | 10,3627354 | 33   |
| 28   | 9,6001181 | 9,9625076 | 9,6376106 | 10,3623894 | 32   |
| 29   | 9,6004090 | 9,9624527 | 9,6379563 | 10,3620437 | 31   |
| 30   | 9,6006997 | 9,9623978 | 9,6383019 | 10,3616981 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 66 Degrees.



# 23 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6006997 | 9,9623978 | 9,6383019 | 10,3616981 | 30   |
| 31   | 9,6009901 | 9,9623428 | 9,6386473 | 10,3613527 | 29   |
| 32   | 9,6012803 | 9,9622878 | 9,6389925 | 10,3610075 | 28   |
| 33   | 9,6015793 | 9,9622328 | 9,6393375 | 10,3606625 | 27   |
| 34   | 9,6018600 | 9,9621777 | 9,6396823 | 10,3603177 | 26   |
| 35   | 9,6021495 | 9,9621226 | 9,6400269 | 10,3599731 | 25   |
| 36   | 9,6024388 | 9,9620674 | 9,6403714 | 10,3596286 | 24   |
| 37   | 9,6027278 | 9,9620122 | 9,6407156 | 10,3592844 | 23   |
| 38   | 9,6030166 | 9,9619569 | 9,6410597 | 10,3589403 | 22   |
| 39   | 9,6033052 | 9,9619016 | 9,6414036 | 10,3585964 | 21   |
| 40   | 9,6035936 | 9,9618463 | 9,6417473 | 10,3582527 | 20   |
| 41   | 9,6038817 | 9,9617909 | 9,6420908 | 10,3579092 | 19   |
| 42   | 9,6041696 | 9,9617355 | 9,6424342 | 10,3575658 | 18   |
| 43   | 9,6044573 | 9,9616800 | 9,6427773 | 10,3572227 | 17   |
| 44   | 9,6047448 | 9,9616245 | 9,6431203 | 10,3568797 | 16   |
| 45   | 9,6050320 | 9,9615689 | 9,6434631 | 10,3565369 | 15   |
| 46   | 9,6053190 | 9,9615133 | 9,6438057 | 10,3561943 | 14   |
| 47   | 9,6056057 | 9,9614576 | 9,6441481 | 10,3558519 | 13   |
| 48   | 9,6058923 | 9,9614020 | 9,6444903 | 10,3555097 | 12   |
| 49   | 9,6061786 | 9,9613462 | 9,6448324 | 10,3551676 | 11   |
| 50   | 9,6064647 | 9,9612904 | 9,6451743 | 10,3548257 | 10   |
| 51   | 9,6067506 | 9,9612346 | 9,6455160 | 10,3544840 | 9    |
| 52   | 9,6070362 | 9,9611787 | 9,6458575 | 10,3541425 | 8    |
| 53   | 9,6073216 | 9,9611228 | 9,6461988 | 10,3538012 | 7    |
| 54   | 9,6076068 | 9,9610668 | 9,6465400 | 10,3534600 | 6    |
| 55   | 9,6078918 | 9,9610108 | 9,6468810 | 10,3531190 | 5    |
| 56   | 9,6081765 | 9,9609548 | 9,6472217 | 10,3527783 | 4    |
| 57   | 9,6084611 | 9,9608987 | 9,6475624 | 10,3524376 | 3    |
| 58   | 9,6087454 | 9,9608426 | 9,6479028 | 10,3520972 | 2    |
| 59   | 9,6090294 | 9,9607864 | 9,6482431 | 10,3517569 | 1    |
| 60   | 9,6093133 | 9,9607302 | 9,6485831 | 10,3514169 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 66 Degrees.

# 24 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6093133 | 9,9607302 | 9,6485831 | 10,3514169 | 60   |
| 1    | 9,6095969 | 9,9606739 | 9,6489230 | 10,3510770 | 59   |
| 2    | 9,6098803 | 9,9606176 | 9,6492628 | 10,3507372 | 58   |
| 3    | 9,6101635 | 9,9605612 | 9,6496023 | 10,3503977 | 57   |
| 4    | 9,6104465 | 9,9605048 | 9,6499417 | 10,3500583 | 56   |
| 5    | 9,6107293 | 9,9604484 | 9,6502809 | 10,3497191 | 55   |
| 6    | 9,6110118 | 9,9603919 | 9,6506199 | 10,3493801 | 54   |
| 7    | 9,6112941 | 9,9603354 | 9,6509587 | 10,3490413 | 53   |
| 8    | 9,6115762 | 9,9602788 | 9,6512974 | 10,3487026 | 52   |
| 9    | 9,6118580 | 9,9602222 | 9,6516359 | 10,3483641 | 51   |
| 10   | 9,6121397 | 9,9601655 | 9,6519742 | 10,3480258 | 50   |
| 11   | 9,6124211 | 9,9601088 | 9,6523123 | 10,3476877 | 49   |
| 12   | 9,6127023 | 9,9600520 | 9,6526503 | 10,3473497 | 48   |
| 13   | 9,6129833 | 9,9599952 | 9,6529881 | 10,3470119 | 47   |
| 14   | 9,6132641 | 9,9599384 | 9,6533257 | 10,3466743 | 46   |
| 15   | 9,6135446 | 9,9598815 | 9,6536631 | 10,3463369 | 45   |
| 16   | 9,6138250 | 9,9598246 | 9,6540004 | 10,3459996 | 44   |
| 17   | 9,6141051 | 9,9597676 | 9,6543375 | 10,3456625 | 43   |
| 18   | 9,6143850 | 9,9597106 | 9,6546744 | 10,3453256 | 42   |
| 19   | 9,6146647 | 9,9596535 | 9,6550112 | 10,3449888 | 41   |
| 20   | 9,6149441 | 9,9595964 | 9,6553477 | 10,3446523 | 40   |
| 21   | 9,6152234 | 9,9595393 | 9,6556841 | 10,3443159 | 39   |
| 22   | 9,6155024 | 9,9594821 | 9,6560204 | 10,3439796 | 38   |
| 23   | 9,6157812 | 9,9594248 | 9,6563564 | 10,3436436 | 37   |
| 24   | 9,6160599 | 9,9593675 | 9,6566923 | 10,3433077 | 36   |
| 25   | 9,6163382 | 9,9593102 | 9,6570280 | 10,3429720 | 35   |
| 26   | 9,6166164 | 9,9592528 | 9,6573636 | 10,3426364 | 34   |
| 27   | 9,6168944 | 9,9591954 | 9,6576989 | 10,3423011 | 33   |
| 28   | 9,6171741 | 9,9591380 | 9,6580341 | 10,3419659 | 32   |
| 29   | 9,6174496 | 9,9590805 | 9,6583692 | 10,3416308 | 31   |
| 30   | 9,6177270 | 9,9590229 | 9,6587041 | 10,3412959 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 65 Degrees.

# 24 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6177270 | 9,9590229 | 9,6587041 | 10,3412959 | 30   |
| 31   | 9,6180041 | 9,9589653 | 9,6590387 | 10,3409613 | 29   |
| 32   | 9,6182809 | 9,9589077 | 9,6593733 | 10,3406267 | 28   |
| 33   | 9,6185576 | 9,9588500 | 9,6597076 | 10,3402924 | 27   |
| 34   | 9,6188341 | 9,9587923 | 9,6600418 | 10,3399582 | 26   |
| 35   | 9,6191103 | 9,9587345 | 9,6603758 | 10,3396242 | 25   |
| 36   | 9,6193864 | 9,9586767 | 9,6607097 | 10,3392903 | 24   |
| 37   | 9,6196622 | 9,9586188 | 9,6610434 | 10,3389566 | 23   |
| 38   | 9,6199378 | 9,9585609 | 9,6613769 | 10,3386231 | 22   |
| 39   | 9,6202132 | 9,9585030 | 9,6617103 | 10,3382897 | 21   |
| 40   | 9,6204884 | 9,9584450 | 9,6620434 | 10,3379566 | 20   |
| 41   | 9,6207634 | 9,9583869 | 9,6623765 | 10,3376235 | 19   |
| 42   | 9,6210382 | 9,9583288 | 9,6627093 | 10,3372907 | 18   |
| 43   | 9,6213127 | 9,9582707 | 9,6630420 | 10,3369580 | 17   |
| 44   | 9,6215871 | 9,9582125 | 9,6633745 | 10,3366255 | 16   |
| 45   | 9,6218612 | 9,9581543 | 9,6637069 | 10,3362931 | 15   |
| 46   | 9,6221351 | 9,9580961 | 9,6640391 | 10,3359609 | 14   |
| 47   | 9,6224088 | 9,9580378 | 9,6643711 | 10,3356289 | 13   |
| 48   | 9,6226824 | 9,9579794 | 9,6647030 | 10,3352970 | 12   |
| 49   | 9,6229557 | 9,9579210 | 9,6650346 | 10,3349654 | 11   |
| 50   | 9,6232287 | 9,9578626 | 9,6653662 | 10,3346338 | 10   |
| 51   | 9,6235016 | 9,9578041 | 9,6656975 | 10,3343025 | 9    |
| 52   | 9,6237743 | 9,9577456 | 9,6660288 | 10,3339712 | 8    |
| 53   | 9,6240468 | 9,9576870 | 9,6663598 | 10,3336402 | 7    |
| 54   | 9,6243190 | 9,9576284 | 9,6666907 | 10,3333093 | 6    |
| 55   | 9,6245911 | 9,9575697 | 9,6670214 | 10,3329786 | 5    |
| 56   | 9,6248629 | 9,9575110 | 9,6673519 | 10,3326481 | 4    |
| 57   | 9,6251346 | 9,9574522 | 9,6676823 | 10,3323177 | 3    |
| 58   | 9,6254060 | 9,9573934 | 9,6680126 | 10,3319874 | 2    |
| 59   | 9,6256772 | 9,9573346 | 9,6683426 | 10,3316574 | 1    |
| 60   | 9,6259483 | 9,9572757 | 9,6686725 | 10,3313275 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 65 Degrees.

Q.

# 25 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6259483 | 9,9572757 | 9,6686725 | 10,3313275 | 60   |
| 1    | 9,6262191 | 9,9572168 | 9,6690023 | 10,3309977 | 59   |
| 2    | 9,6264897 | 9,9571578 | 9,6693319 | 10,3306681 | 58   |
| 3    | 9,6267601 | 9,9570988 | 9,6696613 | 10,3303387 | 57   |
| 4    | 9,6270303 | 9,9570397 | 9,6699906 | 10,3300094 | 56   |
| 5    | 9,6273003 | 9,9569806 | 9,6703197 | 10,3296803 | 55   |
| 6    | 9,6275701 | 9,9569215 | 9,6706486 | 10,3293514 | 54   |
| 7    | 9,6278397 | 9,9568623 | 9,6709774 | 10,3290226 | 53   |
| 8    | 9,6281090 | 9,9568030 | 9,6713060 | 10,3286940 | 52   |
| 9    | 9,6283782 | 9,9567437 | 9,6716345 | 10,3283655 | 51   |
| 10   | 9,6286472 | 9,9566844 | 9,6719628 | 10,3280372 | 50   |
| 11   | 9,6289160 | 9,9566250 | 9,6722910 | 10,3277090 | 49   |
| 12   | 9,6291845 | 9,9565656 | 9,6726190 | 10,3273810 | 48   |
| 13   | 9,6294529 | 9,9565061 | 9,6729468 | 10,3270532 | 47   |
| 14   | 9,6297211 | 9,9564466 | 9,6732745 | 10,3267255 | 46   |
| 15   | 9,6299890 | 9,9563870 | 9,6736020 | 10,3263980 | 45   |
| 16   | 9,6302568 | 9,9563274 | 9,6739294 | 10,3260706 | 44   |
| 17   | 9,6305243 | 9,9562678 | 9,6742566 | 10,3257434 | 43   |
| 18   | 9,6307917 | 9,9562081 | 9,6745836 | 10,3254164 | 42   |
| 19   | 9,6310589 | 9,9561483 | 9,6749105 | 10,3250895 | 41   |
| 20   | 9,6313258 | 9,9560886 | 9,6752372 | 10,3247628 | 40   |
| 21   | 9,6315926 | 9,9560287 | 9,6755638 | 10,3244362 | 39   |
| 22   | 9,6318591 | 9,9559689 | 9,6758903 | 10,3241097 | 38   |
| 23   | 9,6321255 | 9,9559089 | 9,6762165 | 10,3237835 | 37   |
| 24   | 9,6323916 | 9,9558490 | 9,6765426 | 10,3234574 | 36   |
| 25   | 9,6326576 | 9,9557890 | 9,6768686 | 10,3231314 | 35   |
| 26   | 9,6329233 | 9,9557289 | 9,6771944 | 10,3228056 | 34   |
| 27   | 9,6331889 | 9,9556688 | 9,6775201 | 10,3224799 | 33   |
| 28   | 9,6334542 | 9,9556087 | 9,6778456 | 10,3221544 | 32   |
| 29   | 9,6337194 | 9,9555485 | 9,6781709 | 10,3218291 | 31   |
| 30   | 9,6339844 | 9,9554882 | 9,6784961 | 10,3215039 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

64 Degrees.

# 25 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6339844 | 9,9554882 | 9,6784961 | 10,3215039 | 30   |
| 31   | 9,6342491 | 9,9554280 | 9,6788211 | 10,3211789 | 29   |
| 32   | 9,6345137 | 9,9553676 | 9,6791460 | 10,3208540 | 28   |
| 33   | 9,6347780 | 9,9553073 | 9,6794708 | 10,3205292 | 27   |
| 34   | 9,6350422 | 9,9552469 | 9,6797953 | 10,3202047 | 26   |
| 35   | 9,6353062 | 9,9551864 | 9,6801198 | 10,3198802 | 25   |
| 36   | 9,6355699 | 9,9551259 | 9,6804440 | 10,3195560 | 24   |
| 37   | 9,6358335 | 9,9550653 | 9,6807682 | 10,3192318 | 23   |
| 38   | 9,6360969 | 9,9550047 | 9,6810921 | 10,3189079 | 22   |
| 39   | 9,6363601 | 9,9549441 | 9,6814160 | 10,3185840 | 21   |
| 40   | 9,6366231 | 9,9548834 | 9,6817396 | 10,3182604 | 20   |
| 41   | 9,6368859 | 9,9548227 | 9,6820632 | 10,3179368 | 19   |
| 42   | 9,6371484 | 9,9547619 | 9,6823855 | 10,3176135 | 18   |
| 43   | 9,6374108 | 9,9547011 | 9,6827098 | 10,3172902 | 17   |
| 44   | 9,6376731 | 9,9546402 | 9,6830328 | 10,3169672 | 16   |
| 45   | 9,6379351 | 9,9545793 | 9,6833557 | 10,3166443 | 15   |
| 46   | 9,6381969 | 9,9545184 | 9,6836785 | 10,3163215 | 14   |
| 47   | 9,6384585 | 9,9544574 | 9,6840011 | 10,3159989 | 13   |
| 48   | 9,6387199 | 9,9543963 | 9,6843236 | 10,3156764 | 12   |
| 49   | 9,6389812 | 9,9543352 | 9,6846459 | 10,3153541 | 11   |
| 50   | 9,6392422 | 9,9542741 | 9,6849681 | 10,3150319 | 10   |
| 51   | 9,6395030 | 9,9542129 | 9,6852901 | 10,3147099 | 9    |
| 52   | 9,6397637 | 9,9541517 | 9,6856120 | 10,3143880 | 8    |
| 53   | 9,6400241 | 9,9540904 | 9,6859338 | 10,3140662 | 7    |
| 54   | 9,6402844 | 9,9540291 | 9,6862553 | 10,3137447 | 6    |
| 55   | 9,6405445 | 9,9539677 | 9,6865768 | 10,3134232 | 5    |
| 56   | 9,6408044 | 9,9539063 | 9,6868981 | 10,3131019 | 4    |
| 57   | 9,6410640 | 9,9538448 | 9,6872192 | 10,3127808 | 3    |
| 58   | 9,6413233 | 9,9537833 | 9,6875402 | 10,3124598 | 2    |
| 59   | 9,6415828 | 9,9537218 | 9,6878611 | 10,3121389 | 1    |
| 60   | 9,6418420 | 9,9536602 | 9,6881818 | 10,3118182 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

## 64 Degrees.

# 26 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6418420 | 9,9536602 | 9,6881818 | 10,3118182 | 60   |
| 1    | 9,6421009 | 9,9535985 | 9,6885023 | 10,3114977 | 59   |
| 2    | 9,6423596 | 9,9535369 | 9,6888227 | 10,3111773 | 58   |
| 3    | 9,6426182 | 9,9534751 | 9,6891430 | 10,3108570 | 57   |
| 4    | 9,6428765 | 9,9534134 | 9,6894631 | 10,3105369 | 56   |
| 5    | 9,6431347 | 9,9533515 | 9,6897831 | 10,3102169 | 55   |
| 6    | 9,6433926 | 9,9532897 | 9,6901030 | 10,3098970 | 54   |
| 7    | 9,6436504 | 9,9532278 | 9,6904226 | 10,3095774 | 53   |
| 8    | 9,6439080 | 9,9531658 | 9,6907422 | 10,3092578 | 52   |
| 9    | 9,6441654 | 9,9531038 | 9,6910616 | 10,3089384 | 51   |
| 10   | 9,6444226 | 9,9530418 | 9,6913809 | 10,3086191 | 50   |
| 11   | 9,6446796 | 9,9529797 | 9,6917000 | 10,3083000 | 49   |
| 12   | 9,6449365 | 9,9529175 | 9,6920189 | 10,3079811 | 48   |
| 13   | 9,6451931 | 9,9528553 | 9,6923378 | 10,3076622 | 47   |
| 14   | 9,6454496 | 9,9527931 | 9,6926565 | 10,3073435 | 46   |
| 15   | 9,6457058 | 9,9527308 | 9,6929750 | 10,3070250 | 45   |
| 16   | 9,6459619 | 9,9526685 | 9,6932934 | 10,3067066 | 44   |
| 17   | 9,6462178 | 9,9526061 | 9,6936117 | 10,3063883 | 43   |
| 18   | 9,6464735 | 9,9525437 | 9,6939298 | 10,3060702 | 42   |
| 19   | 9,6467290 | 9,9524813 | 9,6942478 | 10,3057522 | 41   |
| 20   | 9,6469844 | 9,9524188 | 9,6945656 | 10,3054344 | 40   |
| 21   | 9,6472395 | 9,9523562 | 9,6948833 | 10,3051167 | 39   |
| 22   | 9,6474945 | 9,9522936 | 9,6952009 | 10,3047991 | 38   |
| 23   | 9,6477492 | 9,9522310 | 9,6955183 | 10,3044817 | 37   |
| 24   | 9,6480038 | 9,9521683 | 9,6958355 | 10,3041645 | 36   |
| 25   | 9,6482582 | 9,9521055 | 9,6961527 | 10,3038473 | 35   |
| 26   | 9,6485124 | 9,9520428 | 9,6964697 | 10,3035303 | 34   |
| 27   | 9,6487665 | 9,9519799 | 9,6967865 | 10,3032135 | 33   |
| 28   | 9,6490203 | 9,9519171 | 9,6971032 | 10,3028968 | 32   |
| 29   | 9,6492740 | 9,9518541 | 9,6974198 | 10,3025802 | 31   |
| 30   | 9,6495274 | 9,9517912 | 9,6977363 | 10,3022637 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 63 Degrees.

# 26 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6495274 | 9,9517912 | 9,6977363 | 10,3022637 | 30   |
| 31   | 9,6497807 | 9,9517282 | 9,6980520 | 10,3019474 | 29   |
| 32   | 9,6500338 | 9,9516651 | 9,6983687 | 10,3016313 | 28   |
| 33   | 9,6502868 | 9,9516020 | 9,6986847 | 10,3013153 | 27   |
| 34   | 9,6505395 | 9,9515389 | 9,6990006 | 10,3009994 | 26   |
| 35   | 9,6507920 | 9,9514757 | 9,6993164 | 10,3006836 | 25   |
| 36   | 9,6510444 | 9,9514124 | 9,6996320 | 10,3003680 | 24   |
| 37   | 9,6512966 | 9,9513492 | 9,6999474 | 10,3000526 | 23   |
| 38   | 9,6515486 | 9,9512858 | 9,7002628 | 10,2997372 | 22   |
| 39   | 9,6518004 | 9,9512224 | 9,7005780 | 10,2994220 | 21   |
| 40   | 9,6520521 | 9,9511590 | 9,7008930 | 10,2991070 | 20   |
| 41   | 9,6523035 | 9,9510956 | 9,7012080 | 10,2987920 | 19   |
| 42   | 9,6525548 | 9,9510320 | 9,7015227 | 10,2984773 | 18   |
| 43   | 9,6528059 | 9,9509685 | 9,7018374 | 10,2981626 | 17   |
| 44   | 9,6530568 | 9,9509049 | 9,7021519 | 10,2978481 | 16   |
| 45   | 9,6533075 | 9,9508412 | 9,7024663 | 10,2975337 | 15   |
| 46   | 9,6535581 | 9,9507775 | 9,7027805 | 10,2972195 | 14   |
| 47   | 9,6538084 | 9,9507138 | 9,7030946 | 10,2969054 | 13   |
| 48   | 9,6540586 | 9,9506500 | 9,7034086 | 10,2965914 | 12   |
| 49   | 9,6543086 | 9,9505861 | 9,7037225 | 10,2962775 | 11   |
| 50   | 9,6545584 | 9,9505223 | 9,7040362 | 10,2959638 | 10   |
| 51   | 9,6548081 | 9,9504583 | 9,7043497 | 10,2956503 | 9    |
| 52   | 9,6550575 | 9,9503944 | 9,7046632 | 10,2953368 | 8    |
| 53   | 9,6553068 | 9,9503303 | 9,7049765 | 10,2950235 | 7    |
| 54   | 9,6555559 | 9,9502663 | 9,7052897 | 10,2947103 | 6    |
| 55   | 9,6558048 | 9,9502022 | 9,7056027 | 10,2943973 | 5    |
| 56   | 9,6560536 | 9,9501380 | 9,7059150 | 10,2940844 | 4    |
| 57   | 9,6563021 | 9,9500738 | 9,7062284 | 10,2937716 | 3    |
| 58   | 9,6565505 | 9,9500095 | 9,7065410 | 10,2934590 | 2    |
| 59   | 9,6567987 | 9,9499452 | 9,7068535 | 10,2931465 | 1    |
| 60   | 9,6570468 | 9,9498809 | 9,7071659 | 10,2928341 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 63 Degrees.

# 27 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6570468 | 9,9498809 | 9,7071659 | 10,2928341 | 60   |
| 1    | 9,6572946 | 9,9498165 | 9,7074781 | 10,2925219 | 59   |
| 2    | 9,6575423 | 9,9497521 | 9,7077902 | 10,2922098 | 58   |
| 3    | 9,6577898 | 9,9496876 | 9,7081022 | 10,2918978 | 57   |
| 4    | 9,6580371 | 9,9496230 | 9,7084141 | 10,2915859 | 56   |
| 5    | 9,6582842 | 9,9495585 | 9,7087258 | 10,2912742 | 55   |
| 6    | 9,6585312 | 9,9494938 | 9,7090374 | 10,2909626 | 54   |
| 7    | 9,6587785 | 9,9494292 | 9,7093488 | 10,2906512 | 53   |
| 8    | 9,6590246 | 9,9493645 | 9,7096601 | 10,2903399 | 52   |
| 9    | 9,6592710 | 9,9492997 | 9,7099713 | 10,2905287 | 51   |
| 10   | 9,6595173 | 9,9492349 | 9,7102824 | 10,2897176 | 50   |
| 11   | 9,6597634 | 9,9491700 | 9,7105933 | 10,2894067 | 49   |
| 12   | 9,6600093 | 9,9491051 | 9,7109041 | 10,2890959 | 48   |
| 13   | 9,6602530 | 9,9490402 | 9,7112148 | 10,2887852 | 47   |
| 14   | 9,6605005 | 9,9489752 | 9,7115254 | 10,2884746 | 46   |
| 15   | 9,6607459 | 9,9489101 | 9,7118358 | 10,2881642 | 45   |
| 16   | 9,6609911 | 9,9488450 | 9,7121461 | 10,2878539 | 44   |
| 17   | 9,6612361 | 9,9487799 | 9,7124562 | 10,2875438 | 43   |
| 18   | 9,6614810 | 9,9487147 | 9,7127662 | 10,2872338 | 42   |
| 19   | 9,6617257 | 9,9486495 | 9,7130761 | 10,2869239 | 41   |
| 20   | 9,6619702 | 9,9485842 | 9,7133859 | 10,2866141 | 40   |
| 21   | 9,6622145 | 9,9485189 | 9,7136956 | 10,2863044 | 39   |
| 22   | 9,6624586 | 9,9484535 | 9,7140051 | 10,2859949 | 38   |
| 23   | 9,6627026 | 9,9483881 | 9,7143145 | 10,2856855 | 37   |
| 24   | 9,6629464 | 9,9483227 | 9,7146237 | 10,2853763 | 36   |
| 25   | 9,6631900 | 9,9482572 | 9,7149329 | 10,2850671 | 35   |
| 26   | 9,6634335 | 9,9481916 | 9,7152419 | 10,2847581 | 34   |
| 27   | 9,6636768 | 9,9481260 | 9,7155508 | 10,2844492 | 33   |
| 28   | 9,6639199 | 9,9480604 | 9,7158595 | 10,2841405 | 32   |
| 29   | 9,6641628 | 9,9479947 | 9,7161682 | 10,2838318 | 31   |
| 30   | 9,6644056 | 9,9479289 | 9,7164767 | 10,2835233 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 62 Degrees.



# 27 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6644056 | 9,9479287 | 9,7164767 | 10,2835233 | 30   |
| 31   | 9,6646482 | 9,9478631 | 9,7167851 | 10,2832149 | 29   |
| 32   | 9,6648906 | 9,9477973 | 9,7170933 | 10,2829067 | 28   |
| 33   | 9,6651329 | 9,9477314 | 9,7174014 | 10,2825986 | 27   |
| 34   | 9,6653749 | 9,9476655 | 9,7177095 | 10,2822906 | 26   |
| 35   | 9,6656168 | 9,9475995 | 9,7180173 | 10,2819827 | 25   |
| 36   | 9,6658586 | 9,9475335 | 9,7183251 | 10,2816749 | 24   |
| 37   | 9,6661001 | 9,9474674 | 9,7186327 | 10,2813673 | 23   |
| 38   | 9,6663415 | 9,9474013 | 9,7189402 | 10,2810598 | 22   |
| 39   | 9,6665828 | 9,9473352 | 9,7192476 | 10,2807524 | 21   |
| 40   | 9,6668238 | 9,9472689 | 9,7195549 | 10,2804451 | 20   |
| 41   | 9,6670647 | 9,9472027 | 9,7198620 | 10,2801380 | 19   |
| 42   | 9,6673054 | 9,9471364 | 9,7201690 | 10,2798310 | 18   |
| 43   | 9,6675459 | 9,9470700 | 9,7204759 | 10,2795241 | 17   |
| 44   | 9,6677863 | 9,9470036 | 9,7207827 | 10,2792173 | 16   |
| 45   | 9,6680265 | 9,9469372 | 9,7210893 | 10,2789107 | 15   |
| 46   | 9,6682665 | 9,9468707 | 9,7213958 | 10,2786042 | 14   |
| 47   | 9,6685064 | 9,9468042 | 9,7217022 | 10,2782978 | 13   |
| 48   | 9,6687461 | 9,9467376 | 9,7220085 | 10,2779915 | 12   |
| 49   | 9,6689856 | 9,9466710 | 9,7223147 | 10,2776853 | 11   |
| 50   | 9,6692252 | 9,9466043 | 9,7226207 | 10,2773793 | 10   |
| 51   | 9,6694642 | 9,9465376 | 9,7229266 | 10,2770734 | 9    |
| 52   | 9,6697032 | 9,9464708 | 9,7232324 | 10,2767676 | 8    |
| 53   | 9,6699420 | 9,9464040 | 9,7235381 | 10,2764619 | 7    |
| 54   | 9,6701807 | 9,9463371 | 9,7238436 | 10,2761564 | 6    |
| 55   | 9,6704192 | 9,9462702 | 9,7241490 | 10,2758510 | 5    |
| 56   | 9,6706576 | 9,9462032 | 9,7244543 | 10,2755457 | 4    |
| 57   | 9,6708958 | 9,9461362 | 9,7247595 | 10,2752405 | 3    |
| 58   | 9,6711338 | 9,9460692 | 9,7250646 | 10,2749354 | 2    |
| 59   | 9,6713716 | 9,9460021 | 9,7253695 | 10,2746305 | 1    |
| 60   | 9,6716093 | 9,9459349 | 9,7256774 | 10,2743256 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 62 Degrees.

# 28 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6716093 | 9,9459349 | 9,7256744 | 10,2743256 | 60   |
| 1    | 9,6718468 | 9,9458677 | 9,7259791 | 10,2740209 | 59   |
| 2    | 9,6720841 | 9,9458005 | 9,7262837 | 10,2737163 | 58   |
| 3    | 9,6723213 | 9,9457332 | 9,7265881 | 10,2734119 | 57   |
| 4    | 9,6725583 | 9,9456659 | 9,7268925 | 10,2731075 | 56   |
| 5    | 9,6727952 | 9,9455985 | 9,7271967 | 10,2728033 | 55   |
| 6    | 9,6730319 | 9,9455310 | 9,7275008 | 10,2724992 | 54   |
| 7    | 9,6732684 | 9,9454636 | 9,7278048 | 10,2721952 | 53   |
| 8    | 9,6735047 | 9,9453960 | 9,7281087 | 10,2718913 | 52   |
| 9    | 9,6737409 | 9,9453285 | 9,7284124 | 10,2715876 | 51   |
| 10   | 9,6739769 | 9,9452609 | 9,7287161 | 10,2712839 | 50   |
| 11   | 9,6742128 | 9,9451932 | 9,7290196 | 10,2709804 | 49   |
| 12   | 9,6744485 | 9,9451255 | 9,7293230 | 10,2706770 | 48   |
| 13   | 9,6746840 | 9,9450577 | 9,7296263 | 10,2703737 | 47   |
| 14   | 9,6749194 | 9,9449899 | 9,7299295 | 10,2700705 | 46   |
| 15   | 9,6751546 | 9,9449220 | 9,7302325 | 10,2697675 | 45   |
| 16   | 9,6753896 | 9,9448541 | 9,7305354 | 10,2694646 | 44   |
| 17   | 9,6756245 | 9,9447862 | 9,7308383 | 10,2691617 | 43   |
| 18   | 9,6758592 | 9,9447182 | 9,7311410 | 10,2688590 | 42   |
| 19   | 9,6760937 | 9,9446501 | 9,7314436 | 10,2685564 | 41   |
| 20   | 9,6763281 | 9,9445821 | 9,7317460 | 10,2682540 | 40   |
| 21   | 9,6765623 | 9,9445139 | 9,7320484 | 10,2679516 | 39   |
| 22   | 9,6767963 | 9,9444457 | 9,7323506 | 10,2676494 | 38   |
| 23   | 9,6770302 | 9,9443775 | 9,7326527 | 10,2673473 | 37   |
| 24   | 9,6772640 | 9,9443092 | 9,7329547 | 10,2670453 | 36   |
| 25   | 9,6774975 | 9,9442409 | 9,7332566 | 10,2667434 | 35   |
| 26   | 9,6777309 | 9,9441725 | 9,7335584 | 10,2664416 | 34   |
| 27   | 9,6779642 | 9,9441041 | 9,7338601 | 10,2661399 | 33   |
| 28   | 9,6781972 | 9,9440356 | 9,7341616 | 10,2658384 | 32   |
| 29   | 9,6784301 | 9,9439671 | 9,7344631 | 10,2655369 | 31   |
| 30   | 9,6786629 | 9,9438985 | 9,7347644 | 10,2652356 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 61 Degrees.

# 28 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6786629 | 9,9438985 | 9,7347644 | 10,2652356 | 30   |
| 31   | 9,6788955 | 9,9438299 | 9,7350656 | 10,2649344 | 29   |
| 32   | 9,6791279 | 9,9437612 | 9,7353667 | 10,2646333 | 28   |
| 33   | 9,6793602 | 9,9436925 | 9,7356677 | 10,2643323 | 27   |
| 34   | 9,6795923 | 9,9436238 | 9,7359685 | 10,2640315 | 26   |
| 35   | 9,6798243 | 9,9435549 | 9,7362693 | 10,2637307 | 25   |
| 36   | 9,6800560 | 9,9434861 | 9,7365699 | 10,2634301 | 24   |
| 37   | 9,6802877 | 9,9434172 | 9,7368705 | 10,2631295 | 23   |
| 38   | 9,6805191 | 9,9433482 | 9,7371709 | 10,2628291 | 22   |
| 39   | 9,6807504 | 9,9432792 | 9,7374712 | 10,2625288 | 21   |
| 40   | 9,6809816 | 9,9432102 | 9,7377714 | 10,2622286 | 20   |
| 41   | 9,6812126 | 9,9431411 | 9,7380715 | 10,2619285 | 19   |
| 42   | 9,6814434 | 9,9430720 | 9,7383714 | 10,2616286 | 18   |
| 43   | 9,6816741 | 9,9430028 | 9,7386713 | 10,2613287 | 17   |
| 44   | 9,6819046 | 9,9429335 | 9,7389710 | 10,2610290 | 16   |
| 45   | 9,6821349 | 9,9428643 | 9,7392707 | 10,2607293 | 15   |
| 46   | 9,6823651 | 9,9427949 | 9,7395702 | 10,2604298 | 14   |
| 47   | 9,6825952 | 9,9427255 | 9,7398696 | 10,2601304 | 13   |
| 48   | 9,6828250 | 9,9426561 | 9,7401689 | 10,2598311 | 12   |
| 49   | 9,6830548 | 9,9425866 | 9,7404681 | 10,2595319 | 11   |
| 50   | 9,6832843 | 9,9425171 | 9,7407672 | 10,2592328 | 10   |
| 51   | 9,6835137 | 9,9424476 | 9,7410662 | 10,2589338 | 9    |
| 52   | 9,6837430 | 9,9423779 | 9,7413650 | 10,2586350 | 8    |
| 53   | 9,6839720 | 9,9423083 | 9,7416638 | 10,2583362 | 7    |
| 54   | 9,6842010 | 9,9422386 | 9,7419624 | 10,2580376 | 6    |
| 55   | 9,6844297 | 9,9421688 | 9,7422609 | 10,2577391 | 5    |
| 56   | 9,6846583 | 9,9420990 | 9,7425594 | 10,2574406 | 4    |
| 57   | 9,6848868 | 9,9420291 | 9,7428577 | 10,2571423 | 3    |
| 58   | 9,6851151 | 9,9419592 | 9,7431559 | 10,2568441 | 2    |
| 59   | 9,6853432 | 9,9418893 | 9,7434540 | 10,2565460 | 1    |
| 60   | 9,6855712 | 9,9418193 | 9,7437520 | 10,2562480 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

61 Degrees.

29 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6855712 | 9,9418193 | 9,7437520 | 10,2562480 | 60   |
| 1    | 9,6857991 | 9,9417492 | 9,7440499 | 10,2559501 | 59   |
| 2    | 9,6860267 | 9,9416791 | 9,7443476 | 10,2556524 | 58   |
| 3    | 9,6862542 | 9,9416090 | 9,7446453 | 10,2553547 | 57   |
| 4    | 9,6864816 | 9,9415388 | 9,7449428 | 10,2550572 | 56   |
| 5    | 9,6867088 | 9,9414685 | 9,7452403 | 10,2547597 | 55   |
| 6    | 9,6869359 | 9,9413982 | 9,7455376 | 10,2544624 | 54   |
| 7    | 9,6871628 | 9,9413279 | 9,7458349 | 10,2541651 | 53   |
| 8    | 9,6873895 | 9,9412575 | 9,7461320 | 10,2538680 | 52   |
| 9    | 9,6876161 | 9,9411871 | 9,7464290 | 10,2535710 | 51   |
| 10   | 9,6878425 | 9,9411166 | 9,7467259 | 10,2532741 | 50   |
| 11   | 9,6880688 | 9,9410461 | 9,7470227 | 10,2529773 | 49   |
| 12   | 9,6882949 | 9,9409755 | 9,7473194 | 10,2526806 | 48   |
| 13   | 9,6885209 | 9,9409048 | 9,7476160 | 10,2523840 | 47   |
| 14   | 9,6887467 | 9,9408342 | 9,7479125 | 10,2520875 | 46   |
| 15   | 9,6889723 | 9,9407634 | 9,7482089 | 10,2517911 | 45   |
| 16   | 9,6891978 | 9,9406927 | 9,7485052 | 10,2514948 | 44   |
| 17   | 9,6894232 | 9,9406219 | 9,7488013 | 10,2511987 | 43   |
| 18   | 9,6896484 | 9,9405510 | 9,7490974 | 10,2509026 | 42   |
| 19   | 9,6898734 | 9,9404801 | 9,7493934 | 10,2506066 | 41   |
| 20   | 9,6900983 | 9,9404091 | 9,7496892 | 10,2503108 | 40   |
| 21   | 9,6903231 | 9,9403381 | 9,7499850 | 10,2500150 | 39   |
| 22   | 9,6905476 | 9,9402670 | 9,7502806 | 10,2497194 | 38   |
| 23   | 9,6907721 | 9,9401959 | 9,7505762 | 10,2494238 | 37   |
| 24   | 9,6909964 | 9,9401248 | 9,7508716 | 10,2491284 | 36   |
| 25   | 9,6912205 | 9,9400535 | 9,7511669 | 10,2488331 | 35   |
| 26   | 9,6914445 | 9,9399823 | 9,7514622 | 10,2485378 | 34   |
| 27   | 9,6916683 | 9,9399110 | 9,7517573 | 10,2482427 | 33   |
| 28   | 9,6918919 | 9,9398396 | 9,7520523 | 10,2479477 | 32   |
| 29   | 9,6921155 | 9,9397682 | 9,7523472 | 10,2476528 | 31   |
| 30   | 9,6923388 | 9,9396968 | 9,7526420 | 10,2473580 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

60 Degrees.

# 29 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,6923388 | 9,9396968 | 9,7526420 | 10,2473580 | 30   |
| 31   | 9,6925620 | 9,9396253 | 9,7529368 | 10,2470632 | 29   |
| 32   | 9,6927851 | 9,9395537 | 9,7532314 | 10,2467686 | 28   |
| 33   | 9,6930080 | 9,9394821 | 9,7535259 | 10,2464741 | 27   |
| 34   | 9,6932308 | 9,9394105 | 9,7538203 | 10,2461797 | 26   |
| 35   | 9,6934534 | 9,9393388 | 9,7541146 | 10,2458854 | 25   |
| 36   | 9,6936758 | 9,9392671 | 9,7544088 | 10,2455912 | 24   |
| 37   | 9,6938981 | 9,9391953 | 9,7547029 | 10,2452971 | 23   |
| 38   | 9,6941203 | 9,9391234 | 9,7549969 | 10,2450031 | 22   |
| 39   | 9,6943423 | 9,9390515 | 9,7552908 | 10,2447092 | 21   |
| 40   | 9,6945642 | 9,9389796 | 9,7555846 | 10,2444154 | 20   |
| 41   | 9,6947859 | 9,9389076 | 9,7558783 | 10,2441217 | 19   |
| 42   | 9,6950074 | 9,9388356 | 9,7561718 | 10,2438282 | 18   |
| 43   | 9,6952288 | 9,9387635 | 9,7564653 | 10,2435347 | 17   |
| 44   | 9,6954501 | 9,9386914 | 9,7567587 | 10,2432413 | 16   |
| 45   | 9,6956712 | 9,9386192 | 9,7570520 | 10,2429480 | 15   |
| 46   | 9,6958922 | 9,9385470 | 9,7573452 | 10,2426548 | 14   |
| 47   | 9,6961130 | 9,9384747 | 9,7576383 | 10,2423617 | 13   |
| 48   | 9,6963336 | 9,9384024 | 9,7579313 | 10,2420689 | 12   |
| 49   | 9,6965541 | 9,9383300 | 9,7582242 | 10,2417758 | 11   |
| 50   | 9,6967745 | 9,9382576 | 9,7585170 | 10,2414830 | 10   |
| 51   | 9,6969947 | 9,9381851 | 9,7588096 | 10,2411904 | 9    |
| 52   | 9,6972148 | 9,9381126 | 9,7591022 | 10,2408978 | 8    |
| 53   | 9,6974347 | 9,9380400 | 9,7593947 | 10,2406053 | 7    |
| 54   | 9,6976545 | 9,9379674 | 9,7596871 | 10,2403129 | 6    |
| 55   | 9,6978741 | 9,9378947 | 9,7599794 | 10,2400206 | 5    |
| 56   | 9,6980936 | 9,9378220 | 9,7602716 | 10,2397284 | 4    |
| 57   | 9,6983129 | 9,9377492 | 9,7605637 | 10,2394363 | 3    |
| 58   | 9,6985321 | 9,9376764 | 9,7608557 | 10,2391443 | 2    |
| 59   | 9,6987511 | 9,9376035 | 9,7611476 | 10,2388524 | 1    |
| 60   | 9,6989700 | 9,9375306 | 9,7614394 | 10,2385606 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 60 Degrees.

# 30 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,6989700 | 9,9375306 | 9,7614394 | 10,2385606 | 60   |
| 1    | 9,6991887 | 9,9374577 | 9,7617311 | 10,2382689 | 59   |
| 2    | 9,6994073 | 9,9373847 | 9,7620227 | 10,2379773 | 58   |
| 3    | 9,6996258 | 9,9373116 | 9,7623142 | 10,2376858 | 57   |
| 4    | 9,6998441 | 9,9372385 | 9,7626056 | 10,2373944 | 56   |
| 5    | 9,7000622 | 9,9371653 | 9,7628969 | 10,2371031 | 55   |
| 6    | 9,7002802 | 9,9370921 | 9,7631881 | 10,2368119 | 54   |
| 7    | 9,7004981 | 9,9370189 | 9,7634792 | 10,2365208 | 53   |
| 8    | 9,7007158 | 9,9369456 | 9,7637702 | 10,2362298 | 52   |
| 9    | 9,7009334 | 9,9368722 | 9,7640612 | 10,2359388 | 51   |
| 10   | 9,7011508 | 9,9367988 | 9,7643520 | 10,2356480 | 50   |
| 11   | 9,7013681 | 9,9367254 | 9,7646427 | 10,2353573 | 49   |
| 12   | 9,7015852 | 9,9366519 | 9,7649334 | 10,2350666 | 48   |
| 13   | 9,7018022 | 9,9365783 | 9,7652239 | 10,2347761 | 47   |
| 14   | 9,7020190 | 9,9365047 | 9,7655143 | 10,2344857 | 46   |
| 15   | 9,7022357 | 9,9364311 | 9,7658047 | 10,2341953 | 45   |
| 16   | 9,7024523 | 9,9363574 | 9,7660949 | 10,2339051 | 44   |
| 17   | 9,7026687 | 9,9362836 | 9,7663851 | 10,2336149 | 43   |
| 18   | 9,7028849 | 9,9362098 | 9,7666751 | 10,2333249 | 42   |
| 19   | 9,7031011 | 9,9361360 | 9,7669651 | 10,2330349 | 41   |
| 20   | 9,7033170 | 9,9360621 | 9,7672550 | 10,2327450 | 40   |
| 21   | 9,7035329 | 9,9359881 | 9,7675448 | 10,2324552 | 39   |
| 22   | 9,7037486 | 9,9359141 | 9,7678344 | 10,2321656 | 38   |
| 23   | 9,7039641 | 9,9358401 | 9,7681240 | 10,2318760 | 37   |
| 24   | 9,7041795 | 9,9357660 | 9,7684135 | 10,2315865 | 36   |
| 25   | 9,7043947 | 9,9356918 | 9,7687029 | 10,2312971 | 35   |
| 26   | 9,7046099 | 9,9356177 | 9,7689922 | 10,2310078 | 34   |
| 27   | 9,7048248 | 9,9355434 | 9,7692814 | 10,2307186 | 33   |
| 28   | 9,7050397 | 9,9354691 | 9,7695705 | 10,2304295 | 32   |
| 29   | 9,7052543 | 9,9353948 | 9,7698596 | 10,2301404 | 31   |
| 30   | 9,7054689 | 9,9353204 | 9,7701485 | 10,2298515 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 59 Degrees.

# 30 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7054689 | 9,9353204 | 9,7701485 | 10,2298515 | 30   |
| 31   | 9,7056833 | 9,9352459 | 9,7704373 | 10,2295627 | 29   |
| 32   | 9,7058975 | 9,9351715 | 9,7707261 | 10,2292739 | 28   |
| 33   | 9,7061116 | 9,9350969 | 9,7710147 | 10,2289853 | 27   |
| 34   | 9,7063256 | 9,9350223 | 9,7713033 | 10,2286967 | 26   |
| 35   | 9,7065394 | 9,9349477 | 9,7715917 | 10,2284083 | 25   |
| 36   | 9,7067131 | 9,9348730 | 9,7718801 | 10,2281199 | 24   |
| 37   | 9,7069667 | 9,9347983 | 9,7721684 | 10,2278316 | 23   |
| 38   | 9,7071801 | 9,9347235 | 9,7724566 | 10,2275434 | 22   |
| 39   | 9,7073933 | 9,9346486 | 9,7727447 | 10,2272553 | 21   |
| 40   | 9,7076064 | 9,9345738 | 9,7730327 | 10,2269673 | 20   |
| 41   | 9,7078194 | 9,9344988 | 9,7733206 | 10,2266794 | 19   |
| 42   | 9,7080323 | 9,9344238 | 9,7736084 | 10,2263916 | 18   |
| 43   | 9,7082450 | 9,9343488 | 9,7738961 | 10,2261039 | 17   |
| 44   | 9,7084575 | 9,9342737 | 9,7741838 | 10,2258162 | 16   |
| 45   | 9,7086699 | 9,9341986 | 9,7744713 | 10,2255287 | 15   |
| 46   | 9,7088822 | 9,9341234 | 9,7747588 | 10,2252412 | 14   |
| 47   | 9,7090943 | 9,9340482 | 9,7750462 | 10,2249538 | 13   |
| 48   | 9,7093063 | 9,9339729 | 9,7753334 | 10,2246666 | 12   |
| 49   | 9,7095182 | 9,9338976 | 9,7756206 | 10,2243794 | 11   |
| 50   | 9,7097299 | 9,9338222 | 9,7759077 | 10,2240923 | 10   |
| 51   | 9,7099415 | 9,9337467 | 9,7761947 | 10,2238053 | 9    |
| 52   | 9,7101529 | 9,9336713 | 9,7764816 | 10,2235184 | 8    |
| 53   | 9,7103642 | 9,9335957 | 9,7767685 | 10,2232315 | 7    |
| 54   | 9,7105753 | 9,9335201 | 9,7770552 | 10,2229448 | 6    |
| 55   | 9,7107863 | 9,9334445 | 9,7773418 | 10,2226582 | 5    |
| 56   | 9,7109972 | 9,9333688 | 9,7776284 | 10,2223716 | 4    |
| 57   | 9,7112080 | 9,9332931 | 9,7779149 | 10,2220851 | 3    |
| 58   | 9,7114186 | 9,9332173 | 9,7782012 | 10,2217988 | 2    |
| 59   | 9,7116290 | 9,9331415 | 9,7784875 | 10,2215125 | 1    |
| 60   | 9,7118393 | 9,9330656 | 9,7787737 | 10,2212263 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 59 Degrees.

| 31 Degrees. |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 9,7118393 | 9,9330656 | 9,7787737 | 10,2212263 | 60   |
| 1           | 9,7120495 | 9,9329897 | 9,7790599 | 10,2209401 | 59   |
| 2           | 9,7122596 | 9,9329137 | 9,7793459 | 10,2206541 | 58   |
| 3           | 9,7124695 | 9,9328376 | 9,7796318 | 10,2203682 | 57   |
| 4           | 9,7126792 | 9,9327616 | 9,7799177 | 10,2200823 | 56   |
| 5           | 9,7128889 | 9,9326854 | 9,7802034 | 10,2197966 | 55   |
| 6           | 9,7130983 | 9,9326092 | 9,7804891 | 10,2195109 | 54   |
| 7           | 9,7133077 | 9,9325330 | 9,7807747 | 10,2192253 | 53   |
| 8           | 9,7135169 | 9,9324567 | 9,7810602 | 10,2189398 | 52   |
| 9           | 9,7137260 | 9,9323804 | 9,7813456 | 10,2186544 | 51   |
| 10          | 9,7139349 | 9,9323040 | 9,7816309 | 10,2183691 | 50   |
| 11          | 9,7141437 | 9,9322276 | 9,7819162 | 10,2180838 | 49   |
| 12          | 9,7143524 | 9,9321511 | 9,7822013 | 10,2177987 | 48   |
| 13          | 9,7145609 | 9,9320746 | 9,7824864 | 10,2175136 | 47   |
| 14          | 9,7147693 | 9,9319980 | 9,7827713 | 10,2172287 | 46   |
| 15          | 9,7149776 | 9,9319213 | 9,7830562 | 10,2169430 | 45   |
| 16          | 9,7151857 | 9,9318447 | 9,7833410 | 10,2166598 | 44   |
| 17          | 9,7153937 | 9,9317679 | 9,7836258 | 10,2163742 | 43   |
| 18          | 9,7156015 | 9,9316911 | 9,7839104 | 10,2160896 | 42   |
| 19          | 9,7158092 | 9,9316143 | 9,7841949 | 10,2158051 | 41   |
| 20          | 9,7160168 | 9,9315374 | 9,7844794 | 10,2155206 | 40   |
| 21          | 9,7162243 | 9,9314605 | 9,7847638 | 10,2152362 | 39   |
| 22          | 9,7164316 | 9,9313835 | 9,7850481 | 10,2149519 | 38   |
| 23          | 9,7166387 | 9,9313065 | 9,7853323 | 10,2146677 | 37   |
| 24          | 9,7168458 | 9,9312294 | 9,7856164 | 10,2143836 | 36   |
| 25          | 9,7170526 | 9,9311522 | 9,7859004 | 10,2140996 | 35   |
| 26          | 9,7172594 | 9,9310750 | 9,7861844 | 10,2138156 | 34   |
| 27          | 9,7174660 | 9,9309978 | 9,7864682 | 10,2135318 | 33   |
| 28          | 9,7176725 | 9,9309205 | 9,7867520 | 10,2132480 | 32   |
| 29          | 9,7178789 | 9,9308432 | 9,7870357 | 10,2129643 | 31   |
| 30          | 9,7180851 | 9,9307658 | 9,7873193 | 10,2126807 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 58 Degrees. |           |           |           |            |      |



# 31 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7180851 | 9,9307658 | 9,7873193 | 10,2126807 | 30   |
| 31   | 9,7182912 | 9,9306883 | 9,7876028 | 10,2123972 | 29   |
| 32   | 9,7184971 | 9,9306109 | 9,7878863 | 10,2121137 | 28   |
| 33   | 9,7187030 | 9,9305333 | 9,7881696 | 10,2118304 | 27   |
| 34   | 9,7189086 | 9,9304557 | 9,7884529 | 10,2115471 | 26   |
| 35   | 9,7191142 | 9,9303781 | 9,7887361 | 10,2112639 | 25   |
| 36   | 9,7193196 | 9,9303004 | 9,7890192 | 10,2109808 | 24   |
| 37   | 9,7195249 | 9,9302226 | 9,7893023 | 10,2106977 | 23   |
| 38   | 9,7197300 | 9,9301448 | 9,7895852 | 10,2104148 | 22   |
| 39   | 9,7199350 | 9,9300670 | 9,7898681 | 10,2101319 | 21   |
| 40   | 9,7201399 | 9,9299891 | 9,7901508 | 10,2098492 | 20   |
| 41   | 9,7203447 | 9,9299112 | 9,7904335 | 10,2095665 | 19   |
| 42   | 9,7205493 | 9,9298332 | 9,7907161 | 10,2092839 | 18   |
| 43   | 9,7207538 | 9,9297551 | 9,7909987 | 10,2090013 | 17   |
| 44   | 9,7209581 | 9,9296770 | 9,7912811 | 10,2087189 | 16   |
| 45   | 9,7211623 | 9,9295989 | 9,7915635 | 10,2084365 | 15   |
| 46   | 9,7213664 | 9,9295207 | 9,7918458 | 10,2081542 | 14   |
| 47   | 9,7215704 | 9,9294424 | 9,7921280 | 10,2078720 | 13   |
| 48   | 9,7217742 | 9,9293641 | 9,7924101 | 10,2075899 | 12   |
| 49   | 9,7219779 | 9,9292857 | 9,7926921 | 10,2073079 | 11   |
| 50   | 9,7221814 | 9,9292073 | 9,7929741 | 10,2070259 | 10   |
| 51   | 9,7223848 | 9,9291289 | 9,7932560 | 10,2067440 | 9    |
| 52   | 9,7225881 | 9,9290504 | 9,7935378 | 10,2064622 | 8    |
| 53   | 9,7227913 | 9,9289718 | 9,7938195 | 10,2061805 | 7    |
| 54   | 9,7229943 | 9,9288932 | 9,7941011 | 10,2058989 | 6    |
| 55   | 9,7231972 | 9,9288145 | 9,7943827 | 10,2056173 | 5    |
| 56   | 9,7234000 | 9,9287358 | 9,7946641 | 10,2053359 | 4    |
| 57   | 9,7236026 | 9,9286571 | 9,7949455 | 10,2050545 | 3    |
| 58   | 9,7238051 | 9,9285783 | 9,7952268 | 10,2047732 | 2    |
| 59   | 9,7240075 | 9,9284994 | 9,7955081 | 10,2044919 | 1    |
| 60   | 9,7242097 | 9,9284205 | 9,7957892 | 10,2042108 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 58 Degrees.

# 32 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7242097 | 9,9284205 | 9,7957892 | 10,2042108 | 60   |
| 1    | 9,7244118 | 9,9283415 | 9,7960703 | 10,2039297 | 59   |
| 2    | 9,7246138 | 9,9282625 | 9,7963513 | 10,2036487 | 58   |
| 3    | 9,7248156 | 9,9281834 | 9,7966322 | 10,2033678 | 57   |
| 4    | 9,7250174 | 9,9281043 | 9,7969130 | 10,2030870 | 56   |
| 5    | 9,7252189 | 9,9280251 | 9,7971938 | 10,2028062 | 55   |
| 6    | 9,7254204 | 9,9279459 | 9,7974745 | 10,2025255 | 54   |
| 7    | 9,7256217 | 9,9278666 | 9,7977551 | 10,2022449 | 53   |
| 8    | 9,7258229 | 9,9277873 | 9,7980356 | 10,2019644 | 52   |
| 9    | 9,7260240 | 9,9277079 | 9,7983160 | 10,2016840 | 51   |
| 10   | 9,7262249 | 9,9276285 | 9,7985964 | 10,2014036 | 50   |
| 11   | 9,7264257 | 9,9275490 | 9,7988767 | 10,2011233 | 49   |
| 12   | 9,7266264 | 9,9274695 | 9,7991569 | 10,2008431 | 48   |
| 13   | 9,7268269 | 9,9273899 | 9,7994370 | 10,2005630 | 47   |
| 14   | 9,7270273 | 9,9273103 | 9,7997170 | 10,2002830 | 46   |
| 15   | 9,7272276 | 9,9272306 | 9,7999970 | 10,2000030 | 45   |
| 16   | 9,7274278 | 9,9271509 | 9,8002769 | 10,1997231 | 44   |
| 17   | 9,7276278 | 9,9270711 | 9,8005567 | 10,1994433 | 43   |
| 18   | 9,7278277 | 9,9269913 | 9,8008365 | 10,1991635 | 42   |
| 19   | 9,7280275 | 9,9269114 | 9,8011161 | 10,1988839 | 41   |
| 20   | 9,7282271 | 9,9268314 | 9,8013957 | 10,1986043 | 40   |
| 21   | 9,7284267 | 9,9267514 | 9,8016752 | 10,1983248 | 39   |
| 22   | 9,7286260 | 9,9266714 | 9,8019546 | 10,1980454 | 38   |
| 23   | 9,7288253 | 9,9265913 | 9,8022340 | 10,1977660 | 37   |
| 24   | 9,7290244 | 9,9265112 | 9,8025133 | 10,1974867 | 36   |
| 25   | 9,7292234 | 9,9264310 | 9,8027925 | 10,1972075 | 35   |
| 26   | 9,7294223 | 9,9263507 | 9,8030716 | 10,1969284 | 34   |
| 27   | 9,7296211 | 9,9262704 | 9,8033506 | 10,1966494 | 33   |
| 28   | 9,7298197 | 9,9261901 | 9,8036296 | 10,1963704 | 32   |
| 29   | 9,7300182 | 9,9261096 | 9,8039085 | 10,1960915 | 31   |
| 30   | 9,7302165 | 9,9260292 | 9,8041873 | 10,1958127 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

57 Degrees.

# 32. Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7302165 | 9,9260292 | 9,8041873 | 10,1958127 | 30   |
| 31   | 9,7304148 | 9,9259487 | 9,8044661 | 10,1955339 | 29   |
| 32   | 9,7306129 | 9,9258681 | 9,8047447 | 10,1952553 | 28   |
| 33   | 9,7308109 | 9,9257875 | 9,8050233 | 10,1949767 | 27   |
| 34   | 9,7310087 | 9,9257069 | 9,8053019 | 10,1946981 | 26   |
| 35   | 9,7312064 | 9,9256261 | 9,8055803 | 10,1944197 | 25   |
| 36   | 9,7314040 | 9,9255454 | 9,8058587 | 10,1941413 | 24   |
| 37   | 9,7316015 | 9,9254646 | 9,8061370 | 10,1938630 | 23   |
| 38   | 9,7317989 | 9,9253837 | 9,8064152 | 10,1935848 | 22   |
| 39   | 9,7319961 | 9,9253028 | 9,8066933 | 10,1933067 | 21   |
| 40   | 9,7321932 | 9,9252218 | 9,8069714 | 10,1930286 | 20   |
| 41   | 9,7323902 | 9,9251408 | 9,8072494 | 10,1927506 | 19   |
| 42   | 9,7325870 | 9,9250597 | 9,8075273 | 10,1924727 | 18   |
| 43   | 9,7327837 | 9,9249786 | 9,8078052 | 10,1921948 | 17   |
| 44   | 9,7329803 | 9,9248974 | 9,8080829 | 10,1919171 | 16   |
| 45   | 9,7331768 | 9,9248161 | 9,8083606 | 10,1916394 | 15   |
| 46   | 9,7333731 | 9,9247349 | 9,8086383 | 10,1913617 | 14   |
| 47   | 9,7335693 | 9,9246535 | 9,8089158 | 10,1910842 | 13   |
| 48   | 9,7337654 | 9,9245721 | 9,8091933 | 10,1908067 | 12   |
| 49   | 9,7339614 | 9,9244907 | 9,8094707 | 10,1905293 | 11   |
| 50   | 9,7341572 | 9,9244092 | 9,8097480 | 10,1902520 | 10   |
| 51   | 9,7343529 | 9,9243277 | 9,8100253 | 10,1899747 | 9    |
| 52   | 9,7345485 | 9,9242461 | 9,8103025 | 10,1896975 | 8    |
| 53   | 9,7347440 | 9,9241644 | 9,8105796 | 10,1894204 | 7    |
| 54   | 9,7349393 | 9,9240827 | 9,8108566 | 10,1891434 | 6    |
| 55   | 9,7351345 | 9,9240010 | 9,8111336 | 10,1888664 | 5    |
| 56   | 9,7353296 | 9,9239191 | 9,8114105 | 10,1885895 | 4    |
| 57   | 9,7355246 | 9,9238373 | 9,8116873 | 10,1883127 | 3    |
| 58   | 9,7357195 | 9,9237554 | 9,8119641 | 10,1880359 | 2    |
| 59   | 9,7359142 | 9,9236734 | 9,8122404 | 10,1877592 | 1    |
| 60   | 9,7361088 | 9,9235914 | 9,8125174 | 10,1874826 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 57 Degrees.

Q

# 33 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7361088 | 9,9235914 | 9,8125174 | 10,1874826 | 60   |
| 1    | 9,7363032 | 9,9235093 | 9,8127939 | 10,1872061 | 59   |
| 2    | 9,7364976 | 9,9234272 | 9,8130704 | 10,1869296 | 58   |
| 3    | 9,7366918 | 9,9233450 | 9,8133468 | 10,1866532 | 57   |
| 4    | 9,7368859 | 9,9232628 | 9,8136231 | 10,1863769 | 56   |
| 5    | 9,7370799 | 9,9231805 | 9,8138993 | 10,1861007 | 55   |
| 6    | 9,7372737 | 9,9230982 | 9,8141755 | 10,1858245 | 54   |
| 7    | 9,7374675 | 9,9230158 | 9,8144516 | 10,1855484 | 53   |
| 8    | 9,7376611 | 9,9229334 | 9,8147277 | 10,1852723 | 52   |
| 9    | 9,7378546 | 9,9228509 | 9,8150036 | 10,1849964 | 51   |
| 10   | 9,7380479 | 9,9227684 | 9,8152795 | 10,1847205 | 50   |
| 11   | 9,7382412 | 9,9226858 | 9,8155554 | 10,1844446 | 49   |
| 12   | 9,7384343 | 9,9226032 | 9,8158311 | 10,1841689 | 48   |
| 13   | 9,7386273 | 9,9225205 | 9,8161068 | 10,1838932 | 47   |
| 14   | 9,7388201 | 9,9224377 | 9,8163824 | 10,1836176 | 46   |
| 15   | 9,7390129 | 9,9223549 | 9,8166580 | 10,1833420 | 45   |
| 16   | 9,7392055 | 9,9222721 | 9,8169335 | 10,1830665 | 44   |
| 17   | 9,7393980 | 9,9221891 | 9,8172089 | 10,1827911 | 43   |
| 18   | 9,7395904 | 9,9221062 | 9,8174842 | 10,1825158 | 42   |
| 19   | 9,7397827 | 9,9220232 | 9,8177595 | 10,1822405 | 41   |
| 20   | 9,7399748 | 9,9219401 | 9,8180347 | 10,1819653 | 40   |
| 21   | 9,7401668 | 9,9218570 | 9,8183098 | 10,1816902 | 39   |
| 22   | 9,7403587 | 9,9217738 | 9,8185849 | 10,1814151 | 38   |
| 23   | 9,7405505 | 9,9216906 | 9,8188599 | 10,1811401 | 37   |
| 24   | 9,7407421 | 9,9216073 | 9,8191348 | 10,1808652 | 36   |
| 25   | 9,7409337 | 9,9215240 | 9,8194096 | 10,1805904 | 35   |
| 26   | 9,7411251 | 9,9214406 | 9,8196844 | 10,1803156 | 34   |
| 27   | 9,7413164 | 9,9213572 | 9,8199592 | 10,1800408 | 33   |
| 28   | 9,7415075 | 9,9212737 | 9,8202338 | 10,1797662 | 32   |
| 29   | 9,7416986 | 9,9211902 | 9,8205084 | 10,1794916 | 31   |
| 30   | 9,7418895 | 9,9211066 | 9,8207829 | 10,1792171 | 30   |
|      | Sine Com. | Sine.     | Tan. Com  | Tangent.   | Min. |

56 Degrees.

# 33 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7418895 | 9,9211066 | 9,8207829 | 10,1792171 | 30.  |
| 31   | 9,7420803 | 9,9210229 | 9,8210574 | 10,1789426 | 29.  |
| 32   | 9,7422710 | 9,9209393 | 9,8213317 | 10,1786683 | 28.  |
| 33   | 9,7424616 | 9,9208555 | 9,8216060 | 10,1783940 | 27.  |
| 34   | 9,7426520 | 9,9207717 | 9,8218803 | 10,1781197 | 26.  |
| 35   | 9,7428423 | 9,9206878 | 9,8221545 | 10,1778455 | 25.  |
| 36   | 9,7430325 | 9,9206039 | 9,8224286 | 10,1775714 | 24.  |
| 37   | 9,7432226 | 9,9205200 | 9,8227026 | 10,1772974 | 23.  |
| 38   | 9,7434126 | 9,9204360 | 9,8229766 | 10,1770234 | 22.  |
| 39   | 9,7436024 | 9,9203519 | 9,8232505 | 10,1767495 | 21.  |
| 40   | 9,7437921 | 9,9202678 | 9,8235244 | 10,1764756 | 20.  |
| 41   | 9,7439817 | 9,9201836 | 9,8237981 | 10,1762019 | 19.  |
| 42   | 9,7441712 | 9,9200994 | 9,8240719 | 10,1759281 | 18.  |
| 43   | 9,7443606 | 9,9200151 | 9,8243455 | 10,1756545 | 17.  |
| 44   | 9,7445498 | 9,9199308 | 9,8246191 | 10,1753809 | 16.  |
| 45   | 9,7447390 | 9,9198464 | 9,8248926 | 10,1751074 | 15.  |
| 46   | 9,7449280 | 9,9197619 | 9,8251660 | 10,1748340 | 14.  |
| 47   | 9,7451169 | 9,9196775 | 9,8254394 | 10,1745606 | 13.  |
| 48   | 9,7453056 | 9,9195929 | 9,8257127 | 10,1742873 | 12.  |
| 49   | 9,7454943 | 9,9195083 | 9,8259860 | 10,1740140 | 11.  |
| 50   | 9,7456828 | 9,9194237 | 9,8262592 | 10,1737408 | 10.  |
| 51   | 9,7458712 | 9,9193390 | 9,8265323 | 10,1734677 | 9.   |
| 52   | 9,7460595 | 9,9192542 | 9,8268053 | 10,1731947 | 8.   |
| 53   | 9,7462477 | 9,9191694 | 9,8270783 | 10,1729217 | 7.   |
| 54   | 9,7464358 | 9,9190845 | 9,8273513 | 10,1726487 | 6.   |
| 55   | 9,7466237 | 9,9189996 | 9,8276241 | 10,1723759 | 5.   |
| 56   | 9,7468115 | 9,9189146 | 9,8278969 | 10,1721031 | 4.   |
| 57   | 9,7469992 | 9,9188296 | 9,8281696 | 10,1718304 | 3.   |
| 58   | 9,7471868 | 9,9187445 | 9,8284423 | 10,1715577 | 2.   |
| 59   | 9,7473743 | 9,9186594 | 9,8287149 | 10,1712851 | 1.   |
| 60   | 9,7475617 | 9,9185742 | 9,8289874 | 10,1710126 | 0.   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 56 Degrees.

# 34 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7475617 | 9,9185742 | 9,8289874 | 10,1710126 | 60   |
| 1    | 9,7477489 | 9,9184896 | 9,8292599 | 10,1707401 | 59   |
| 2    | 9,7479360 | 9,9184037 | 9,8295323 | 10,1704677 | 58   |
| 3    | 9,7481230 | 9,9183183 | 9,8298047 | 10,1701953 | 57   |
| 4    | 9,7483099 | 9,9182329 | 9,8300769 | 10,1699231 | 56   |
| 5    | 9,7484967 | 9,9181475 | 9,8303492 | 10,1696508 | 55   |
| 6    | 9,7486833 | 9,9180620 | 9,8306213 | 10,1693787 | 54   |
| 7    | 9,7488698 | 9,9179764 | 9,8308934 | 10,1691066 | 53   |
| 8    | 9,7490562 | 9,9178908 | 9,8311654 | 10,1688346 | 52   |
| 9    | 9,7492425 | 9,9178051 | 9,8314374 | 10,1685626 | 51   |
| 10   | 9,7494287 | 9,9177194 | 9,8317093 | 10,1682907 | 50   |
| 11   | 9,7496148 | 9,9176336 | 9,8319811 | 10,1680189 | 49   |
| 12   | 9,7498007 | 9,9175478 | 9,8322529 | 10,1677471 | 48   |
| 13   | 9,7499866 | 9,9174619 | 9,8325246 | 10,1674754 | 47   |
| 14   | 9,7501723 | 9,9173760 | 9,8327963 | 10,1672037 | 46   |
| 15   | 9,7503579 | 9,9172900 | 9,8330679 | 10,1669321 | 45   |
| 16   | 9,7505434 | 9,9172040 | 9,8333394 | 10,1666606 | 44   |
| 17   | 9,7507287 | 9,9171179 | 9,8336109 | 10,1663891 | 43   |
| 18   | 9,7509140 | 9,9170317 | 9,8338823 | 10,1661177 | 42   |
| 19   | 9,7510991 | 9,9169455 | 9,8341536 | 10,1658464 | 41   |
| 20   | 9,7512842 | 9,9168593 | 9,8344249 | 10,1655751 | 40   |
| 21   | 9,7514691 | 9,9167730 | 9,8346961 | 10,1653039 | 39   |
| 22   | 9,7516538 | 9,9166866 | 9,8349673 | 10,1650327 | 38   |
| 23   | 9,7518385 | 9,9166002 | 9,8352384 | 10,1647616 | 37   |
| 24   | 9,7520231 | 9,9165137 | 9,8355094 | 10,1644906 | 36   |
| 25   | 9,7522075 | 9,9164272 | 9,8357804 | 10,1642196 | 35   |
| 26   | 9,7523919 | 9,9163406 | 9,8360513 | 10,1639487 | 34   |
| 27   | 9,7525761 | 9,9162539 | 9,8363221 | 10,1636779 | 33   |
| 28   | 9,7527602 | 9,9161673 | 9,8365929 | 10,1634071 | 32   |
| 29   | 9,7529442 | 9,9160805 | 9,8368636 | 10,1631364 | 31   |
| 30   | 9,7531280 | 9,9159937 | 9,8371343 | 10,1628657 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 55 Degrees.

# 34 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7531280 | 9,9159937 | 9,8371343 | 10,1628657 | 30   |
| 31   | 9,7533118 | 9,9159069 | 9,8374049 | 10,1625951 | 29   |
| 32   | 9,7534954 | 9,9158200 | 9,8376755 | 10,1623245 | 28   |
| 33   | 9,7536790 | 9,9157330 | 9,8379460 | 10,1620540 | 27   |
| 34   | 9,7538624 | 9,9156460 | 9,8382164 | 10,1617836 | 26   |
| 35   | 9,7540457 | 9,9155589 | 9,8384867 | 10,1615133 | 25   |
| 36   | 9,7542288 | 9,9154718 | 9,8387571 | 10,1612429 | 24   |
| 37   | 9,7544119 | 9,9153846 | 9,8390273 | 10,1609727 | 23   |
| 38   | 9,7545948 | 9,9152974 | 9,8392975 | 10,1607025 | 22   |
| 39   | 9,7547777 | 9,9152101 | 9,8395676 | 10,1604324 | 21   |
| 40   | 9,7549604 | 9,9151228 | 9,8398377 | 10,1601623 | 20   |
| 41   | 9,7551431 | 9,9150354 | 9,8401077 | 10,1598923 | 19   |
| 42   | 9,7553256 | 9,9149479 | 9,8403776 | 10,1596224 | 18   |
| 43   | 9,7555080 | 9,9148604 | 9,8406475 | 10,1593525 | 17   |
| 44   | 9,7556902 | 9,9147729 | 9,8409174 | 10,1590826 | 16   |
| 45   | 9,7558724 | 9,9146852 | 9,8411871 | 10,1588129 | 15   |
| 46   | 9,7560544 | 9,9145976 | 9,8414569 | 10,1585431 | 14   |
| 47   | 9,7562364 | 9,9145099 | 9,8417265 | 10,1582735 | 13   |
| 48   | 9,7564182 | 9,9144221 | 9,8419961 | 10,1580039 | 12   |
| 49   | 9,7565999 | 9,9143342 | 9,8422657 | 10,1577343 | 11   |
| 50   | 9,7567815 | 9,9142464 | 9,8425351 | 10,1574649 | 10   |
| 51   | 9,7569630 | 9,9141584 | 9,8428046 | 10,1571954 | 9    |
| 52   | 9,7571444 | 9,9140704 | 9,8430739 | 10,1569261 | 8    |
| 53   | 9,7573256 | 9,9139824 | 9,8433432 | 10,1566568 | 7    |
| 54   | 9,7575068 | 9,9138943 | 9,8436125 | 10,1563875 | 6    |
| 55   | 9,7576878 | 9,9138061 | 9,8438817 | 10,1561183 | 5    |
| 56   | 9,7578687 | 9,9137179 | 9,8441508 | 10,1558492 | 4    |
| 57   | 9,7580495 | 9,9136296 | 9,8444199 | 10,1555801 | 3    |
| 58   | 9,7582302 | 9,9135413 | 9,8446889 | 10,1553111 | 2    |
| 59   | 9,7584108 | 9,9134530 | 9,8449579 | 10,1550421 | 1    |
| 60   | 9,7585913 | 9,9133645 | 9,8452268 | 10,1547732 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 55 Degrees.

| 35 Degrees. |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 9,7585913 | 9,9133645 | 9,8452268 | 10,1547732 | 60   |
| 1           | 9,7587717 | 9,9132760 | 9,8454956 | 10,1545044 | 59   |
| 2           | 9,7589519 | 9,9131875 | 9,8457644 | 10,1542356 | 58   |
| 3           | 9,7591321 | 9,9130989 | 9,8460332 | 10,1539668 | 57   |
| 4           | 9,7593121 | 9,9130102 | 9,8463018 | 10,1536982 | 56   |
| 5           | 9,7594920 | 9,9129215 | 9,8465705 | 10,1534295 | 55   |
| 6           | 9,7596718 | 9,9128328 | 9,8468390 | 10,1531610 | 54   |
| 7           | 9,7598515 | 9,9127440 | 9,8471075 | 10,1528925 | 53   |
| 8           | 9,7600311 | 9,9126551 | 9,8473760 | 10,1526240 | 52   |
| 9           | 9,7602106 | 9,9125662 | 9,8476444 | 10,1523556 | 51   |
| 10          | 9,7603899 | 9,9124772 | 9,8479127 | 10,1520873 | 50   |
| 11          | 9,7605692 | 9,9123882 | 9,8481810 | 10,1518190 | 49   |
| 12          | 9,7607483 | 9,9122991 | 9,8484492 | 10,1515508 | 48   |
| 13          | 9,7609274 | 9,9122099 | 9,8487174 | 10,1512826 | 47   |
| 14          | 9,7611063 | 9,9121207 | 9,8489855 | 10,1510145 | 46   |
| 15          | 9,7612851 | 9,9120315 | 9,8492536 | 10,1507464 | 45   |
| 16          | 9,7614638 | 9,9119422 | 9,8495216 | 10,1504784 | 44   |
| 17          | 9,7616424 | 9,9118528 | 9,8497896 | 10,1502104 | 43   |
| 18          | 9,7618208 | 9,9117634 | 9,8500575 | 10,1499425 | 42   |
| 19          | 9,7619992 | 9,9116739 | 9,8503253 | 10,1496747 | 41   |
| 20          | 9,7621775 | 9,9115844 | 9,8505931 | 10,1494069 | 40   |
| 21          | 9,7623556 | 9,9114948 | 9,8508608 | 10,1491392 | 39   |
| 22          | 9,7625337 | 9,9114051 | 9,8511285 | 10,1488715 | 38   |
| 23          | 9,7627116 | 9,9113155 | 9,8513961 | 10,1486039 | 37   |
| 24          | 9,7628894 | 9,9112257 | 9,8516637 | 10,1483363 | 36   |
| 25          | 9,7630671 | 9,9111359 | 9,8519312 | 10,1480688 | 35   |
| 26          | 9,7632447 | 9,9110460 | 9,8521987 | 10,1478013 | 34   |
| 27          | 9,7634222 | 9,9109561 | 9,8524661 | 10,1475339 | 33   |
| 28          | 9,7635996 | 9,9108661 | 9,8527335 | 10,1472665 | 32   |
| 29          | 9,7637769 | 9,9107761 | 9,8530008 | 10,1469992 | 31   |
| 30          | 9,7639540 | 9,9106860 | 9,8532680 | 10,1467320 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

54 Degrees.



# 35 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Con.  | Min. |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7639540 | 9,9106860 | 9,8532680 | 10,1467320 | 30   |
| 31   | 9,7641311 | 9,9105959 | 9,8535352 | 10,1464648 | 29   |
| 32   | 9,7643080 | 9,9105057 | 9,8538023 | 10,1461977 | 28   |
| 33   | 9,7644849 | 9,9104155 | 9,8540694 | 10,1459306 | 27   |
| 34   | 9,7646616 | 9,9103251 | 9,8543365 | 10,1456635 | 26   |
| 35   | 9,7648382 | 9,9102348 | 9,8546034 | 10,1453966 | 25   |
| 36   | 9,7650147 | 9,9101444 | 9,8548704 | 10,1451296 | 24   |
| 37   | 9,7651911 | 9,9100539 | 9,8551372 | 10,1448628 | 23   |
| 38   | 9,7653674 | 9,9099634 | 9,8554041 | 10,1445959 | 22   |
| 39   | 9,7655436 | 9,9098728 | 9,8556708 | 10,1443292 | 21   |
| 40   | 9,7657197 | 9,9097821 | 9,8559376 | 10,1440624 | 20   |
| 41   | 9,7658957 | 9,9096915 | 9,8562042 | 10,1437958 | 19   |
| 42   | 9,7660715 | 9,9096007 | 9,8564708 | 10,1435292 | 18   |
| 43   | 9,7662473 | 9,9095090 | 9,8567374 | 10,1432626 | 17   |
| 44   | 9,7664229 | 9,9094190 | 9,8570039 | 10,1429961 | 16   |
| 45   | 9,7665985 | 9,9093281 | 9,8572704 | 10,1427296 | 15   |
| 46   | 9,7667739 | 9,9092371 | 9,8575368 | 10,1424632 | 14   |
| 47   | 9,7669492 | 9,9091461 | 9,8578031 | 10,1421969 | 13   |
| 48   | 9,7671244 | 9,9090550 | 9,8580694 | 10,1419306 | 12   |
| 49   | 9,7672996 | 9,9089639 | 9,8583357 | 10,1416643 | 11   |
| 50   | 9,7674746 | 9,9088727 | 9,8586019 | 10,1413981 | 10   |
| 51   | 9,7676494 | 9,9087814 | 9,8588680 | 10,1411320 | 9    |
| 52   | 9,7678242 | 9,9086901 | 9,8591341 | 10,1408659 | 8    |
| 53   | 9,7679989 | 9,9085988 | 9,8594002 | 10,1405998 | 7    |
| 54   | 9,7681735 | 9,9085073 | 9,8596661 | 10,1403339 | 6    |
| 55   | 9,7683480 | 9,9084159 | 9,8599321 | 10,1400679 | 5    |
| 56   | 9,7685223 | 9,9083243 | 9,8601980 | 10,1398020 | 4    |
| 57   | 9,7686966 | 9,9082327 | 9,8604638 | 10,1395362 | 3    |
| 58   | 9,7688707 | 9,9081411 | 9,8607296 | 10,1392704 | 2    |
| 59   | 9,7690448 | 9,9080494 | 9,8609954 | 10,1390046 | 1    |
| 60   | 9,7692187 | 9,9079576 | 9,8612610 | 10,1387390 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 54 Degrees.

# 36 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7692187 | 9,9079576 | 9,8612610 | 10,1387390 | 60   |
| 1    | 9,7693925 | 9,9078658 | 9,8615267 | 10,1384733 | 59   |
| 2    | 9,7695662 | 9,9077740 | 9,8617923 | 10,1382077 | 58   |
| 3    | 9,7697398 | 9,9076820 | 9,8620578 | 10,1379422 | 57   |
| 4    | 9,7699134 | 9,9075901 | 9,8623233 | 10,1376767 | 56   |
| 5    | 9,7700808 | 9,9074980 | 9,8625887 | 10,1374113 | 55   |
| 6    | 9,7702601 | 9,9074059 | 9,8628541 | 10,1371459 | 54   |
| 7    | 9,7704332 | 9,9073138 | 9,8631195 | 10,1368805 | 53   |
| 8    | 9,7706063 | 9,9072216 | 9,8633848 | 10,1366152 | 52   |
| 9    | 9,7707793 | 9,9071293 | 9,8636500 | 10,1363500 | 51   |
| 10   | 9,7709522 | 9,9070370 | 9,8639152 | 10,1360848 | 50   |
| 11   | 9,7711249 | 9,9069446 | 9,8641803 | 10,1358197 | 49   |
| 12   | 9,7712976 | 9,9068522 | 9,8644454 | 10,1355546 | 48   |
| 13   | 9,7714702 | 9,9067597 | 9,8647105 | 10,1352895 | 47   |
| 14   | 9,7716426 | 9,9066671 | 9,8649755 | 10,1350245 | 46   |
| 15   | 9,7718150 | 9,9065745 | 9,8652404 | 10,1347596 | 45   |
| 16   | 9,7719872 | 9,9064819 | 9,8655053 | 10,1344947 | 44   |
| 17   | 9,7721593 | 9,9063892 | 9,8657702 | 10,1342298 | 43   |
| 18   | 9,7723314 | 9,9062964 | 9,8660350 | 10,1339650 | 42   |
| 19   | 9,7725033 | 9,9062036 | 9,8662997 | 10,1337003 | 41   |
| 20   | 9,7726751 | 9,9061107 | 9,8665644 | 10,1334356 | 40   |
| 21   | 9,7728468 | 9,9060177 | 9,8668291 | 10,1331709 | 39   |
| 22   | 9,7730185 | 9,9059247 | 9,8670937 | 10,1329063 | 38   |
| 23   | 9,7731900 | 9,9058317 | 9,8673583 | 10,1326417 | 37   |
| 24   | 9,7733614 | 9,9057386 | 9,8676228 | 10,1323772 | 36   |
| 25   | 9,7735327 | 9,9056454 | 9,8678873 | 10,1321127 | 35   |
| 26   | 9,7737039 | 9,9055522 | 9,8681517 | 10,1318483 | 34   |
| 27   | 9,7738749 | 9,9054589 | 9,8684160 | 10,1315840 | 33   |
| 28   | 9,7740459 | 9,9053656 | 9,8686804 | 10,1313196 | 32   |
| 29   | 9,7742168 | 9,9052722 | 9,8689446 | 10,1310554 | 31   |
| 30   | 9,7743876 | 9,9051787 | 9,8692089 | 10,1307911 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 53 Degrees.

# 36 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7743876 | 9,9051787 | 9,8692089 | 10,1307911 | 30   |
| 31   | 9,7745583 | 9,9050852 | 9,8694731 | 10,1305269 | 29   |
| 32   | 9,7747288 | 9,9049916 | 9,8697372 | 10,1302628 | 28   |
| 33   | 9,7748993 | 9,9048980 | 9,8700013 | 10,1299987 | 27   |
| 34   | 9,7750697 | 9,9048043 | 9,8702653 | 10,1297347 | 26   |
| 35   | 9,7752399 | 9,9047106 | 9,8705293 | 10,1294707 | 25   |
| 36   | 9,7754101 | 9,9046168 | 9,8707933 | 10,1292067 | 24   |
| 37   | 9,7755801 | 9,9045230 | 9,8710572 | 10,1289428 | 23   |
| 38   | 9,7757501 | 9,9044291 | 9,8713210 | 10,1286790 | 22   |
| 39   | 9,7759199 | 9,9043351 | 9,8715848 | 10,1284152 | 21   |
| 40   | 9,7760897 | 9,9042411 | 9,8718486 | 10,1281514 | 20   |
| 41   | 9,7762593 | 9,9041470 | 9,8721123 | 10,1278877 | 19   |
| 42   | 9,7764289 | 9,9040529 | 9,8723760 | 10,1276240 | 18   |
| 43   | 9,7765983 | 9,9039587 | 9,8726396 | 10,1273604 | 17   |
| 44   | 9,7767676 | 9,9038644 | 9,8729032 | 10,1270968 | 16   |
| 45   | 9,7769369 | 9,9037701 | 9,8731668 | 10,1268332 | 15   |
| 46   | 9,7771060 | 9,9036757 | 9,8734302 | 10,1265698 | 14   |
| 47   | 9,7772750 | 9,9035813 | 9,8736937 | 10,1263063 | 13   |
| 48   | 9,7774439 | 9,9034868 | 9,8739571 | 10,1260429 | 12   |
| 49   | 9,7776128 | 9,9033923 | 9,8742204 | 10,1257796 | 11   |
| 50   | 9,7777815 | 9,9032977 | 9,8744838 | 10,1255162 | 10   |
| 51   | 9,7779501 | 9,9032031 | 9,8747470 | 10,1252530 | 9    |
| 52   | 9,7781186 | 9,9031084 | 9,8750102 | 10,1249898 | 8    |
| 53   | 9,7782870 | 9,9030136 | 9,8752734 | 10,1247266 | 7    |
| 54   | 9,7784553 | 9,9029188 | 9,8755365 | 10,1244635 | 6    |
| 55   | 9,7786235 | 9,9028239 | 9,8757996 | 10,1242004 | 5    |
| 56   | 9,7787916 | 9,9027289 | 9,8760627 | 10,1239373 | 4    |
| 57   | 9,7789596 | 9,9026339 | 9,8763257 | 10,1236743 | 3    |
| 58   | 9,7791275 | 9,9025389 | 9,8765886 | 10,1234114 | 2    |
| 59   | 9,7792953 | 9,9024438 | 9,8768515 | 10,1231485 | 1    |
| 60   | 9,7794630 | 9,9023486 | 9,8771144 | 10,1228856 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 53 Degrees.

R

# 37 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7794630 | 9,9023486 | 9,8771144 | 10,1228856 | 60   |
| 1    | 9,7796306 | 9,9022534 | 9,8773772 | 10,1226228 | 59   |
| 2    | 9,7797981 | 9,9022581 | 9,8776400 | 10,1223600 | 58   |
| 3    | 9,7799655 | 9,9020628 | 9,8779027 | 10,1220973 | 57   |
| 4    | 9,7801328 | 9,9019674 | 9,8781654 | 10,1218340 | 56   |
| 5    | 9,7803000 | 9,9018719 | 9,8784281 | 10,1215719 | 55   |
| 6    | 9,7804671 | 9,9017764 | 9,8786907 | 10,1213093 | 54   |
| 7    | 9,7806341 | 9,9016808 | 9,8789533 | 10,1210467 | 53   |
| 8    | 9,7808010 | 9,9015852 | 9,8792158 | 10,1207842 | 52   |
| 9    | 9,7809677 | 9,9014895 | 9,8794782 | 10,1205218 | 51   |
| 10   | 9,7811344 | 9,9013938 | 9,8797407 | 10,1202593 | 50   |
| 11   | 9,7813010 | 9,9012980 | 9,8800031 | 10,1199969 | 49   |
| 12   | 9,7814675 | 9,9012021 | 9,8802654 | 10,1197346 | 48   |
| 13   | 9,7816339 | 9,9011062 | 9,8805277 | 10,1194723 | 47   |
| 14   | 9,7818002 | 9,9010102 | 9,8807900 | 10,1192100 | 46   |
| 15   | 9,7819664 | 9,9009142 | 9,8810522 | 10,1189478 | 45   |
| 16   | 9,7821324 | 9,9008181 | 9,8813144 | 10,1186856 | 44   |
| 17   | 9,7822984 | 9,9007219 | 9,8815765 | 10,1184235 | 43   |
| 18   | 9,7824643 | 9,9006257 | 9,8818386 | 10,1181614 | 42   |
| 19   | 9,7826301 | 9,9005294 | 9,8821007 | 10,1178993 | 41   |
| 20   | 9,7827958 | 9,9004331 | 9,8823627 | 10,1176373 | 40   |
| 21   | 9,7829614 | 9,9003367 | 9,8826246 | 10,1173754 | 39   |
| 22   | 9,7831268 | 9,9002403 | 9,8828866 | 10,1171134 | 38   |
| 23   | 9,7832922 | 9,9001438 | 9,8831484 | 10,1168516 | 37   |
| 24   | 9,7834575 | 9,9000472 | 9,8834103 | 10,1165897 | 36   |
| 25   | 9,7836227 | 9,8999506 | 9,8836721 | 10,1163279 | 35   |
| 26   | 9,7837878 | 9,8998539 | 9,8839338 | 10,1160662 | 34   |
| 27   | 9,7839528 | 9,8997572 | 9,8841956 | 10,1158044 | 33   |
| 28   | 9,7841177 | 9,8996604 | 9,8844572 | 10,1155428 | 32   |
| 29   | 9,7842824 | 9,8995636 | 9,8847189 | 10,1152811 | 31   |
| 30   | 9,7844471 | 9,8994667 | 9,8849805 | 10,1150195 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 52 Degrees.

# 37 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7844471 | 9,8994667 | 9,8849805 | 10,1150195 | 30   |
| 31   | 9,7846117 | 9,8993697 | 9,8852420 | 10,1147580 | 29   |
| 32   | 9,7847762 | 9,8992727 | 9,8855035 | 10,1144965 | 28   |
| 33   | 9,7849406 | 9,8991756 | 9,8857650 | 10,1142350 | 27   |
| 34   | 9,7851049 | 9,8990784 | 9,8860264 | 10,1139736 | 26   |
| 35   | 9,7852691 | 9,8989812 | 9,8862878 | 10,1137122 | 25   |
| 36   | 9,7854332 | 9,8988840 | 9,8865492 | 10,1134508 | 24   |
| 37   | 9,7855972 | 9,8987867 | 9,8868105 | 10,1131895 | 23   |
| 38   | 9,7857611 | 9,8986893 | 9,8870718 | 10,1129282 | 22   |
| 39   | 9,7859249 | 9,8985919 | 9,8873330 | 10,1126670 | 21   |
| 40   | 9,7860886 | 9,8984944 | 9,8875942 | 10,1124058 | 20   |
| 41   | 9,7862522 | 9,8983968 | 9,8878554 | 10,1121446 | 19   |
| 42   | 9,7864157 | 9,8982992 | 9,8881165 | 10,1118835 | 18   |
| 43   | 9,7865791 | 9,8982015 | 9,8883775 | 10,1116225 | 17   |
| 44   | 9,7867424 | 9,8981038 | 9,8886386 | 10,1113614 | 16   |
| 45   | 9,7869056 | 9,8980060 | 9,8888996 | 10,1111004 | 15   |
| 46   | 9,7870687 | 9,8979082 | 9,8891605 | 10,1108395 | 14   |
| 47   | 9,7872317 | 9,8978103 | 9,8894214 | 10,1105786 | 13   |
| 48   | 9,7873946 | 9,8977123 | 9,8896823 | 10,1103177 | 12   |
| 49   | 9,7875574 | 9,8976143 | 9,8899432 | 10,1100568 | 11   |
| 50   | 9,7877202 | 9,8975162 | 9,8902040 | 10,1097960 | 10   |
| 51   | 9,7878828 | 9,8974181 | 9,8904647 | 10,1095353 | 9    |
| 52   | 9,7880453 | 9,8973199 | 9,8907254 | 10,1092746 | 8    |
| 53   | 9,7882077 | 9,8972216 | 9,8909861 | 10,1090139 | 7    |
| 54   | 9,7883701 | 9,8971233 | 9,8912468 | 10,1087532 | 6    |
| 55   | 9,7885323 | 9,8970249 | 9,8915074 | 10,1084926 | 5    |
| 56   | 9,7886944 | 9,8969265 | 9,8917679 | 10,1082321 | 4    |
| 57   | 9,7888565 | 9,8968280 | 9,8920285 | 10,1079715 | 3    |
| 58   | 9,7890184 | 9,8967294 | 9,8922890 | 10,1077110 | 2    |
| 59   | 9,7891802 | 9,8966308 | 9,8925494 | 10,1074506 | 1    |
| 60   | 9,7893420 | 9,8965321 | 9,8928098 | 10,1071902 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 52 Degrees.

# 38 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7893420 | 9,8965321 | 9,8928098 | 10,1071902 | 60   |
| 1    | 9,7895036 | 9,8964334 | 9,8930702 | 10,1069298 | 59   |
| 2    | 9,7896652 | 9,8963346 | 9,8933306 | 10,1066694 | 58   |
| 3    | 9,7898266 | 9,8962358 | 9,8935909 | 10,1064091 | 57   |
| 4    | 9,7899880 | 9,8961369 | 9,8938511 | 10,1061489 | 56   |
| 5    | 9,7901493 | 9,8960379 | 9,8941114 | 10,1058886 | 55   |
| 6    | 9,7903104 | 9,8959389 | 9,8943715 | 10,1056285 | 54   |
| 7    | 9,7904715 | 9,8958398 | 9,8946317 | 10,1053683 | 53   |
| 8    | 9,7906325 | 9,8957406 | 9,8948918 | 10,1051082 | 52   |
| 9    | 9,7907933 | 9,8956414 | 9,8951519 | 10,1048481 | 51   |
| 10   | 9,7909541 | 9,8955422 | 9,8954119 | 10,1045881 | 50   |
| 11   | 9,7911148 | 9,8954429 | 9,8956719 | 10,1043281 | 49   |
| 12   | 9,7912754 | 9,8953435 | 9,8959319 | 10,1040681 | 48   |
| 13   | 9,7914359 | 9,8952440 | 9,8961918 | 10,1038082 | 47   |
| 14   | 9,7915963 | 9,8951445 | 9,8964517 | 10,1035483 | 46   |
| 15   | 9,7917566 | 9,8950450 | 9,8967116 | 10,1032884 | 45   |
| 16   | 9,7919168 | 9,8949453 | 9,8969714 | 10,1030286 | 44   |
| 17   | 9,7920769 | 9,8948457 | 9,8972312 | 10,1027688 | 43   |
| 18   | 9,7922369 | 9,8947459 | 9,8974910 | 10,1025090 | 42   |
| 19   | 9,7923968 | 9,8946461 | 9,8977507 | 10,1022493 | 41   |
| 20   | 9,7925566 | 9,8945463 | 9,8980104 | 10,1019896 | 40   |
| 21   | 9,7927163 | 9,8944463 | 9,8982700 | 10,1017300 | 39   |
| 22   | 9,7928760 | 9,8943464 | 9,8985296 | 10,1014704 | 38   |
| 23   | 9,7930355 | 9,8942463 | 9,8987892 | 10,1012108 | 37   |
| 24   | 9,7931949 | 9,8941462 | 9,8990487 | 10,1009513 | 36   |
| 25   | 9,7933543 | 9,8940461 | 9,8993082 | 10,1006918 | 35   |
| 26   | 9,7935135 | 9,8939458 | 9,8995677 | 10,1004323 | 34   |
| 27   | 9,7936727 | 9,8938456 | 9,8998271 | 10,1001729 | 33   |
| 28   | 9,7938317 | 9,8937452 | 9,9000865 | 10,0999135 | 32   |
| 29   | 9,7939907 | 9,8936448 | 9,9003459 | 10,0996541 | 31   |
| 30   | 9,7941496 | 9,8935444 | 9,9006052 | 10,0993948 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 51 Degrees.

# 38 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,7941496 | 9,8935444 | 9,9006052 | 10,0993948 | 30   |
| 31   | 9,7943083 | 9,8934439 | 9,9008645 | 10,0991355 | 29   |
| 32   | 9,7944670 | 9,8933433 | 9,9011237 | 10,0988763 | 28   |
| 33   | 9,7946256 | 9,8932426 | 9,9013830 | 10,0986170 | 27   |
| 34   | 9,7947841 | 9,8931419 | 9,9016422 | 10,0983578 | 26   |
| 35   | 9,7949425 | 9,8930412 | 9,9019013 | 10,0980987 | 25   |
| 36   | 9,7951008 | 9,8929404 | 9,9021604 | 10,0978396 | 24   |
| 37   | 9,7952590 | 9,8928395 | 9,9024195 | 10,0975805 | 23   |
| 38   | 9,7954171 | 9,8927385 | 9,9026786 | 10,0973214 | 22   |
| 39   | 9,7955751 | 9,8926375 | 9,9029376 | 10,0970624 | 21   |
| 40   | 9,7957330 | 9,8925305 | 9,9031966 | 10,0968034 | 20   |
| 41   | 9,7958909 | 9,8924354 | 9,9034555 | 10,0965445 | 19   |
| 42   | 9,7960486 | 9,8923342 | 9,9037144 | 10,0962856 | 18   |
| 43   | 9,7962062 | 9,8922329 | 9,9039733 | 10,0960267 | 17   |
| 44   | 9,7963638 | 9,8921316 | 9,9042321 | 10,0957679 | 16   |
| 45   | 9,7965212 | 9,8920303 | 9,9044910 | 10,0955090 | 15   |
| 46   | 9,7966786 | 9,8919289 | 9,9047497 | 10,0952503 | 14   |
| 47   | 9,7968359 | 9,8918274 | 9,9050085 | 10,0949915 | 13   |
| 48   | 9,7969930 | 9,8917258 | 9,9052672 | 10,0947328 | 12   |
| 49   | 9,7971501 | 9,8916242 | 9,9055259 | 10,0944741 | 11   |
| 50   | 9,7973071 | 9,8915226 | 9,9057845 | 10,0942155 | 10   |
| 51   | 9,7974640 | 9,8914208 | 9,9060431 | 10,0939569 | 9    |
| 52   | 9,7976208 | 9,8913191 | 9,9063017 | 10,0936983 | 8    |
| 53   | 9,7977775 | 9,8912172 | 9,9065603 | 10,0934397 | 7    |
| 54   | 9,7979341 | 9,8911153 | 9,9068188 | 10,0931812 | 6    |
| 55   | 9,7980906 | 9,8910133 | 9,9070773 | 10,0929227 | 5    |
| 56   | 9,7982470 | 9,8909113 | 9,9073357 | 10,0926643 | 4    |
| 57   | 9,7984034 | 9,8908092 | 9,9075941 | 10,0924059 | 3    |
| 58   | 9,7985596 | 9,8907071 | 9,9078525 | 10,0921475 | 2    |
| 59   | 9,7987158 | 9,8906049 | 9,9081109 | 10,0918891 | 1    |
| 60   | 9,7988718 | 9,8905026 | 9,9083692 | 10,0916308 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 51 Degrees.

# 39 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,7988718 | 9,8905026 | 9,9083692 | 10,0916308 | 60.  |
| 1    | 9,7990278 | 9,8904003 | 9,9086275 | 10,0913725 | 59   |
| 2    | 9,7991836 | 9,8902979 | 9,9088858 | 10,0911142 | 58   |
| 3    | 9,7993394 | 9,8901954 | 9,9091440 | 10,0908560 | 57.  |
| 4    | 9,7994951 | 9,8900929 | 9,9094022 | 10,0905978 | 56.  |
| 5    | 9,7996507 | 9,8899903 | 9,9096603 | 10,0903397 | 55   |
| 6    | 9,7998062 | 9,8898877 | 9,9099185 | 10,0900815 | 54.  |
| 7    | 9,7999616 | 9,8897850 | 9,9101766 | 10,0898234 | 53   |
| 8    | 9,8001169 | 9,8896822 | 9,9104347 | 10,0895653 | 52.  |
| 9    | 9,8002721 | 9,8895794 | 9,9106927 | 10,0893073 | 51   |
| 10   | 9,8004272 | 9,8894765 | 9,9109507 | 10,0890493 | 50   |
| 11   | 9,8005823 | 9,8893736 | 9,9112087 | 10,0887913 | 49.  |
| 12   | 9,8007372 | 9,8892706 | 9,9114666 | 10,0885334 | 48   |
| 13   | 9,8008921 | 9,8891675 | 9,9117245 | 10,0882755 | 47.  |
| 14   | 9,8010468 | 9,8890644 | 9,9119824 | 10,0880176 | 46.  |
| 15   | 9,8012015 | 9,8889612 | 9,9122403 | 10,0877597 | 45.  |
| 16   | 9,8013561 | 9,8888580 | 9,9124981 | 10,0875019 | 44.  |
| 17   | 9,8015106 | 9,8887547 | 9,9127559 | 10,0872441 | 43   |
| 18   | 9,8016649 | 9,8886513 | 9,9130137 | 10,0869863 | 42.  |
| 19   | 9,8018192 | 9,8885479 | 9,9132714 | 10,0867286 | 41.  |
| 20   | 9,8019735 | 9,8884444 | 9,9135291 | 10,0864709 | 40   |
| 21   | 9,8021276 | 9,8883408 | 9,9137868 | 10,0862132 | 39.  |
| 22   | 9,8022816 | 9,8882372 | 9,9140444 | 10,0859556 | 38.  |
| 23   | 9,8024355 | 9,8881335 | 9,9143020 | 10,0856980 | 37.  |
| 24   | 9,8025894 | 9,8880298 | 9,9145596 | 10,0854404 | 36.  |
| 25   | 9,8027431 | 9,8879260 | 9,9148171 | 10,0851829 | 35   |
| 26   | 9,8028968 | 9,8878221 | 9,9150747 | 10,0849253 | 34.  |
| 27   | 9,8030504 | 9,8877182 | 9,9153322 | 10,0846678 | 33   |
| 28   | 9,8032038 | 9,8876142 | 9,9155896 | 10,0844104 | 32.  |
| 29   | 9,8033572 | 9,8875102 | 9,9158471 | 10,0841529 | 31   |
| 30   | 9,8035105 | 9,8874061 | 9,9161045 | 10,0838955 | 30.  |
|      | Sine Com. | S ne.     | Tan. Com. | Tangent.   | Min. |

# 50 Degrees.



# 39 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp  |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,8035105 | 9,8874061 | 9,9161045 | 10,0838955 | 30   |
| 31   | 9,8036637 | 9,8873019 | 9,9163618 | 10,0836382 | 29   |
| 32   | 9,8038168 | 9,8871977 | 9,9166102 | 10,0833808 | 28   |
| 33   | 9,8039699 | 9,8870934 | 9,916875  | 10,0831235 | 27   |
| 34   | 9,8041228 | 9,8869890 | 9,9171338 | 10,0828662 | 26   |
| 35   | 9,8042757 | 9,8868846 | 9,9173911 | 10,0826089 | 25   |
| 36   | 9,8044284 | 9,8867801 | 9,9176483 | 10,0823517 | 24   |
| 37   | 9,8045811 | 9,8866756 | 9,9179055 | 10,0820945 | 23   |
| 38   | 9,8047336 | 9,8865710 | 9,9181627 | 10,0818373 | 22   |
| 39   | 9,8048861 | 9,8864663 | 9,9184198 | 10,0815802 | 21   |
| 40   | 9,8050385 | 9,8863616 | 9,9186769 | 10,0813231 | 20   |
| 41   | 9,8051908 | 9,8862568 | 9,9189340 | 10,0810660 | 19   |
| 42   | 9,8053430 | 9,8861519 | 9,9191911 | 10,0808089 | 18   |
| 43   | 9,8054951 | 9,8860470 | 9,9194481 | 10,0805519 | 17   |
| 44   | 9,8056472 | 9,8859420 | 9,9197051 | 10,0802949 | 16   |
| 45   | 9,8057991 | 9,8858370 | 9,9199621 | 10,0800379 | 15   |
| 46   | 9,8059510 | 9,8857319 | 9,9202191 | 10,0797809 | 14   |
| 47   | 9,8061027 | 9,8856267 | 9,9204760 | 10,0795240 | 13   |
| 48   | 9,8062544 | 9,8855215 | 9,9207329 | 10,0792671 | 12   |
| 49   | 9,8064060 | 9,8854162 | 9,9209898 | 10,0790102 | 11   |
| 50   | 9,8065575 | 9,8853109 | 9,9212466 | 10,0787534 | 10   |
| 51   | 9,8067089 | 9,8852055 | 9,9215034 | 10,0784966 | 9    |
| 52   | 9,8068602 | 9,8851000 | 9,9217602 | 10,0782398 | 8    |
| 53   | 9,8070114 | 9,8849945 | 9,9220170 | 10,0779830 | 7    |
| 54   | 9,8071626 | 9,8848889 | 9,9222737 | 10,0777263 | 6    |
| 55   | 9,8073136 | 9,8847832 | 9,9225304 | 10,0774696 | 5    |
| 56   | 9,8074646 | 9,8846775 | 9,9227871 | 10,0772129 | 4    |
| 57   | 9,8076154 | 9,8845717 | 9,9230437 | 10,0769563 | 3    |
| 58   | 9,8077662 | 9,8844659 | 9,9233004 | 10,0766996 | 2    |
| 59   | 9,8079169 | 9,8843599 | 9,9235570 | 10,0764430 | 1    |
| 60   | 9,8080675 | 9,8842540 | 9,9238135 | 10,0761865 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 50 Degrees.

# 40 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,8080675 | 9,8842540 | 9,9238135 | 10,0761865 | 60   |
| 1    | 9,8082180 | 9,8841479 | 9,9240701 | 10,0759299 | 59   |
| 2    | 9,8083684 | 9,8840418 | 9,9243266 | 10,0756734 | 58   |
| 3    | 9,8085188 | 9,8839357 | 9,9245831 | 10,0754169 | 57   |
| 4    | 9,8086690 | 9,8838294 | 9,9248396 | 10,0751604 | 56   |
| 5    | 9,8088192 | 9,8837232 | 9,9250960 | 10,0749040 | 55   |
| 6    | 9,8089692 | 9,8836168 | 9,9253524 | 10,0746476 | 54   |
| 7    | 9,8091192 | 9,8835104 | 9,9256088 | 10,0743912 | 53   |
| 8    | 9,8092691 | 9,8834039 | 9,9258652 | 10,0741348 | 52   |
| 9    | 9,8094189 | 9,8832974 | 9,9261215 | 10,0738785 | 51   |
| 10   | 9,8095686 | 9,8831908 | 9,9263778 | 10,0736222 | 50   |
| 11   | 9,8097182 | 9,8830841 | 9,9266341 | 10,0733659 | 49   |
| 12   | 9,8098678 | 9,8829774 | 9,9268904 | 10,0731096 | 48   |
| 13   | 9,8100172 | 9,8828706 | 9,9271466 | 10,0728534 | 47   |
| 14   | 9,8101666 | 9,8827638 | 9,9274028 | 10,0725972 | 46   |
| 15   | 9,8103159 | 9,8826568 | 9,9276590 | 10,0723410 | 45   |
| 16   | 9,8104650 | 9,8825499 | 9,9279152 | 10,0720848 | 44   |
| 17   | 9,8106141 | 9,8824428 | 9,9281713 | 10,0718287 | 43   |
| 18   | 9,8107631 | 9,8823357 | 9,9284274 | 10,0715726 | 42   |
| 19   | 9,8109121 | 9,8822285 | 9,9286835 | 10,0713165 | 41   |
| 20   | 9,8110609 | 9,8821213 | 9,9289396 | 10,0710604 | 40   |
| 21   | 9,8112096 | 9,8820140 | 9,9291956 | 10,0708044 | 39   |
| 22   | 9,8113583 | 9,8819067 | 9,9294516 | 10,0705484 | 38   |
| 23   | 9,8115069 | 9,8817992 | 9,9297076 | 10,0702924 | 37   |
| 24   | 9,8116554 | 9,8816918 | 9,9299636 | 10,0700364 | 36   |
| 25   | 9,8118038 | 9,8815842 | 9,9302195 | 10,0697805 | 35   |
| 26   | 9,8119521 | 9,8814766 | 9,9304755 | 10,0695245 | 34   |
| 27   | 9,8121003 | 9,8813689 | 9,9307314 | 10,0692686 | 33   |
| 28   | 9,8122484 | 9,8812612 | 9,9309872 | 10,0690128 | 32   |
| 29   | 9,8123965 | 9,8811534 | 9,9312431 | 10,0687569 | 31   |
| 30   | 9,8125444 | 9,8810455 | 9,9314989 | 10,0685011 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 49 Degrees.

# 40 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,8125444 | 9,8810455 | 9,9314989 | 10,0685011 | 30   |
| 31   | 9,8126923 | 9,8809376 | 9,9317547 | 10,0682453 | 29   |
| 32   | 9,8128401 | 9,8808296 | 9,9320105 | 10,0679895 | 28   |
| 33   | 9,8129878 | 9,8807215 | 9,9322662 | 10,0677338 | 27   |
| 34   | 9,8131354 | 9,8806134 | 9,9325220 | 10,0674780 | 26   |
| 35   | 9,8132829 | 9,8805052 | 9,9327777 | 10,0672223 | 25   |
| 36   | 9,8134303 | 9,8803970 | 9,9330334 | 10,0669666 | 24   |
| 37   | 9,8135777 | 9,8802887 | 9,9332890 | 10,0667110 | 23   |
| 38   | 9,8137250 | 9,8801803 | 9,9335446 | 10,0664554 | 22   |
| 39   | 9,8138721 | 9,8800719 | 9,9338003 | 10,0661997 | 21   |
| 40   | 9,8140192 | 9,8799634 | 9,9340559 | 10,0659441 | 20   |
| 41   | 9,8141662 | 9,8798548 | 9,9343114 | 10,0656886 | 19   |
| 42   | 9,8143131 | 9,8797462 | 9,9345670 | 10,0654330 | 18   |
| 43   | 9,8144600 | 9,8796375 | 9,9348225 | 10,0651775 | 17   |
| 44   | 9,8146067 | 9,8795287 | 9,9350780 | 10,0649220 | 16   |
| 45   | 9,8147534 | 9,8794199 | 9,9353335 | 10,0646665 | 15   |
| 46   | 9,8148999 | 9,8793110 | 9,9355889 | 10,0644111 | 14   |
| 47   | 9,8150464 | 9,8792021 | 9,9358444 | 10,0641556 | 13   |
| 48   | 9,8151928 | 9,8790930 | 9,9360998 | 10,0639002 | 12   |
| 49   | 9,8153391 | 9,8789840 | 9,9363552 | 10,0636448 | 11   |
| 50   | 9,8154854 | 9,8788748 | 9,9366105 | 10,0633895 | 10   |
| 51   | 9,8156315 | 9,8787656 | 9,9368659 | 10,0631341 | 9    |
| 52   | 9,8157776 | 9,8786563 | 9,9371212 | 10,0628788 | 8    |
| 53   | 9,8159235 | 9,8785470 | 9,9373765 | 10,0626235 | 7    |
| 54   | 9,8160694 | 9,8784376 | 9,9376318 | 10,0623682 | 6    |
| 55   | 9,8162152 | 9,8783281 | 9,9378871 | 10,0621129 | 5    |
| 56   | 9,8163609 | 9,8782186 | 9,9381423 | 10,0618577 | 4    |
| 57   | 9,8165066 | 9,8781090 | 9,9383975 | 10,0616025 | 3    |
| 58   | 9,8166521 | 9,8779994 | 9,9386527 | 10,0613473 | 2    |
| 59   | 9,8167975 | 9,8778896 | 9,9389079 | 10,0610921 | 1    |
| 60   | 9,8169429 | 9,8777799 | 9,9391631 | 10,0608369 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 49 Degrees.

S

## 41 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,8169429 | 9,8777799 | 9,9391631 | 10,0608369 | 60   |
| 1    | 9,8170882 | 9,8776700 | 9,9394182 | 10,0605818 | 59   |
| 2    | 9,8172334 | 9,8775601 | 9,9396733 | 10,0603267 | 58   |
| 3    | 9,8173785 | 9,8774501 | 9,9399284 | 10,0600716 | 57   |
| 4    | 9,8175235 | 9,8773401 | 9,9401835 | 10,0598165 | 56   |
| 5    | 9,8176685 | 9,8772300 | 9,9404385 | 10,0595615 | 55   |
| 6    | 9,8178133 | 9,8771198 | 9,9406936 | 10,0593064 | 54   |
| 7    | 9,8179581 | 9,8770096 | 9,9409486 | 10,0590514 | 53   |
| 8    | 9,8181028 | 9,8768993 | 9,9412036 | 10,0587964 | 52   |
| 9    | 9,8182474 | 9,8767889 | 9,9414585 | 10,0585415 | 51   |
| 10   | 9,8183919 | 9,8766785 | 9,9417135 | 10,0582865 | 50   |
| 11   | 9,8185364 | 9,8765680 | 9,9419684 | 10,0580316 | 49   |
| 12   | 9,8186807 | 9,8764574 | 9,9422233 | 10,0577767 | 48   |
| 13   | 9,8188250 | 9,8763468 | 9,9424782 | 10,0575218 | 47   |
| 14   | 9,8189692 | 9,8762361 | 9,9427331 | 10,0572669 | 46   |
| 15   | 9,8191133 | 9,8761253 | 9,9429879 | 10,0570121 | 45   |
| 16   | 9,8192573 | 9,8760145 | 9,9432428 | 10,0567572 | 44   |
| 17   | 9,8194012 | 9,8759036 | 9,9434976 | 10,0565024 | 43   |
| 18   | 9,8195450 | 9,8757927 | 9,9437524 | 10,0562476 | 42   |
| 19   | 9,8196888 | 9,8756816 | 9,9440072 | 10,0559928 | 41   |
| 20   | 9,8198325 | 9,8755706 | 9,9442619 | 10,0557381 | 40   |
| 21   | 9,8199761 | 9,8754594 | 9,9445166 | 10,0554834 | 39   |
| 22   | 9,8201196 | 9,8753482 | 9,9447714 | 10,0552286 | 38   |
| 23   | 9,8202630 | 9,8752369 | 9,9450261 | 10,0549739 | 37   |
| 24   | 9,8204063 | 9,8751256 | 9,9452807 | 10,0547193 | 36   |
| 25   | 9,8205496 | 9,8750142 | 9,9455354 | 10,0544646 | 35   |
| 26   | 9,8206927 | 9,8749027 | 9,9457900 | 10,0542100 | 34   |
| 27   | 9,8208358 | 9,8747912 | 9,9460447 | 10,0539553 | 33   |
| 28   | 9,8209788 | 9,8746795 | 9,9462993 | 10,0537007 | 32   |
| 29   | 9,8211217 | 9,8745679 | 9,9465539 | 10,0534461 | 31   |
| 30   | 9,8212646 | 9,8744561 | 9,9468084 | 10,0531916 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

## 48 Degrees.

# 41 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,8212646 | 9,8744561 | 9,9468084 | 10,0531916 | 30   |
| 31   | 9,8214073 | 9,8743443 | 9,9470630 | 10,0529370 | 29   |
| 32   | 9,8215500 | 9,8742325 | 9,9473175 | 10,0526825 | 28   |
| 33   | 9,8216926 | 9,8741205 | 9,947572  | 10,0524280 | 27   |
| 34   | 9,8218351 | 9,8740085 | 9,9478265 | 10,0521735 | 26   |
| 35   | 9,8219775 | 9,8738965 | 9,9480810 | 10,0519190 | 25   |
| 36   | 9,8221198 | 9,8737844 | 9,9483355 | 10,0516645 | 24   |
| 37   | 9,8222621 | 9,8736722 | 9,9485899 | 10,0514101 | 23   |
| 38   | 9,8224042 | 9,8735599 | 9,9488443 | 10,0511557 | 22   |
| 39   | 9,8225463 | 9,8734476 | 9,9490987 | 10,0509013 | 21   |
| 40   | 9,8226883 | 9,8733352 | 9,9493531 | 10,0506469 | 20   |
| 41   | 9,8228302 | 9,8732227 | 9,9496075 | 10,0503925 | 19   |
| 42   | 9,8229721 | 9,8731102 | 9,9498619 | 10,0501381 | 18   |
| 43   | 9,8231138 | 9,8729976 | 9,9501162 | 10,0498838 | 17   |
| 44   | 9,8232555 | 9,8728849 | 9,9503705 | 10,0496295 | 16   |
| 45   | 9,8233971 | 9,8727722 | 9,9506248 | 10,0493752 | 15   |
| 46   | 9,8235386 | 9,8726594 | 9,9508791 | 10,0491209 | 14   |
| 47   | 9,8236800 | 9,8725466 | 9,9511334 | 10,0488666 | 13   |
| 48   | 9,8238213 | 9,8724337 | 9,9513876 | 10,0486124 | 12   |
| 49   | 9,8239626 | 9,8723207 | 9,9516419 | 10,0483581 | 11   |
| 50   | 9,8241037 | 9,8722076 | 9,9518961 | 10,0481039 | 10   |
| 51   | 9,8242448 | 9,8720945 | 9,9521503 | 10,0478497 | 9    |
| 52   | 9,8243858 | 9,8719813 | 9,9524045 | 10,0475955 | 8    |
| 53   | 9,8245267 | 9,8718681 | 9,9526587 | 10,0473413 | 7    |
| 54   | 9,8246676 | 9,8717548 | 9,9529128 | 10,0470872 | 6    |
| 55   | 9,8248083 | 9,8716414 | 9,9531670 | 10,0468330 | 5    |
| 56   | 9,8249490 | 9,8715279 | 9,9534211 | 10,0465789 | 4    |
| 57   | 9,8250896 | 9,8714144 | 9,9536752 | 10,0463248 | 3    |
| 58   | 9,8252301 | 9,8713008 | 9,9539293 | 10,0460707 | 2    |
| 59   | 9,8253705 | 9,8711872 | 9,9541834 | 10,0458166 | 1    |
| 60   | 9,8255109 | 9,8710735 | 9,9544374 | 10,0455626 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 48 Degrees.

## 42 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,8255109 | 9,8710735 | 9,9544374 | 10,0455626 | 60   |
| 1    | 9,8256512 | 9,8709597 | 9,9546915 | 10,0453085 | 59   |
| 2    | 9,8257913 | 9,8708458 | 9,9549455 | 10,0450545 | 58   |
| 3    | 9,8259314 | 9,8707319 | 9,9551995 | 10,0448005 | 57   |
| 4    | 9,8260715 | 9,8706179 | 9,9554535 | 10,0445465 | 56   |
| 5    | 9,8262114 | 9,8705039 | 9,9557075 | 10,0442925 | 55   |
| 6    | 9,8263512 | 9,8703898 | 9,9559615 | 10,0440385 | 54   |
| 7    | 9,8264910 | 9,8702756 | 9,9562154 | 10,0437846 | 53   |
| 8    | 9,8266307 | 9,8701613 | 9,9564694 | 10,0435306 | 52   |
| 9    | 9,8267703 | 9,8700470 | 9,9567233 | 10,0432767 | 51   |
| 10   | 9,8269098 | 9,8699326 | 9,9569772 | 10,0430228 | 50   |
| 11   | 9,8270493 | 9,8698182 | 9,9572311 | 10,0427689 | 49   |
| 12   | 9,8271887 | 9,8697037 | 9,9574850 | 10,0425150 | 48   |
| 13   | 9,8273279 | 9,8695891 | 9,9577389 | 10,0422611 | 47   |
| 14   | 9,8274671 | 9,8694744 | 9,9579927 | 10,0420073 | 46   |
| 15   | 9,8276063 | 9,8693597 | 9,9582465 | 10,0417535 | 45   |
| 16   | 9,8277453 | 9,8692449 | 9,9585004 | 10,0414996 | 44   |
| 17   | 9,8278843 | 9,8691301 | 9,9587542 | 10,0412458 | 43   |
| 18   | 9,8280231 | 9,8690152 | 9,9590080 | 10,0409920 | 42   |
| 19   | 9,8281619 | 9,8689002 | 9,9592618 | 10,0407382 | 41   |
| 20   | 9,8283006 | 9,8687851 | 9,9595155 | 10,0404845 | 40   |
| 21   | 9,8284393 | 9,8686700 | 9,9597693 | 10,0402307 | 39   |
| 22   | 9,8285778 | 9,8685548 | 9,9600230 | 10,0399770 | 38   |
| 23   | 9,8287163 | 9,8684396 | 9,9602767 | 10,0397233 | 37   |
| 24   | 9,8288547 | 9,8683242 | 9,9605305 | 10,0394695 | 36   |
| 25   | 9,8289930 | 9,8682088 | 9,9607842 | 10,0392158 | 35   |
| 26   | 9,8291312 | 9,8680934 | 9,9610378 | 10,0389622 | 34   |
| 27   | 9,8292694 | 9,8679779 | 9,9612915 | 10,0387085 | 33   |
| 28   | 9,8294075 | 9,8678623 | 9,9615452 | 10,0384548 | 32   |
| 29   | 9,8295454 | 9,8677466 | 9,9617988 | 10,0382012 | 31   |
| 30   | 9,8296833 | 9,8676309 | 9,9620525 | 10,0379475 | 30   |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

## 47 Degrees.

# 42 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,8296833 | 9,8676309 | 9,9620525 | 10,0379475 | 30   |
| 31   | 9,8298212 | 9,8675151 | 9,9623061 | 10,0376939 | 29   |
| 32   | 9,8299589 | 9,8673992 | 9,9625597 | 10,0374403 | 28   |
| 33   | 9,8300966 | 9,8672833 | 9,9628133 | 10,0371867 | 27   |
| 34   | 9,8302342 | 9,8671673 | 9,9630669 | 10,0369331 | 26   |
| 35   | 9,8303717 | 9,8670512 | 9,9633204 | 10,0366796 | 25   |
| 36   | 9,8305091 | 9,8669351 | 9,9635740 | 10,0364260 | 24   |
| 37   | 9,8306464 | 9,8668189 | 9,9638275 | 10,0361725 | 23   |
| 38   | 9,8307837 | 9,8667026 | 9,9640811 | 10,0359186 | 22   |
| 39   | 9,8309209 | 9,8665863 | 9,9643346 | 10,0356654 | 21   |
| 40   | 9,8310580 | 9,8664699 | 9,9645881 | 10,0354119 | 20   |
| 41   | 9,8311950 | 9,8663534 | 9,9648416 | 10,0351584 | 19   |
| 42   | 9,8313320 | 9,8662369 | 9,9650951 | 10,0349049 | 18   |
| 43   | 9,8314688 | 9,8661203 | 9,9653486 | 10,0346514 | 17   |
| 44   | 9,8316056 | 9,8660036 | 9,9656020 | 10,0343980 | 16   |
| 45   | 9,8317423 | 9,8658868 | 9,9658555 | 10,0341445 | 15   |
| 46   | 9,8318789 | 9,8657700 | 9,9661089 | 10,0338911 | 14   |
| 47   | 9,8320155 | 9,8656531 | 9,9663623 | 10,0336377 | 13   |
| 48   | 9,8321519 | 9,8655362 | 9,9666157 | 10,0333843 | 12   |
| 49   | 9,8322883 | 9,8654192 | 9,9668692 | 10,0331308 | 11   |
| 50   | 9,8324246 | 9,8653021 | 9,9671225 | 10,0328775 | 10   |
| 51   | 9,8325609 | 9,8651849 | 9,9673759 | 10,0326241 | 9    |
| 52   | 9,8326970 | 9,8650677 | 9,9676293 | 10,0323707 | 8    |
| 53   | 9,8328331 | 9,8649504 | 9,9678827 | 10,0321173 | 7    |
| 54   | 9,8329691 | 9,8648331 | 9,9681360 | 10,0318640 | 6    |
| 55   | 9,8331050 | 9,8647156 | 9,9683893 | 10,0316107 | 5    |
| 56   | 9,8332408 | 9,8645981 | 9,9686427 | 10,0313573 | 4    |
| 57   | 9,8333766 | 9,8644806 | 9,9688960 | 10,0311040 | 3    |
| 58   | 9,8335122 | 9,8643629 | 9,9691493 | 10,0308507 | 2    |
| 59   | 9,8336478 | 9,8642452 | 9,9694026 | 10,0305974 | 1    |
| 60   | 9,8337833 | 9,8641275 | 9,9696559 | 10,0303441 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 43 Degrees.

| 43 Degrees. |           |           |           |            |      |
|-------------|-----------|-----------|-----------|------------|------|
| Min.        | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
| 0           | 9,8337833 | 9,8641275 | 9,9696559 | 10,0303441 | 60   |
| 1           | 9,8339188 | 9,8640096 | 9,9699091 | 10,0300909 | 59   |
| 2           | 9,8340541 | 9,8638917 | 9,9701624 | 10,0298376 | 58   |
| 3           | 9,8341894 | 9,8637737 | 9,9704157 | 10,0295843 | 57   |
| 4           | 9,8343246 | 9,8636557 | 9,9706689 | 10,0293311 | 56   |
| 5           | 9,8344597 | 9,8635376 | 9,9709221 | 10,0290779 | 55   |
| 6           | 9,8345948 | 9,8634194 | 9,9711754 | 10,0288246 | 54   |
| 7           | 9,8347297 | 9,8633011 | 9,9714286 | 10,0285714 | 53   |
| 8           | 9,8348646 | 9,8631828 | 9,9716818 | 10,0283182 | 52   |
| 9           | 9,8349994 | 9,8630644 | 9,9719350 | 10,0280650 | 51   |
| 10          | 9,8351341 | 9,8629460 | 9,9721882 | 10,0278118 | 50   |
| 11          | 9,8352688 | 9,8628274 | 9,9724413 | 10,0275587 | 49   |
| 12          | 9,8354033 | 9,8627088 | 9,9726945 | 10,0273055 | 48   |
| 13          | 9,8355378 | 9,8625902 | 9,9729477 | 10,0270523 | 47   |
| 14          | 9,8356722 | 9,8624714 | 9,9732008 | 10,0267992 | 46   |
| 15          | 9,8358066 | 9,8623526 | 9,9734539 | 10,0265461 | 45   |
| 16          | 9,8359408 | 9,8622338 | 9,9737071 | 10,0262929 | 44   |
| 17          | 9,8360750 | 9,8621148 | 9,9739602 | 10,0260398 | 43   |
| 18          | 9,8362091 | 9,8619958 | 9,9742133 | 10,0257867 | 42   |
| 19          | 9,8363431 | 9,8618767 | 9,9744664 | 10,0255336 | 41   |
| 20          | 9,8364771 | 9,8617576 | 9,9747195 | 10,0252805 | 40   |
| 21          | 9,8366109 | 9,8616383 | 9,9749726 | 10,0250274 | 39   |
| 22          | 9,8367447 | 9,8615190 | 9,9752257 | 10,0247743 | 38   |
| 23          | 9,8368784 | 9,8613997 | 9,9754787 | 10,0245213 | 37   |
| 24          | 9,8370121 | 9,8612803 | 9,9757318 | 10,0242682 | 36   |
| 25          | 9,8371456 | 9,8611608 | 9,9759849 | 10,0240151 | 35   |
| 26          | 9,8372791 | 9,8610412 | 9,9762379 | 10,0237621 | 34   |
| 27          | 9,8374125 | 9,8609215 | 9,9764909 | 10,0235091 | 33   |
| 28          | 9,8375458 | 9,8608018 | 9,9767440 | 10,0232560 | 32   |
| 29          | 9,8376790 | 9,8606821 | 9,9769970 | 10,0230030 | 31   |
| 30          | 9,8378122 | 9,8605622 | 9,9772500 | 10,0227500 | 30   |
|             | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |
| 46 Degrees. |           |           |           |            |      |



# 43 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. |      |
|------|-----------|-----------|-----------|------------|------|
| 30   | 9,8378122 | 9,8605622 | 9,9772500 | 10,0227500 | 30   |
| 31   | 9,8379453 | 9,8604423 | 9,9775030 | 10,0224970 | 29   |
| 32   | 9,8380783 | 9,8603223 | 9,9777560 | 10,0222440 | 28   |
| 33   | 9,8382112 | 9,8602022 | 9,9780090 | 10,0219910 | 27   |
| 34   | 9,8383441 | 9,8600821 | 9,9782620 | 10,0217380 | 26   |
| 35   | 9,8384769 | 9,8599619 | 9,9785149 | 10,0214851 | 25   |
| 36   | 9,8386096 | 9,8598416 | 9,9787679 | 10,0212321 | 24   |
| 37   | 9,8387422 | 9,8597213 | 9,9790209 | 10,0209791 | 23   |
| 38   | 9,8388747 | 9,8596009 | 9,9792738 | 10,0207262 | 22   |
| 39   | 9,8390072 | 9,8594804 | 9,9795268 | 10,0204732 | 21   |
| 40   | 9,8391396 | 9,8593599 | 9,9797797 | 10,0202203 | 20   |
| 41   | 9,8392719 | 9,8592393 | 9,9800326 | 10,0199674 | 19   |
| 42   | 9,8394041 | 9,8591186 | 9,9802856 | 10,0197144 | 18   |
| 43   | 9,8395363 | 9,8589978 | 9,9805385 | 10,0194615 | 17   |
| 44   | 9,8396684 | 9,8588770 | 9,9807914 | 10,0192086 | 16   |
| 45   | 9,8398004 | 9,8587561 | 9,9810443 | 10,0189557 | 15   |
| 46   | 9,8399323 | 9,8586351 | 9,9812972 | 10,0187028 | 14   |
| 47   | 9,8400642 | 9,8585141 | 9,9815501 | 10,0184499 | 13   |
| 48   | 9,8401959 | 9,8583929 | 9,9818030 | 10,0181970 | 12   |
| 49   | 9,8403276 | 9,8582718 | 9,9820559 | 10,0179441 | 11   |
| 50   | 9,8404593 | 9,8581505 | 9,9823087 | 10,0176913 | 10   |
| 51   | 9,8405908 | 9,8580202 | 9,9825616 | 10,0174384 | 9    |
| 52   | 9,8407223 | 9,8579078 | 9,9828145 | 10,0171855 | 8    |
| 53   | 9,8408537 | 9,8577863 | 9,9830673 | 10,0169327 | 7    |
| 54   | 9,8409850 | 9,8576648 | 9,9833202 | 10,0166798 | 6    |
| 55   | 9,8411162 | 9,8575432 | 9,9835730 | 10,0164270 | 5    |
| 56   | 9,8412474 | 9,8574215 | 9,9838259 | 10,0161741 | 4    |
| 57   | 9,8413785 | 9,8572998 | 9,9840787 | 10,0159213 | 3    |
| 58   | 9,8415095 | 9,8571779 | 9,9843315 | 10,0156685 | 2    |
| 59   | 9,8416404 | 9,8570561 | 9,9845844 | 10,0154156 | 1    |
| 60   | 9,8417713 | 9,8569341 | 9,9848372 | 10,0151628 | 0    |
|      | Sine Com. | Sine.     | Tan. Com. | Tangent.   | Min. |

# 46 Degrees.

# 44 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.  | Tan. Comp. | Min. |
|------|-----------|-----------|-----------|------------|------|
| 0    | 9,8417713 | 9,8569341 | 9,9848372 | 10,0151628 | 60   |
| 1    | 9,8419021 | 9,8568121 | 9,9850900 | 10,0149100 | 59   |
| 2    | 9,8420328 | 9,8566900 | 9,9853428 | 10,0146572 | 58   |
| 3    | 9,8424634 | 9,8565678 | 9,9855956 | 10,0144044 | 57   |
| 4    | 9,8422939 | 9,8564455 | 9,9858484 | 10,0141516 | 56   |
| 5    | 9,8424244 | 9,8563232 | 9,9861012 | 10,0138988 | 55   |
| 6    | 9,8425548 | 9,8562008 | 9,9863540 | 10,0136460 | 54   |
| 7    | 9,8426851 | 9,8560784 | 9,9866068 | 10,0133932 | 53   |
| 8    | 9,8428154 | 9,8559558 | 9,9868596 | 10,0131404 | 52   |
| 9    | 9,8429456 | 9,8558332 | 9,9871123 | 10,0128877 | 51   |
| 10   | 9,8430757 | 9,8557106 | 9,9873651 | 10,0126349 | 50   |
| 11   | 9,8432057 | 9,8555878 | 9,9876179 | 10,0123821 | 49   |
| 12   | 9,8433356 | 9,8554650 | 9,9878706 | 10,0121294 | 48   |
| 13   | 9,8434655 | 9,8553421 | 9,9881234 | 10,0118766 | 47   |
| 14   | 9,8435953 | 9,8552192 | 9,9883761 | 10,0116239 | 46   |
| 15   | 9,8437250 | 9,8550961 | 9,9886289 | 10,0113711 | 45   |
| 16   | 9,8438547 | 9,8549730 | 9,9888816 | 10,0111184 | 44   |
| 17   | 9,8439842 | 9,8548499 | 9,9891344 | 10,0108656 | 43   |
| 18   | 9,8441137 | 9,8547266 | 9,9893871 | 10,0106129 | 42   |
| 19   | 9,8442432 | 9,8546033 | 9,9896399 | 10,0103601 | 41   |
| 20   | 9,8443725 | 9,8544799 | 9,9898926 | 10,0101074 | 40   |
| 21   | 9,8445018 | 9,8543564 | 9,9901453 | 10,0098547 | 39   |
| 22   | 9,8446310 | 9,8542329 | 9,9903981 | 10,0096019 | 38   |
| 23   | 9,8447601 | 9,8541093 | 9,9906508 | 10,0093492 | 37   |
| 24   | 9,8448891 | 9,8539856 | 9,9909035 | 10,0090965 | 36   |
| 25   | 9,8450181 | 9,8538619 | 9,9911562 | 10,0088438 | 35   |
| 26   | 9,8451470 | 9,8537381 | 9,9914089 | 10,0085911 | 34   |
| 27   | 9,8452758 | 9,8536142 | 9,9916616 | 10,0083384 | 33   |
| 28   | 9,8454045 | 9,8534902 | 9,9919143 | 10,0080857 | 32   |
| 29   | 9,8455332 | 9,8533662 | 9,9921670 | 10,0078330 | 31   |
| 30   | 9,8456618 | 9,8532421 | 9,9924197 | 10,0075803 | 30   |
|      | Tan. Com. | Sine.     | Sine Com. | Tangent.   | Min. |

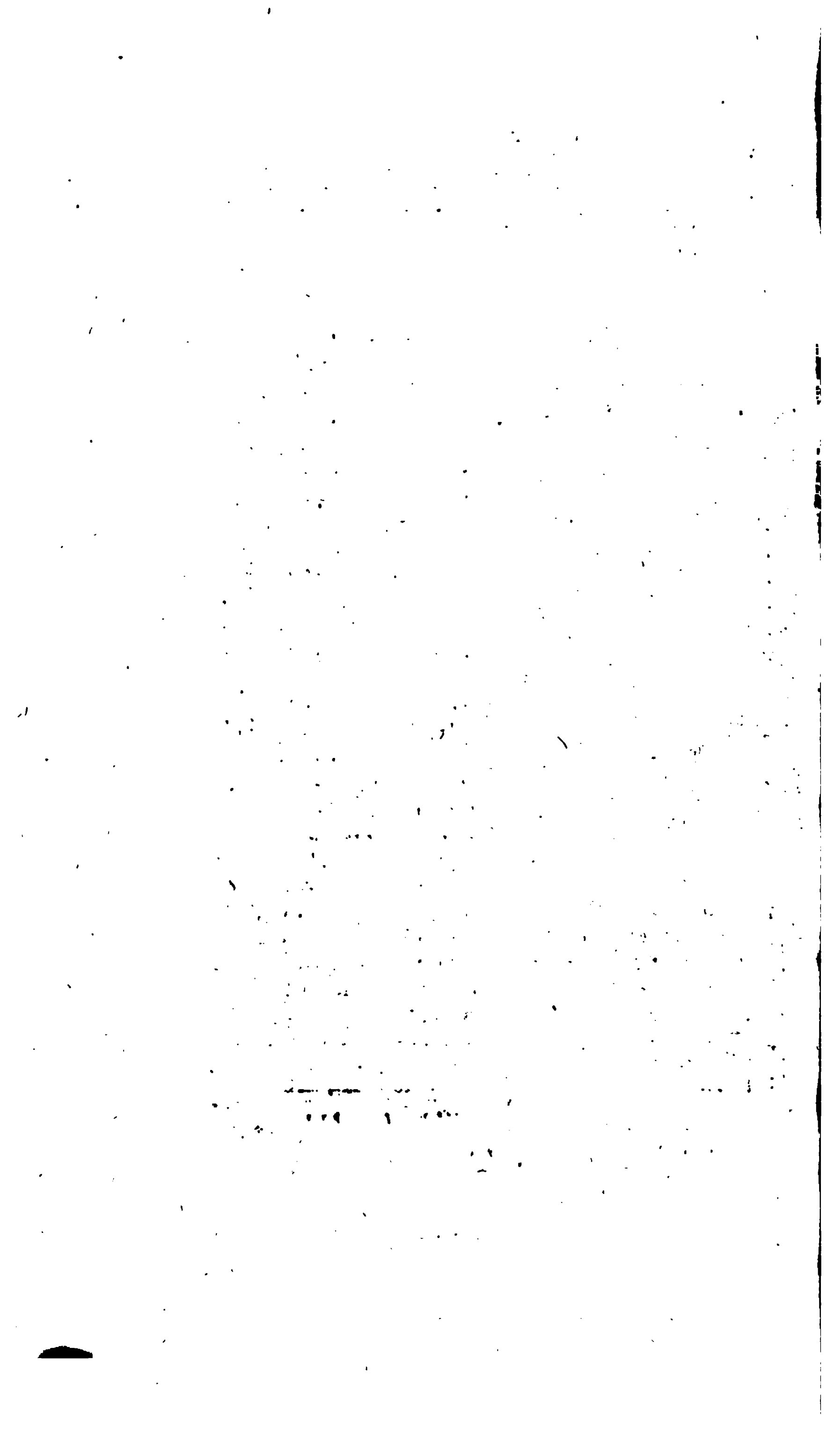
# 45 Degrees.

# 44 Degrees.

| Min. | Sine.     | Sine Com. | Tangent.   | Tan. Comp. |      |
|------|-----------|-----------|------------|------------|------|
| 30   | 9,8456618 | 9,8532421 | 9,9924197  | 10,0075803 | 30   |
| 31   | 9,8457903 | 9,8531179 | 9,9926724  | 10,0073276 | 29   |
| 32   | 9,8459188 | 9,8529936 | 9,9929251  | 10,0070749 | 28   |
| 33   | 9,8460471 | 9,8528693 | 9,9931778  | 10,0068222 | 27   |
| 34   | 9,8461754 | 9,8527449 | 9,9934305  | 10,0065695 | 26   |
| 35   | 9,8463036 | 9,8526204 | 9,9936832  | 10,0063168 | 25   |
| 36   | 9,8464318 | 9,8524959 | 9,9939359  | 10,0060641 | 24   |
| 37   | 9,8465599 | 9,8523713 | 9,9941880  | 10,0058114 | 23   |
| 38   | 9,8466879 | 9,8522466 | 9,9944413  | 10,0055587 | 22   |
| 39   | 9,8468158 | 9,8521218 | 9,9946940  | 10,0053060 | 21   |
| 40   | 9,8469436 | 9,8519970 | 9,9949466  | 10,0050534 | 20   |
| 41   | 9,8470714 | 9,8518721 | 9,9951993  | 10,0048007 | 19   |
| 42   | 9,8471991 | 9,8517471 | 9,9954520  | 10,0045480 | 18   |
| 43   | 9,8473267 | 9,8516220 | 9,9957047  | 10,0042953 | 17   |
| 44   | 9,8474543 | 9,8514969 | 9,9959573  | 10,0040427 | 16   |
| 45   | 9,8475817 | 9,8513717 | 9,9962100  | 10,0037900 | 15   |
| 46   | 9,8477091 | 9,8512465 | 9,9964627  | 10,0035373 | 14   |
| 47   | 9,8478365 | 9,8511211 | 9,9967154  | 10,0032846 | 13   |
| 48   | 9,8479637 | 9,8509957 | 9,9969680  | 10,0030320 | 12   |
| 49   | 9,8480909 | 9,8508702 | 9,9972207  | 10,0027793 | 11   |
| 50   | 9,8482180 | 9,8507446 | 9,9974734  | 10,0025266 | 10   |
| 51   | 9,8483450 | 9,8506190 | 9,9977260  | 10,0022740 | 9    |
| 52   | 9,8484720 | 9,8504933 | 9,9979787  | 10,0020213 | 8    |
| 53   | 9,8485989 | 9,8503675 | 9,9982314  | 10,0017686 | 7    |
| 54   | 9,8487257 | 9,8502417 | 9,9984840  | 10,0015160 | 6    |
| 55   | 9,8488524 | 9,8501157 | 9,9987367  | 10,0012633 | 5    |
| 56   | 9,8489791 | 9,8499897 | 9,9989893  | 10,0010107 | 4    |
| 57   | 9,8491057 | 9,8498637 | 9,9992420  | 10,0007580 | 3    |
| 58   | 9,8492322 | 9,8497375 | 9,9994947  | 10,0005053 | 2    |
| 59   | 9,8493586 | 9,8496113 | 9,9997473  | 10,0002527 | 1    |
| 60   | 9,8494850 | 9,8494850 | 10,0000000 | 10,0000000 | 0    |
|      | Tan. Com. | Sine.     | Sine Com.  | Tangent.   | Min. |

# 45 Degrees.

T

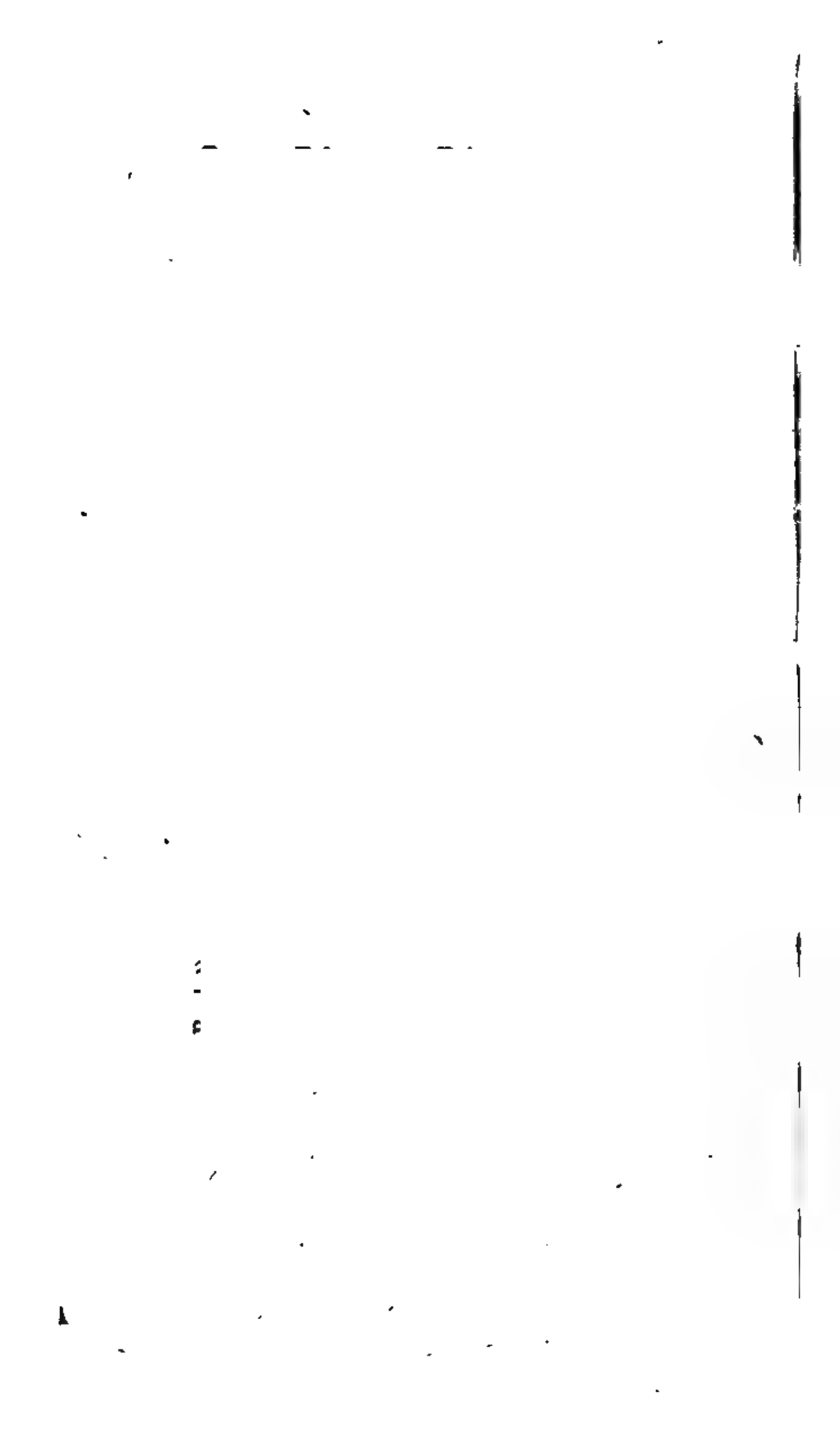


**T A B L E**  
**OF**  
**DIFFERENCE OF LATITUDE**

**AND**  
**D E P A R T U R E**

**To every DEGREE and QUARTER POINT of the**

**C O M P A S S.**





| Cour.  | Dist. 7.  |        | Dist. 8. |        | Dist. 9. |        | Cour.  |
|--------|-----------|--------|----------|--------|----------|--------|--------|
| Pts D. | Lat.      | Dep.   | Lat.     | Dep.   | Lat.     | Dep.   | D. Pts |
|        | 16,99890  | 0,1222 | 7,99880  | 0,1396 | 8,99860  | 0,1571 | 89     |
|        | 26,99570  | 0,2443 | 7,99510  | 0,2792 | 8,99450  | 0,3141 | 88     |
| 0.1    | 6,99160   | 0,3435 | 7,99040  | 0,3925 | 8,98920  | 0,4416 | 7.1    |
|        | 36,99040  | 0,3664 | 7,98900  | 0,4187 | 8,98770  | 0,4710 | 87     |
|        | 46,98290  | 0,4883 | 7,98050  | 0,5580 | 8,97810  | 0,6278 | 86     |
| 0.1    | 56,97340  | 0,6101 | 7,96960  | 0,6972 | 8,96580  | 0,7844 | 85     |
|        | 66,96630  | 0,6861 | 7,96150  | 0,7841 | 8,95670  | 0,8822 | 7.1    |
|        | 66,96170  | 0,7317 | 7,95620  | 0,8362 | 8,95070  | 0,9408 | 84     |
|        | 76,94780  | 0,8531 | 7,94040  | 0,9750 | 8,93291  | 1,0968 | 83     |
|        | 86,93190  | 0,9742 | 7,92211  | 1,1134 | 8,91241  | 1,2526 | 82     |
| 0.3    | 6,92421   | 1,0271 | 7,91341  | 1,1738 | 8,90261  | 1,3206 | 7.1    |
|        | 96,91381  | 1,0950 | 7,90151  | 1,2515 | 8,88921  | 1,4079 | 81     |
| 10     | 6,89371   | 1,2155 | 7,87851  | 1,3892 | 8,86331  | 1,5628 | 80     |
|        | 116,87141 | 1,3357 | 7,85301  | 1,5265 | 8,83461  | 1,7173 | 79     |
| P.     | 6,86551   | 1,3656 | 7,84631  | 1,5607 | 8,82711  | 1,7558 | 7.     |
|        | 126,84701 | 1,4554 | 7,82521  | 1,6633 | 8,80331  | 1,8712 | 78     |
|        | 136,82061 | 1,5746 | 7,79501  | 1,7996 | 8,76932  | 2,0246 | 77     |
|        | 146,79211 | 1,6935 | 7,76241  | 1,9354 | 8,73272  | 2,1773 | 76     |
| 1.1    | 6,79021   | 1,7009 | 7,76021  | 1,9438 | 8,73032  | 2,1868 | 6.1    |
|        | 156,76151 | 1,8117 | 7,72742  | 2,0706 | 8,69332  | 2,3294 | 75     |
|        | 166,72881 | 1,9295 | 7,69012  | 2,2051 | 8,65132  | 2,4807 | 74     |
| 1.1    | 6,69862   | 2,0320 | 7,65552  | 2,3238 | 8,61252  | 2,6126 | 6.1    |
|        | 176,69412 | 2,0466 | 7,65042  | 2,3390 | 8,60672  | 2,6313 | 73     |
|        | 186,65742 | 2,1631 | 7,60842  | 2,4721 | 8,55952  | 2,7812 | 72     |
|        | 196,61862 | 2,2790 | 7,56422  | 2,6045 | 8,50972  | 2,9301 | 71     |
| P.3    | 9,59082   | 2,3582 | 7,53242  | 2,6951 | 8,47392  | 3,0320 | 6.1    |
|        | 206,57792 | 2,3941 | 7,51752  | 2,7362 | 8,45722  | 3,0782 | 70     |
|        | 216,53512 | 2,5086 | 7,46862  | 2,8669 | 8,40222  | 3,2253 | 69     |
|        | 226,49032 | 2,6222 | 7,41752  | 2,9969 | 8,34472  | 3,3715 | 68     |
| 2.     | 6,46722   | 2,6788 | 7,39102  | 3,0615 | 8,31492  | 3,4441 | 6.     |
| Pts    | D.        | Dep.   | Lat.     | Dep.   | Lat.     | Dep.   | D. Pts |
|        | Dist. 7.  |        | Dist. 8. |        | Dist. 9. |        |        |



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